CAUCHY PROBLEM FOR THE COMPLEX GINZBURG-LANDAU TYPE EQUATION WITH L^p-INITIAL DATA

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Abstract. This paper gives the local existence of mild solutions to the Cauchy problem for the complex Ginzburg-Landau type equation

$$\frac{\partial u}{\partial t} - (\lambda + \mathrm{i}\alpha)\Delta u + (\kappa + \mathrm{i}\beta)|u|^{q-1}u - \gamma u = 0$$

in $\mathbb{R}^N \times (0,\infty)$ with L^p-initial data u_0 in the subcritical case $(1 \leqslant q < 1 + 2p/N)$, where u is a complex-valued unknown function, α, β, γ, $\kappa \in \mathbb{R}$, $\lambda > 0$, $p > 1$, $\mathrm{i} = \sqrt{-1}$ and $N \in \mathbb{N}$. The proof is based on the L^p-L^q estimates of the linear semigroup $\{\exp(t(\lambda + \mathrm{i}\alpha)\Delta)\}$ and usual fixed-point argument.

Keywords: local existence; complex Ginzburg-Landau equation

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1. Introduction and main result

We consider the Cauchy problem for the complex Ginzburg-Landau type equation,

$$\text{(CGL)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} - (\lambda + \mathrm{i}\alpha)\Delta u + (\kappa + \mathrm{i}\beta)|u|^{q-1}u - \gamma u = 0 & \text{in } \mathbb{R}^N \times (0,\infty), \\ u(\cdot, 0) = u_0 \quad \text{on } \mathbb{R}^N, \end{cases}$$

where u is a complex-valued unknown function, $\alpha, \beta, \gamma, \kappa \in \mathbb{R}$, $\lambda > 0$, $q \geqslant 1$, $\mathrm{i} = \sqrt{-1}$ and $N \in \mathbb{N}$. Our concern is the local existence of mild solutions to (CGL) with $\kappa \in \mathbb{R}$. We call it the complex Ginzburg-Landau "type" equation because it is usually assumed in (CGL) that $\kappa > 0$. There are many studies on the global

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existence and uniqueness of solutions to (CGL) in various cases, see, e.g., Yang [14], Levermore-Oliver [6], Ginibre-Velo [3], [4], Okazawa-Yokota [10]–[13], Okazawa [9], Yokota-Okazawa [15], Kobayashi-Matsumoto-Tanaka [5], Matsumoto-Tanaka [7], [8], Clément-Okazawa-Sobajima-Yokota [1].

The following table shows more general results for the solvability of (CGL) on $L^p(\Omega)$ with $L^p(\Omega)$-initial data, where $\Omega \subset \mathbb{R}^N$:

Ref.	(p, q)-condition	Ω	solution	other condition
[9]	$1 \leqslant p,\ 1 \leqslant q \leqslant 1 + \dfrac{2p}{N}$	bounded	C^1-in-time, local	$\lambda > 0$
[8]	$1 < p,\ 1 \leqslant q \leqslant 1 + \dfrac{2p}{N}$	general, smooth	C^1-in-time, global	$\dfrac{\lvert\alpha\rvert}{\lambda} < \dfrac{2\sqrt{p-1}}{\lvert p-2\rvert}$

The purpose of this paper is to establish the local existence of solutions to (CGL) in the case $\Omega = \mathbb{R}^N$ under almost the same conditions as in [9, Proposition 1.1].

Before stating our results, we introduce an operator A_p.

Definition 1.1 (CGL-operator)**.** Let $\lambda > 0$, $\alpha \in \mathbb{R}$ and assume that an operator $A_p\colon D(A_p)\ (\subset L^p(\mathbb{R}^N)) \to L^p(\mathbb{R}^N)$ $(1 < p < \infty)$ satisfies

$$D(A_p) := W^{2,p}(\mathbb{R}^N) \cap W_0^{1,p}(\mathbb{R}^N), \quad A_p u := -(\lambda + \mathrm{i}\alpha)\Delta u \quad (u \in D(A_p)).$$

Then we say that A_p is a CGL-*operator*.

Using the operator A_p, we can regard (CGL) as an abstract Cauchy problem on $L^p(\mathbb{R}^N)$:

$$(\mathrm{CGL})_{L^p} \qquad \begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} + A_p u + (\kappa + \mathrm{i}\beta)|u|^{q-1}u - \gamma u = 0 \text{ in } (0,\infty), \\ u(0) = u_0. \end{cases}$$

Strictly speaking, $(\mathrm{CGL})_{L^p}$ might not be called the abstract Cauchy problem, because it still has a concretely nonlinear term $|u|^{q-1}u$.

We will show that there exists a semigroup $\{\mathrm{e}^{-tA_p}\}_{t\geqslant 0}$ which is generated by A_p (see Section 2). This semigroup will be called a CGL-*semigroup*. Then we can define a mild solution to $(\mathrm{CGL})_{L^p}$.

Definition 1.2 (mild solution)**.** Let $u_0 \in L^p(\mathbb{R}^N)$ and $T > 0$. Then $u\colon [0,T) \to L^p(\mathbb{R}^N)$ is called a *mild solution* to $(\mathrm{CGL})_{L^p}$ on $[0,T)$ if $u \in C([0,T); L^p(\mathbb{R}^N))$ and

$$(\mathrm{IE}) \quad u(t) = \mathrm{e}^{-tA_p}u_0 + \int_0^t \mathrm{e}^{-(t-s)A_p}[\gamma u(s) - (\kappa + \mathrm{i}\beta)|u(s)|^{q-1}u(s)]\,\mathrm{d}s, \quad t \in [0,T).$$

We now state our main result in this paper.

Theorem 1.1. *Let $u_0 \in L^p(\mathbb{R}^N)$ $(1 < p < \infty)$ and assume that*

$$1 \leqslant q < 1 + \frac{2p}{N}.$$

Then there exist $T_0 > 0$ and a mild solution to $(\mathrm{CGL})_{L^p}$ *on* $[0, T_0)$.

R e m a r k 1.1. We can show the uniqueness of mild solutions belonging to Y_T which is defined in Section 3. It seems that the global existence can be proved under some conditions for initial data and coefficients. We will discuss these and the critical case $q = 1 + 2p/N$ in our future work.

In Section 2, we give some lemmas and study the CGL-semigroup $\{\mathrm{e}^{-tA_p}\}$, in particular, we derive the L^p-L^q estimates of $\{\mathrm{e}^{-tA_p}\}$. In Section 3, we prove Theorem 1.1.

2. Construction of CGL-semigroup

First we prepare some lemmas to prove the existence of the CGL-semigroup and to obtain its properties.

The following lemma gives a simple equality, which is proved by the methods of complex analysis.

Lemma 2.1. *Let $z \in \mathbb{C} \setminus \{0\}$ with $\operatorname{Re} z \geqslant 0$. Then*

$$\int_{\mathbb{R}^N} \mathrm{e}^{-z|x|^2} \, \mathrm{d}x = \left(\frac{\pi}{z}\right)^{N/2}.$$

P r o o f. It suffices to show that

$$I := \int_0^\infty \mathrm{e}^{-zx^2} \, \mathrm{d}x = \frac{1}{2}\left(\frac{\pi}{z}\right)^{1/2}. \tag{2.1}$$

We show only the case $\operatorname{Im} z \geqslant 0$. Set $f\colon \mathbb{C} \to \mathbb{C}$ satisfying $f(\omega) := \mathrm{e}^{-\omega^2}$ and let $R > 0$ and $\theta := \frac{1}{2}\arg z$. Then it follows from the Cauchy integral theorem that $\int_C f(\omega)\,\mathrm{d}\omega = 0$, where C is the curve in the complex plane formed by $C = C_1 + C_R - C_2$. Here C_1 and C_2 are the directed line segments from the origin 0 to R and $R\mathrm{e}^{\mathrm{i}\theta}$, respectively. C_R is the counterclockwise arc of the circle centered at the origin with radius R and sector $0 \leqslant \arg \omega \leqslant \theta$. Noting $\theta \leqslant \pi/4$, we see from the Cauchy integral theorem that

$$0 = \int_0^R \mathrm{e}^{-x^2} \, \mathrm{d}x + \mathrm{i}R \int_0^\theta \mathrm{e}^{-(R\mathrm{e}^{\mathrm{i}s})^2} \mathrm{e}^{\mathrm{i}s} \, \mathrm{d}s - \sqrt{z} \int_0^{R/\sqrt{|z|}} \mathrm{e}^{-zx^2} \, \mathrm{d}x \to \frac{\sqrt{\pi}}{2} + 0 - \sqrt{z} I$$

as $R \to \infty$. So we obtain (2.1). The case $\operatorname{Im} z \leqslant 0$ is shown analogously. $\square$

The next lemma constitutes the CGL-semigroup e^{-tA_p} and its kernel.

Lemma 2.2. *Let $t > 0$, $\lambda > 0$, $\alpha \in \mathbb{R}$ and set*

$$G_t(x) := [4\pi t(\lambda + \mathrm{i}\alpha)]^{-N/2}\mathrm{e}^{-|x|^2/(4t(\lambda+\mathrm{i}\alpha))}, \quad x \in \mathbb{R}^N.$$

Then a one-parameter family $\{T(t)\}_{t\geqslant 0}$ of operators on $L^p(\mathbb{R}^N)$ defined as

$$T(t)f := \begin{cases} G_t * f = [4\pi t(\lambda + \mathrm{i}\alpha)]^{-N/2} \displaystyle\int_{\mathbb{R}^N} \mathrm{e}^{-|\cdot - y|^2/(4t(\lambda+\mathrm{i}\alpha))} f(y)\,\mathrm{d}y, & t > 0, \\ f, \quad t = 0, \end{cases} \tag{2.2}$$

where $f \in L^p(\mathbb{R}^N)$, is a semigroup which is generated by A_p.

We will see from Lemma 2.2 that $T(t)$ can be represented as e^{-tA_p}.

Proof. We consider the linear Cauchy problem

$$\frac{\partial u}{\partial t}(x,t) = (\lambda + \mathrm{i}\alpha)\Delta u(x,t); \qquad u(\cdot, 0) := u_0 \in L^p(\mathbb{R}^N). \tag{2.3}$$

Using the Fourier transformation in x, we obtain the solution to (2.3) by

$$u(\cdot, t) = \mathcal{F}^{-1}[\mathrm{e}^{-(\lambda+\mathrm{i}\alpha)|\xi|^2 t}(\mathcal{F}u_0)] = \mathcal{F}^{-1}[\mathrm{e}^{-(\lambda+\mathrm{i}\alpha)|\xi|^2 t}] * u_0.$$

We see from Lemma 2.1 that

$$\begin{aligned} \mathcal{F}^{-1}[\mathrm{e}^{-(\lambda+\mathrm{i}\alpha)|\xi|^2 t}] &= \frac{1}{(2\pi)^N}\int_{\mathbb{R}^N} \exp[\mathrm{i}x\cdot\xi - (\lambda+\mathrm{i}\alpha)|\xi|^2 t]\,\mathrm{d}\xi \\ &= \frac{1}{(2\pi)^N}\mathrm{e}^{-|x|^2/(4(\lambda+\mathrm{i}\alpha)t)}\int_{\mathbb{R}^N} \exp\Big[-(\lambda+\mathrm{i}\alpha)t\Big|\xi - \frac{\mathrm{i}x}{2(\lambda+\mathrm{i}\alpha)t}\Big|^2\Big]\,\mathrm{d}\xi \\ &= G_t(x). \end{aligned}$$

Since we can verify that $T(t)u_0$ is a solution to (2.3), the assertion follows. $\square$

The following lemma can be proved in a similar way as in the proof for the heat semigroup (see also Giga-Giga-Saal [2, Section 1.1.2]).

Lemma 2.3 (L^p-L^q estimates). *Let $1 < p < \infty$, $p \leqslant q \leqslant \infty$ and $t > 0$. Then the following L^p-L^q estimate of the* CGL-*semigroup* $\{\mathrm{e}^{-tA_p}\}$ *holds:*

$$\|\mathrm{e}^{-tA_p} f\|_{L^q} \leqslant M_{p,q} t^{-(N/2)\cdot(1/p-1/q)} \|f\|_{L^p}, \quad f \in L^p(\mathbb{R}^N),$$

where

$$M_{p,q} := \Big(\frac{1}{4\pi\sqrt{\lambda^2+\alpha^2}}\Big)^{(N/2)\cdot(1/p-1/q)} \Big(\frac{\sqrt{\lambda^2+\alpha^2}}{\lambda}\Big)^{(N/2)\cdot(1-1/p+1/q)}.$$

P r o o f. Let $1 \leqslant r < \infty$. Then using Lemma 2.1, we have

$$\|G_t\|_{L^r} = \Big(\frac{1}{4\pi t\sqrt{\lambda^2+\alpha^2}}\Big)^{(N/2)\cdot(1-1/r)} \Big(\frac{\sqrt{\lambda^2+\alpha^2}}{\lambda r}\Big)^{N/(2r)}.$$

Noting $r^{-1/r} \leqslant 1$ and applying the Hausdorff-Young inequality to (2.2), we can obtain the assertion. $\square$

3. Local solvability of (CGL)

In this section, we prove Theorem 1.1. The proof is based on the fixed-point theorem.

P r o o f of Theorem 1.1. Let $R > 0$ with $R > (M_{p,p} + M_{p,pq})\|u_0\|_{L^p}$. Set

$$\begin{aligned} Y_T &:= \{u \in L^\infty_{\mathrm{loc}}(0,T; L^{pq}(\mathbb{R}^N)) \cap L^\infty(0,T; L^p(\mathbb{R}^N)); \|u\|_{Y_T} < \infty\}, \\ \|u\|_{Y_T} &:= \sup_{t\in(0,T)} t^\theta \|u(t)\|_{L^{pq}} + \sup_{t\in(0,T)} \|u(t)\|_{L^p}, \\ \theta &:= \frac{N}{2}\Big(\frac{1}{p} - \frac{1}{pq}\Big) \quad (0 \leqslant \theta < 1/q \leqslant 1), \end{aligned}$$

where $T \in (0,1)$. Then Y_T is a complex Banach space. Moreover, set the ball in Y_T;

$$B_{T,R} := \{u \in Y_T;\ \|u\|_{Y_T} < R\}$$

and the operator Φ_{u_0} on $\overline{B_{T,R}}^{Y_T}$

$$\Phi_{u_0}(u)(t) := \mathrm{e}^{-tA_p} u_0 + \int_0^t \mathrm{e}^{-(t-s)A_p} [\gamma u(s) - (\kappa + \mathrm{i}\beta)|u(s)|^{q-1} u(s)]\,\mathrm{d}s. \tag{3.1}$$

Then we show that Φ_{u_0} has a fixed-point u (Step 1, Step 2). After that we confirm that $u(\cdot)$ is a unique solution to $(\mathrm{CGL})_{L^p}$ (Step 3).

Step 1 ($u \in \overline{B_{T,R}}^{Y_T} \Rightarrow \Phi_{u_0}(u) \in \overline{B_{T,R}}^{Y_T}$). Let $u \in \overline{B_{T,R}}^{Y_T}$. Then we have

$$\begin{aligned}\|\Phi_{u_0}(u)(t)\|_{L^{pq}} &\leqslant \|\mathrm{e}^{-tA_p}u_0\|_{L^{pq}} + |\gamma|\int_0^t \|\mathrm{e}^{-(t-s)A_p}u(s)\|_{L^{pq}}\,\mathrm{d}s \\ &\quad + \sqrt{\kappa^2+\beta^2}\int_0^t \|\mathrm{e}^{-(t-s)A_p}|u(s)|^{q-1}u(s)\|_{L^{pq}}\,\mathrm{d}s \\ &=: I_{1,pq} + |\gamma| I_{2,pq} + \sqrt{\kappa^2+\beta^2}\, I_{3,pq}.\end{aligned} \tag{3.2}$$

Since $0 \leqslant \theta \leqslant q\theta < 1$, it follows from Lemma 2.3 that

$$I_{1,pq} \leqslant t^{-\theta} M_{p,pq}\|u_0\|_{L^p}, \tag{3.3}$$

$$\begin{aligned}I_{2,pq} &\leqslant M_{p,pq}\int_0^t (t-s)^{-\theta}\|u(s)\|_{L^p}\,\mathrm{d}s \\ &\leqslant t^{1-\theta}\frac{M_{p,pq}R}{1-\theta},\end{aligned} \tag{3.4}$$

$$\begin{aligned}I_{3,pq} &\leqslant M_{p,pq}\int_0^t (t-s)^{-\theta}\|u(s)\|^q_{L^{pq}}\,\mathrm{d}s \\ &\leqslant M_{p,pq}R^q\int_0^t (t-s)^{-\theta}s^{-q\theta}\,\mathrm{d}s \\ &= t^{1-(q+1)\theta}M_{p,pq}R^q B(1-q\theta, 1-\theta),\end{aligned} \tag{3.5}$$

where B is the Beta function. Since $t \in (0,1)$, we have $t < t^{1-q\theta}$. Combining (3.3)–(3.5) with (3.2), we see that

$$t^{\theta}\|\Phi_{u_0}(u)(t)\|_{L^{pq}} \leqslant M_{p,pq}\|u_0\|_{L^p} + T^{1-q\theta}RC_1, \tag{3.6}$$

where

$$C_1 := M_{p,pq}\Big(\frac{|\gamma|}{1-\theta} + R^{q-1}B(1-q\theta,1-\theta)\sqrt{\kappa^2+\beta^2}\Big).$$

Next we estimate the L^p-norm of (3.1). Similarly as in (3.3)–(3.5), we see

$$\begin{aligned}\|\Phi_{u_0}(u)(t)\|_{L^p} &\leqslant \|\mathrm{e}^{-tA_p}u_0\|_{L^p} + |\gamma|\int_0^t \|\mathrm{e}^{-(t-s)A_p}u(s)\|_{L^p}\,\mathrm{d}s \\ &\quad + \sqrt{\kappa^2+\beta^2}\int_0^t \|\mathrm{e}^{(t-s)A}|u(s)|^{q-1}u(s)\|_{L^p}\,\mathrm{d}s \\ &\leqslant M_{p,p}\|u_0\|_{L^p} + |\gamma| t M_{p,p}R + \sqrt{\kappa^2+\beta^2}\, t^{1-q\theta}\frac{M_{p,p}R^q}{1-q\theta}.\end{aligned}$$

Thus we have

$$\|\Phi_{u_0}(u)(t)\|_{L^p} \leqslant M_{p,p}\|u_0\|_{L^p} + T^{1-q\theta}RC_2, \tag{3.7}$$

where

$$C_2 := M_{p,p}\Big(|\gamma| + \frac{R^{q-1}\sqrt{\kappa^2+\beta^2}}{1-q\theta}\Big).$$

It follows from (3.6) and (3.7) that

$$\|\Phi_{u_0}(u)\|_{Y_T} \leqslant (M_{p,p} + M_{p,pq})\|u_0\|_{L^p} + T^{1-q\theta}R(C_1 + C_2).$$

Therefore we obtain

$$\|\Phi_{u_0}(u)\|_{Y_T} \leqslant R \quad (T \leqslant T_1),$$

where

$$T_1 := \Big[\frac{R - (M_{p,p} + M_{p,pq})\|u_0\|_{L^p}}{R(C_1 + C_2)}\Big]^{1/(1-q\theta)}.$$

Step 2 (Φ_{u_0}: contraction in $\overline{B_{T,R}}^{Y_T}$). Let $u, v \in \overline{B_{T,R}}^{Y_T}$. Then we have

$$\|\Phi_{u_0}(u)(t) - \Phi_{u_0}(v)(t)\|_{L^{pq}} \leqslant |\gamma| J_{1,pq} + \sqrt{\kappa^2+\beta^2}\, J_{2,pq},$$

where

$$J_{1,pq} := \int_0^t \|\mathrm{e}^{-(t-s)A_p}[u(s) - v(s)]\|_{L^{pq}}\,\mathrm{d}s,$$
$$J_{2,pq} := \int_0^t \|\mathrm{e}^{-(t-s)A_p}[|u(s)|^{q-1}u(s) - |v(s)|^{q-1}v(s)]\|_{L^{pq}}\,\mathrm{d}s.$$

Similarly as in (3.4) and (3.5), it follows from Lemma 2.3 that

$$\begin{aligned} J_{1,pq} &\leqslant t^{1-\theta}\frac{M_{p,pq}}{1-\theta}\|u - v\|_{Y_T}, \\ J_{2,pq} &\leqslant M_{p,pq}q\int_0^t (t-s)^{-\theta}(\|u(s)\|_{L^{pq}}^{q-1} + \|v(s)\|_{L^{pq}}^{q-1})\|u(s) - v(s)\|_{L^{pq}}\,\mathrm{d}s \\ &\leqslant t^{1-(q+1)\theta}2qM_{p,pq}R^{q-1}B(1-q\theta, 1-\theta)\|u - v\|_{Y_T}. \end{aligned}$$

Therefore we have for $t \in (0,T)$

$$t^{\theta}\|\Phi_{u_0}(u)(t) - \Phi_{u_0}(v)(t)\|_{L^{pq}} \leqslant T^{1-q\theta}C_1'\|u - v\|_{Y_T}, \tag{3.8}$$

where

$$C_1' := M_{p,pq}\Big(\frac{|\gamma|}{1-\theta} + 2qR^{q-1}B(1-q\theta, 1-\theta)\sqrt{\kappa^2+\beta^2}\Big).$$

As in the proof of (3.7) we see from Lemma 2.3 that

$$\|\Phi_{u_0}(u)(t) - \Phi_{u_0}(v)(t)\|_{L^p} \leqslant T^{1-q\theta}C_2'\|u - v\|_{Y_T} \quad t \in (0,T), \tag{3.9}$$

where

$$C_2' := M_{p,p}\left(|\gamma| + 2q\frac{R^{q-1}\sqrt{\kappa^2+\beta^2}}{1-q\theta}\right).$$

Combining (3.8) and (3.9), we have

$$\|\Phi_{u_0}(u) - \Phi_{u_0}(v)\|_{Y_T} \leqslant \frac{1}{2}\|u-v\|_{Y_T} \quad (T \leqslant T_2),$$

where $T_2 := (2C_1' + 2C_2')^{-1/(1-q\theta)}$.

Hence it follows from the fixed-point theorem that Φ_{u_0} has a unique fixed-point $u \in \overline{B_{T_0,R}}^{Y_{T_0}}$, where $T_0 := \min\{T_1, T_2\}$. Moreover, $u \in \overline{B_{T_0,R}}^{Y_{T_0}}$ satisfies (IE).

Step 3 ($u \in C([0,T_0); L^p(\mathbb{R}^N))$). Finally we prove that $u \in \overline{B_{T_0,R}}^{Y_{T_0}}$ given by Step 2 is a mild solution to $(\mathrm{CGL})_{L^p}$. Set

$$f(s) := \gamma u(s) - (\kappa + \mathrm{i}\beta)|u(s)|^{q-1}u(s).$$

Then it suffices to show that

$$f \in L^1(0,T_0; L^p(\mathbb{R}^N)). \tag{3.10}$$

Since $\|u(s)\|_{L^p} \leqslant R$ and $\|u(s)\|_{L^{pq}} \leqslant Rs^{-\theta}$ for $s \in (0,T_0)$, we have

$$\int_0^{T_0} \|u(s)\|_{L^p}\,\mathrm{d}s \leqslant RT_0,$$

$$\int_0^{T_0} \||u(s)|^q\|_{L^p}\,\mathrm{d}s \leqslant R^q \int_0^{T_0} s^{-q\theta}\,\mathrm{d}s = \frac{R^q T_0^{1-q\theta}}{1-q\theta}.$$

Therefore (3.10) follows. □

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