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Book Reviews

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BOOK REVIEWS

R. L. Herman: A COURSE IN MATHEMATICAL METHODS FOR PHYSICISTS. CRC Press, Boca Raton, 2014, xxvi + 747 pages, paperback, ISBN-13: 978-1-4665-8467-9.

The book introduces basic notions of mathematical methods in Physics. It is aimed at undergraduate students with previous knowledge of general physics. It originated from the notes of a course given by the author for one semester, but it has then expanded beyond that.

It comprises 11 chapters, each accompanied by a number of problems, and one appendix with 9 sections (plus problems). Answers and solutions to the problems, however, are not provided in the book. The viewpoint is to motivate the methods studied directly by problems coming from physics. In this sense, particular emphasis is put on the physics of oscillations and waves.

Chapter 1 is essentially a brief review of basic calculus needed in the following, such as trigonometric and hyperbolic functions, derivatives and integrals, power series etc. Some illustrative examples accompany the text.

Chapter 2 studies some ordinary differential equations (and linear systems) that originate in physics problems. It starts from free fall and then adds air resistance to it, before passing to simple harmonic motion (with applications in mechanics and electromagnetism) and to damped/forced oscillations. Green's functions are also introduced for solving initial value problems involving non-homogeneous differential equations, and Euler's equation is discussed. Some numerical methods for ODEs are also mentioned, with simple examples in Maple and MATLAB.

Chapter 3 is an introduction to basic linear algebra where finite-dimensional vector spaces, linear transformations (with emphasis on rotations), and eigenvalue problems are studied. Applications in, e.g., mechanics and circuits are exemplified.

Chapter 4 is an introduction to basic topics in non-linear dynamics. After presenting the properties of the logistic and Riccati equations, stability of non-linear first order autonomous equations around equilibrium points and bifurcation points are discussed. Applications to the non-linear pendulum follow. Limit cycles and non-autonomous systems are also analyzed.

Chapter 5 studies oscillations (traveling waves) as solutions of the one-dimensional wave equation. Separation of variables (time and space) and Fourier analysis are illustrated in this context, along with boundary value problems. Similar methods are applied to the one-dimensional diffusion equation. The Gibbs phenomenon is discussed. Green's functions for the one-dimensional diffusion and wave equations are also obtained. It is finally demonstrated how to obtain those equations from the fundamental laws of mechanics/thermodynamics, respectively.

Chapter 6 introduces special functions of interest in mathematical physics (classical orthogonal polynomials, Bessel functions) as bases for infinite dimensional function spaces and as solutions of differential equations. The Gamma function is also defined. Those techniques are then employed in the study of Sturm-Liouville eigenvalue problems, including the relevant Green's functions.

Chapter 7 is devoted to introducing complex techniques. Starting from the complex representation of one-dimensional waves, the author reviews useful properties of complex numbers, complex functions and their graphical representation. Next, differential and integral calculus of complex functions (including the Cauchy-Riemann equations, Cauchy's theorem, Cauchy's integral formula and the residue theorem) are explained, with several examples. The Cauchy principal value integral for "infinite" integrals and integrals over multivalued functions are also described.

Chapter 8 describes the Fourier and the Laplace transform methods for differential equations, and their application to physics problems (e.g., the heat equation). In passing, Dirac's delta is also defined and its main properties described.

In Chapter 9 basic properties of vectors calculus and standard operators acting on vectors are reviewed, including Gauss' and Stokes' theorems. Vector calculus is then employed to discuss the two-body problem and Kepler's laws, as well as Maxwell's equations, from which the electromagnetic wave equation is then derived. Further, curvilinear coordinates in three dimensions are analyzed, and the corresponding forms of gradient, divergence and curl derived. Finally, basic properties of tensors are briefly discussed.

Chapter 10 first reviews critical points of real functions of one and several variables. These techniques are then used to study linear regression. Then constraints and Lagrange multipliers are discussed. Finally, calculus of variations is introduced, with discussion of Euler's equation and some simple applications (shortest distance, brachistochrone, etc.). Hamilton's principle is also described, with applications.

Chapter 11 discusses problems in more than one space dimensions, leading to partial differential equations and solutions by separation of variables, along with the use of curvilinear coordinates. In particular, vibrations of membranes and heat flow are considered. Laplace's equation in spherical coordinates is discussed in some detail, leading to properties of spherical harmonics. The Schrödinger equation and the $l = 0$ states of the hydrogen atom, and the 3D Poisson equations are also studied. Green's functions for partial differential equations are finally summarized.

The concluding appendix reviews definitions and basic properties of sequences and infinite series.

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