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CHAOTIC BEHAVIOR AND MODIFIED FUNCTION PROJECTIVE SYNCHRONIZATION OF A SIMPLE SYSTEM WITH ONE STABLE EQUILIBRIUM

Zhouchao Wei and Zhen Wang

By introducing a feedback control to a proposed Sprott E system, an extremely complex chaotic attractor with only one stable equilibrium is derived. The system evolves into periodic and chaotic behaviors by detailed numerical as well as theoretical analysis. Analysis results show that chaos also can be generated via a period-doubling bifurcation when the system has one and only one stable equilibrium. Based on Lyapunov stability theory, the adaptive control law and the parameter update law are derived to achieve modified function projective synchronized between the extended Sprott E system and original Sprott E system. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive controllers.

Keywords: chaotic attractors, stable equilibrium, Shilnikov theorem, Lyapunov exponent, synchronization

Classification: 34H10, 34H20

1. INTRODUCTION

Since Lorenz found the first chaotic attractor in a smooth 3D autonomous system [2], searching for new chaotic attractors has been intensively considered in the past three decades within the scientific, engineering and mathematical communities. Many Lorenz-like or Lorenz-based chaotic systems were proposed and investigated. In 2004, Lü, Chen and Cheng discussed the important problems of classification and normal form of three-dimensional quadratic autonomous chaotic systems [4]. Some classical 3D autonomous chaotic systems have three particular fixed points: one saddle and two unstable saddle-foci [1, 3, 7, 20, 22]. The other 3D chaotic systems have two unstable saddle-foci [16, 17]. In 2008, Yang and Chen found another 3D chaotic system with three fixed points: one saddle and two stable equilibria [23]. Note that the aforementioned methods for generating multi-scroll attractors in autonomous systems are based on the fundamental work of Shilnikov [14, 15] and its subsequent embellishment and slight extension [10]. However, Shilnikov criteria is sufficient but certainly not necessary for the emergence of chaos. Creating a chaotic system with a more complicated topological structure such as chaotic attractors with only one stable equilibrium or several stable equilibria, therefore, becomes a desirable task and sometimes a key issue for many engineering applications.

In the investigation of chaos theory and applications, it is very important to generate new chaotic systems or to enhance complex dynamics and topological structure based on the existing chaotic attractors. In this endeavor, in 2010, an unusual 3D autonomous quadratic Lorenz-like chaotic system with only two stable node-foci was proposed by Yang, Wei and Chen [24]. In 2011, a chaotic system with no equilibria was proposed by Wei [21], which was illustrated in the case of a period-doubling sequence of bifurcations leading to a Feigenbaum-like strange attractor. In 2012, Wang and Chen obtained a kind of chaotic attractors with only one stable node-focus by adding a simple constant control parameter to Sprott E system [19]. However, based on a classification condition formulated by Vanecek and Celikovsky [18], these chaotic systems satisfy $a_{12}a_{21} > 0$, $a_{12}a_{21} < 0$ or $a_{12}a_{21} = 0$. In addition, according to another classification developed in [23], they are classified into the Lorenz system group if $a_{11}a_{22} > 0$ and the Chen system group if $a_{11}a_{22} < 0$, or the Yang–Chen system (transition system) group if $a_{11}a_{22} = 0$. Concerning the conditions on the signs of $a_{11}a_{22}$, a nature question is these signs are essential to the system dynamics, and whether can we construct a chaotic system meets two or three conditions of $a_{11}a_{22} > 0$, $a_{11}a_{22} < 0$ and $a_{11}a_{22} = 0$. In particular, Sprott embarked up on an extensive search for autonomous 3D chaotic systems with less than seven terms in the right hand side of the model equations [11-13]. From the above point of view, we can see that the study of constructing a chaotic system with only one stable equilibrium or several stable equilibriums, and with less than seven terms in the right hand side, also satisfies two or three conditions of $a_{11}a_{22} > 0$, $a_{11}a_{22} < 0$ or $a_{11}a_{22} = 0$ is of high practical importance.

In the qualitative theory of polynomial differential systems, people always find the global structure of the system, the number and distribution of limit cycles etc by polynomial perturbation technique [8, 26]. Following this idea, and motivated by Ref.[19], this paper introduces another generalized system, and utilizes a nonlinear function to create chaotic attractors with only a stable equilibrium, and satisfies two conditions $a_{11}a_{22} > 0$ and $a_{11}a_{22} = 0$, respectively, depending on suitable values of the system's parameters. We analyze the complicated dynamics by theoretical analysis, numerical simulation and Lyapunov exponent spectrum. The evolution processes of this system and dynamics behaviors will be presented when parameters vary.

2. THE NOVEL CHAOTIC SYSTEM FROM SPROTT E SYSTEM

2.1. Chaotic attractor

Based on the Sprott E system, a new chaotic system

$$\begin{cases} \dot{x} = yz + h(x) \\ \dot{y} = x^2 - y \\ \dot{z} = 1 - 4x \end{cases} \tag{1}$$

is proposed. Here $h(x) = ex^2 + fx + g$ and e, f, g are real parameters. This system (1) has one equilibrium $E = \left(\frac{1}{4}, \frac{1}{16}, -e - 4f - 16g\right)$. In particular, if $e = f = g = 0$, system (1) is the Sprott E system [11]; if $e = f = 0$, system (1) is the changed Sprott E system [19].

Note that for $e = -0.14$ and $f = g = 0$, system (1) is chaotic and displays a chaotic attractor like two-scroll, as shown in Figures 1(a) and 1(b). Interestingly, the chaotic

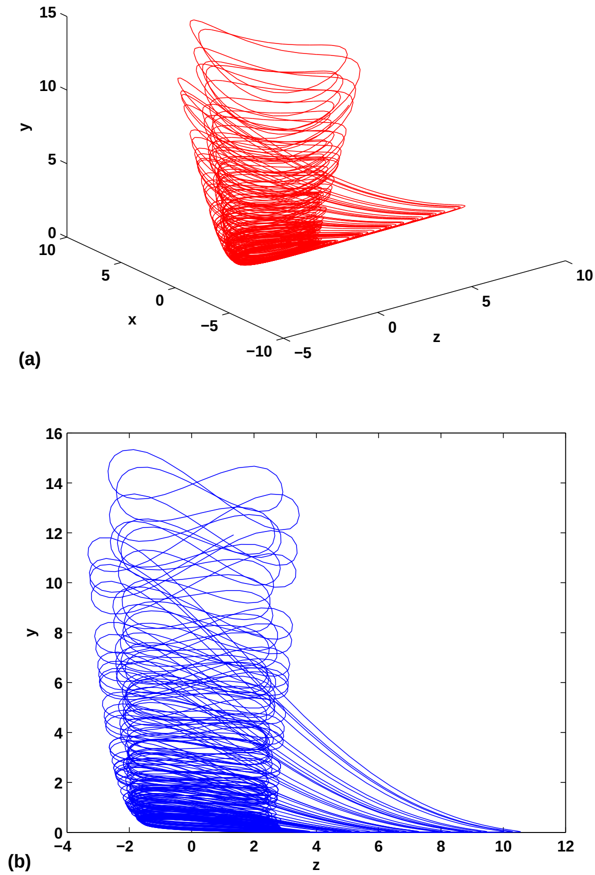


Fig. 1. Parameters values $(e, f, g) = (-0.14, 0, 0)$ of system (1): (a) chaotic attractor in 3-D space; (b) chaotic attractor projected in y-z plane.

attractor is different from that of the Lorenz system or any existing systems, because the only one equilibrium E is stable, whose characteristic values are $\lambda_1 = -1.0572$, $\lambda_{2,3} = -0.0064 \pm 0.4862i$. Therefore, system (1) has no homoclinic orbits joining E. The Lyapunov exponents of system (1) are $L_1 = 0.1020$, $L_2 = 0$, $L_3 = -1.1719$, and the Lyapunov dimension is $DL = 2.0870$ for initial value (-0.6, 0.9, -1.7). Figure 2(a) shows the Poincaré mapping on the plane $z = 0$. Figure 2(b) displays the times series of state variable $z(t)$ of system (1). Furthermore, when $f = 0.03$ and $e = g = 0$, system (1) also can displays a chaotic attractor with one and only one stable equilibrium $E = (0.25, 0.0625, -0.12)$, as shown in Figures 3(a) and 3(b).

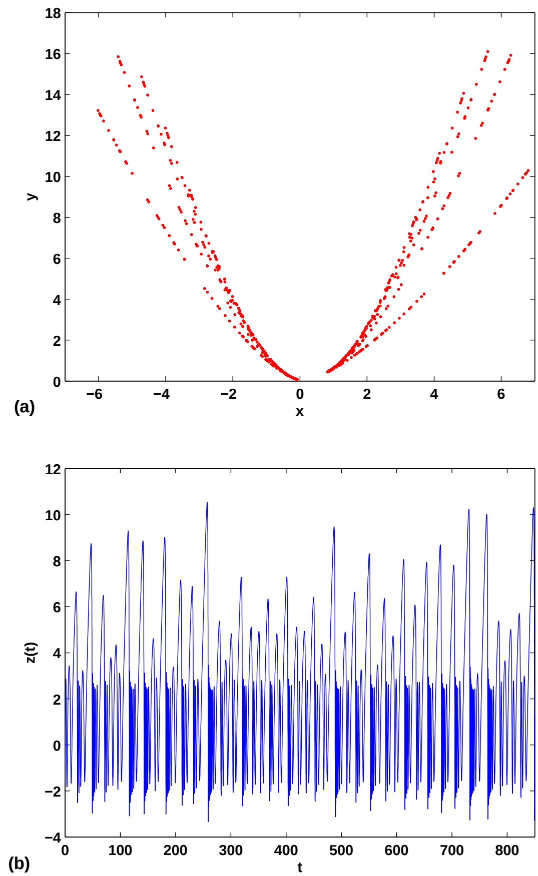


Fig. 2. (a) Poincaré mapping on $y = 1$ section; (b) times series of state variable $z(t)$.

Remark 2.1. In addition to the choice $f(x) = g$ [19], we can also get this result obtained by Wang and Chen if the function $f(x) = ex^2$ or fx. In other words, the following two simple systems both generate chaotic attractors when the following system (2) and system (3) have one and only one stable equilibrium, respectively.

$$\begin{cases} \dot{x} = yz + ex^2 \\ \dot{y} = x^2 - y \\ \dot{z} = 1 - 4x, \end{cases} \tag{2}$$

$$\begin{cases} \dot{x} = yz + fx \\ \dot{y} = x^2 - y \\ \dot{z} = 1 - 4x. \end{cases} \tag{3}$$

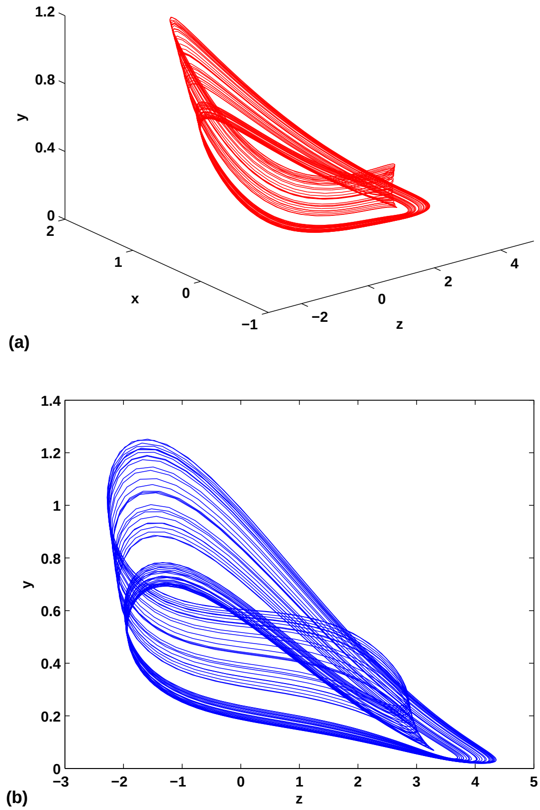


Fig. 3. Parameters values $(e, f, g) = (0, 0.03, 0)$ of system (1): (a) chaotic attractor in 3-D space; (b) chaotic attractor projected in y-z plane.

2.2. Some basic properties of the new system (1)

By linearization around the equilibrium E, the Jacobian matrix of system (1) is given by

$$J(E) = \begin{pmatrix} \frac{e}{2} + f & -e - 4f - 16g & \frac{1}{16} \\ \frac{1}{2} & -1 & 0 \\ -4 & 0 & 0 \end{pmatrix}. \tag{4}$$

Obviously, the characteristic equation about the equilibrium E is :

$$\det\left(\lambda I - J(E)\right) = \lambda^3 + \left(1 - f - \frac{e}{2}\right)\lambda^2 + \left(\frac{1}{4} + f + 8g\right)\lambda + \frac{1}{4} = 0. \tag{5}$$

According to the Routh–Hurwitz criterion, the real parts of all the roots λ are negative if and only if

$$\Delta_1 = 1 - f - \frac{e}{2} > 0,$$
$$\Delta_2 = \left(1 - f - \frac{e}{2}\right)\left(\frac{1}{4} + f + 8g\right) - \frac{1}{4} > 0.$$

From these inequalities, there are

$$f < 1 - \frac{e}{2}, \quad e < \frac{2(3f - 4f^2 + 32g - 32fg)}{1 + 4f + 32g}. \tag{6}$$

Therefore, the equilibrium E is asymptotically stable when the above conditions are met.

3. ANALYSIS OF HOPF BIFURCATION FOR $F = 0$

In this section, we deal with another kind of bifurcation at the E of system (1) using an analytical method.

Suppose the characteristic equation (5) has pure imaginary roots $\lambda_{1,2}$. It is easy to show that when $g = g_0 = \frac{e}{32(2-e)}$ and $e < 2$, the Jacobian matrix of system (1) has a pair of imaginary eigenvalues and one negative real eigenvalue, i. e.

$$\lambda_{1,2} = \pm i\sqrt{\frac{1}{4-2e}}, \quad \lambda_3 = \frac{e-2}{2},$$

with the corresponding eigenvectors (denoting $w = 4 - 2e$)

$$\begin{aligned} v_1 &= \left(1, \frac{\sqrt{w}}{2i + 2\sqrt{w}}, 4i\sqrt{w}\right), \\ v_2 &= \left(1, \frac{\sqrt{w}}{-2i + 2\sqrt{w}}, -4i\sqrt{w}\right), \\ v_3 &= \left(1, -\frac{2}{-4+w}, \frac{16}{w}\right). \end{aligned}$$

By utilizing $(x, y, z)' = P(x_1, y_1, z_1)'$, where

$$P = \begin{pmatrix} 1 & 0 & 1 \\ \frac{w}{2(1+w)} & \frac{\sqrt{w}}{2(1+w)} & -\frac{2}{-4+w} \\ 0 & -4\sqrt{w} & \frac{16}{w} \end{pmatrix}, \tag{7}$$

the real system (1) becomes

$$P = \begin{pmatrix} \dot{x_1} \\ \dot{y_1} \\ \dot{z_1} \end{pmatrix} = P^{-1}JP\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + P^{-1}\begin{pmatrix} ex^2 + yz \\ x^2 \\ 0 \end{pmatrix},$$

or, equivalently

$$\begin{cases} \dot{x_1} = -\frac{1}{\sqrt{w}}y_1 + f^1 \\ \dot{y_1} = \frac{1}{\sqrt{w}}x_1 + f^2 \\ \dot{z_1} = -\frac{w}{4}z_1 + f^3, \end{cases} \tag{8}$$

where

$$\begin{aligned} f^1 &= f^{11}(64wx_1^2 + 16w^2x_1^2 - 36w^3x_1^2 + 11w^4x_1^2 - w^5x_1^2 - 64w^{5/2}x_1x_2 \\ &\quad +16w^{7/2}x_1x_2 - 16w^{9/2}x_1x_2 + 4w^{11/2}x_1x_2 - 64w^2x_2^2 + 16w^3x_2^2 \\ &\quad -16w^4x_2^2 + 4w^5x_2^2 + 384wx_1x_3 - 32w^2x_1x_3 - 8w^3x_1x_3 + 6w^4x_1x_3 \\ &\quad -2w^5x_1x_3 + 256\sqrt{w}x_2x_3 - 128w^{3/2}x_2x_3 - 32w^{7/2}x_2x_3 - 16w^{9/2}x_2x_3 \\ &\quad +256x_3^2 + 320wx_3^2 + 80w^2x_3^2 + 28w^3x_3^2 + 11w^4x_3^2 - w^5x_3^2), \\ f^2 &= f^{22}(-16x_1^2 - 12wx_1^2 - 3w^3x_1^2 + w^4x_1^2 - 16w^{5/2}x_1x_2 + 4w^{7/2}x_1x_2 \\ &\quad -16w^2x_2^2 + 4w^3x_2^2 - 32x_1x_3 + 40wx_1x_3 - 16w^2x_1x_3 - 6w^3x_1x_3 \\ &\quad +2w^4x_1x_3 + 64\sqrt{w}x_2x_3 - 32w^{3/2}x_2x_3 - 16w^{5/2}x_2x_3 + 48x_3^2 \\ &\quad +52wx_3^2 - 3w^3x_3^2 + w^4x_3^2), \\ f^3 &= f^{33}(-16x_1^2 - 12wx_1^2 - 3w^3x_1^2 + w^4x_1^2 - 16w^{5/2}x_1x_2 + 4w^{7/2}x_1x_2 \\ &\quad -16w^2x_2^2 + 4w^3x_2^2 - 32x_1x_3 + 40wx_1x_3 - 16w^2x_1x_3 - 6w^3x_1x_3 \\ &\quad +2w^4x_1x_3 + 64\sqrt{w}x_2x_3 - 32w^{3/2}x_2x_3 - 16w^{5/2}x_2x_3 + 48x_3^2 \\ &\quad +52wx_3^2 - 3w^3x_3^2 + w^4x_3^2) \end{aligned}$$

and

$$\begin{aligned} f^{11} &= -\frac{2}{(w-4)w(1+w)(16+w^3)}, \\ f^{22} &= -\frac{2}{\sqrt{w}(1+w)(16+w^3)}, \\ f^{33} &= -\frac{w}{2(1+w)(16+w^3)}. \end{aligned}$$

According to the center manifold theorem, there exists a center manifold for Eq. (8), which could be represented locally by

$$W_c = \{(x_1, y_1, z_1) \in R^3 | z_1 = h(x_1, y_1),\ |x_1| < \delta,\ |y_1| < \delta,\ h(0,0) = 0,\ Dh(0,0) = 0\}$$

where $|\delta|$ is sufficiently small. We assume that

$$z_1 = h(x_1, y_1) = Ax_1^2 + Bx_1y_1 + Cy_1^2 + \cdots$$

Comparing the coefficients of the first equation and second equation of Eq. (8), we obtain

$$\begin{aligned} A &= -\frac{2(-4+w)(128+128w+160w^2+36w^3-12w^4+w^5+w^6)}{(1+w)(4+w)(16-4w+w^2)(16+w^3)}, \\ B &= -\frac{8(-4+w)w^{3/2}(8+8w-6w^2+2w^3+w^4)}{(1+w)(4+w)(16-4w+w^2)(16+w^3)}, \\ C &= -\frac{8(-4+w)(32+32w+40w^2+8w^3+4w^4+w^5)}{(1+w)(4+w)(16-4w+w^2)(16+w^3)}. \end{aligned}$$

Hence, the system (8) restricted to the central manifold is given by:

$$\begin{cases} \dot{x_1} = -\frac{1}{\sqrt{w}} y_1 + f_1(x_1, y_1, h(x_1, y_1)) \\ \dot{y_1} = \frac{1}{\sqrt{w}} x_1 + f_2(x_1, y_1, h(x_1, y_1)). \end{cases} \tag{9}$$

Now we calculate index number K by the following formula:

$$\begin{aligned} K &= \frac{1}{16}[f^1_{x_1x_1x_1} + f^1_{x_1y_1y_1} + f^2_{x_1x_1y_1} + f^2_{y_1y_1y_1}] \\ &\quad + \frac{1}{16\omega_0}[f^1_{x_1y_1}(f^1_{x_1x_1} + f^1_{y_1y_1}) - f^2_{x_1y_1}(f^2_{x_1x_1} + f^2_{y_1y_1}) - f^1_{x_1x_1}f^2_{x_1x_1} + f^1_{y_1y_1}f^2_{y_1y_1}]. \end{aligned}$$

Then

$$K = K(0,0) = \frac{2(512 + 1344w + 960w^2 + 300w^3 - 84w^4 - 19w^5 + 3w^6)}{(1+w)(16+w^3)(64+w^3)}.$$

From Eq. (5), we have

$$\lambda'(g_0) = -\frac{8(2-e)^2}{2+(2-e)^3} < 0. \tag{10}$$

Therefore, we have the theorem:

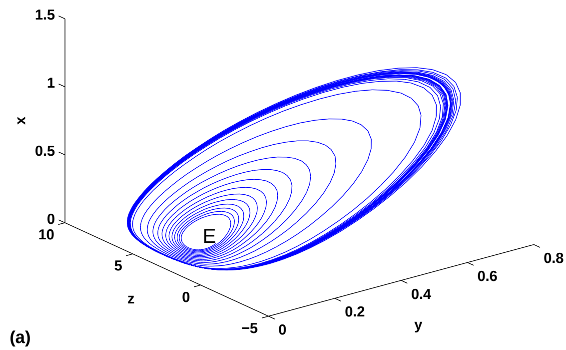
Theorem 3.1. If $g = g_0 = \frac{e}{32(2-e)}$ and $e < 2$, the system (1) undergoes a Poincare–Anddronov–Hopf bifurcation (Hopf bifurcation) at the $E = \left(\frac{1}{4}, \frac{1}{16}, -e - 4f - 16g\right)$. In addition, the periodic orbits that bifurcate from the equilibrium E for a in the neighborhood of as, are stable if $K < 0$, and unstable if $K > 0$. The direction of bifurcation are above (bellow) g_0 if $K > 0$ ($K < 0$).

Remark 3.2. Denoting by e_i $(i = 1, 2)$ the only two roots of $G(u)$ for which $u > 0$, we find that $e_1 \approx -1.65331$ and $e_2 \approx -0.65080$. Moreover, the following results are also obtained:

(i) When $g = g_h$, $e_1 < e < e_2$, system (1) undergoes a transversal Hopf bifurcation at a stable weak focus E for the flow restricted to the center manifold. Moreover, for each $g < g_h(e_1)$, but close to $g_h(e_1)$, there exists a stable limit cycle near the unstable equilibrium point E.

(ii) When $g = g_h$, $e < e_1$ or $e_2 < e < 2$, system (1) undergoes a transversal Hopf bifurcation at an unstable weak focus E for the flow restricted to the center manifold. Moreover, for each $g > g_h(e_2)$, but close to $g_h(e_2)$, there exists a unstable limit cycle near the stable equilibrium point E.

In order to justify the above theoretical analysis of the first Lyapunov coefficient for the Hopf bifurcation of system (1), we chose one set of parameters with $f = 0$, $e = -1.2$ and $g = -0.0142 < g_h(e_1)$. According to Theorem 3.1, a stable periodic solution should be found near the unstable equilibrium point E. This is indeed the case, as shown in Figure 4. For $g > g_h(e_2)$, the equilibrium point E is asymptotically stable. Note that for these parameter values, we have the bifurcation value $g = g_h(e_2) \approx -0.00767$. Therefore, the system (1) undergoes a Hopf bifurcation when the parameter g crosses the critical value $g_h(e_2)$, and a unstable periodic orbit emerges from E with $g > g_h(e_2)$. Choosing $f = 0$, $e = -0.4$ and $g = 0 > g_h(e_2)$, we take initial values $(0.28, 0.032, 0.1)$ near the equilibrium E, the solution of system (1) eventually close to 0. However, if we take initial values $(-0.6, 0.9, -1.7)$ 'outside' the unstable periodic orbit (it does exist from the Hopf bifurcation), a chaotic attractor exists near the unstable equilibrium E. The results are shown in the Figure 4. Therefore, it seems when the parameter g moves away from the critical value $g = g_h(e_2)$, chaotic attractor is generated occurring from the unstable limit cycle that arose in the Hopf bifurcation.



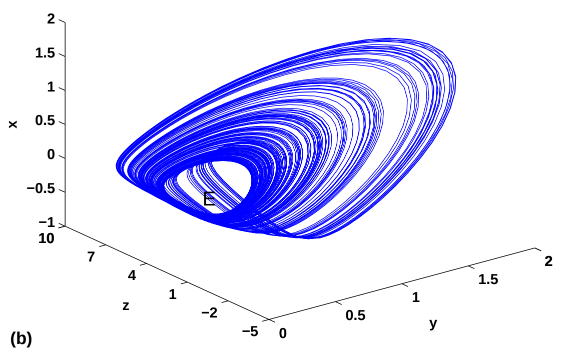


Fig. 4. Attractors of system (1) with parameter values : (a) Parameters values $(f, g, e) = (0, -0.0142, -1.2)$, stable periodic solution in 3-D space for starting initial values $(0.28, 0.032, 0.1)$; (b) Parameters values $(f, g, e) = (0, 0, -0.4)$, chaotic attractor for starting initial values $(-0.6, 0.9, -1.7)$.

4. DYNAMICAL STRUCTURE OF THE NEW CHAOTIC SYSTEM

The basic dynamics of the new chaotic system can be summarized in the following Lyapunov exponent spectrum, bifurcation diagrams, and so on. Here, we only consider two cases: (1): $fg \neq 0$; (2) $f = g = 0$.

4.1. e increasing when $f = -0.1,\ g = 0.02$

Figure 5 (a) shows the bifurcation diagram of the state variable $z(t)$ for initial value (-0.6, 0.9, -1.7) versus parameter e when $f = -0.1$, $g = 0.02$ and $e \in [-0.4, 0.5]$. Figure 5 (b) shows the corresponding Lyapunov exponent spectra. According to the characteristic polynomial (5) and condition (6), E is asymptotically stable. Obviously, the maximum Lyapunov exponent is negative when $e \in [-0.4, -0.303)$, implying that the new system (1) is attracted into a sink. Symmetric dynamical behaviors can be clearly observed. The forward bifurcation appears in the region (-0.303,-0.198), and the reverse bifurcation appears in the region [0.267, 0.5]. From Figure 5 (a), it is clear that $-0.015 \leq e < 0.08$ is a periodic window: $-0.015 \leq e < 0.029$ is period-2 orbit region; $0.029 < e < 0.048$ is period-4 orbit region. As e increases in the range of $0.048 < e < 0.08$, system (1) is chaotic. And we can see that the transition to chaos via period-doubling bifurcations.

4.2. e increasing when $f = 0,\ g = 0$

Now we fix $f = 0,\ g = 0$ and vary $e \in [-0.6, 0.3]$. It is presented that two different type of chaotic attractors coexist for certain parameter conditions. Figure 6 (a) shows the bifurcation diagram of the state variable $z(t)$. Figure 6 (b) shows the corresponding Lyapunov exponent spectra in which E is asymptotically stable for $e < 0$. The maximum Lyapunov exponent is negative when $e \in (-0.6, -0.508)$, implying that the new system (1) is attracted into a sink. When e passes through -0.508, topology structure of the system changes dramatically and the maximum Lyapunov exponent rapidly becomes positive. As e increases further in the region $e > 0.087$, the reverse bifurcation appears.

When $e \in [-0.508, 0)$, the max Lyapunov exponent $L_1 > 0$ and the novel system has chaotic state with stable equilibrium. For $e = 0$, system (1) is also chaotic with a positive Lyapunov exponent (see Figure 6 (b)) and corresponding equilibrium is non-hyperbolic. Note that for any $e > 0$, E is unstable, but the chaotic attractor can also be obtained in the range $(0, 0.087]$.

5. MODIFIED FUNCTION PROJECTIVE SYNCHRONIZATION BETWEEN SYSTEM (1) WITH SPROTT E SYSTEM

In this section, we will study the synchronization behavior between system (1) with Sprott E system. The drive and response systems are described by the following equations, respectively,

$$\begin{cases} \dot{x_1} = y_1 z_1 + e x_1^2 + f x_1 + g \\ \dot{y_1} = x_1^2 - y_1 \\ \dot{z_1} = 1 - 4x_1, \end{cases} \tag{11}$$

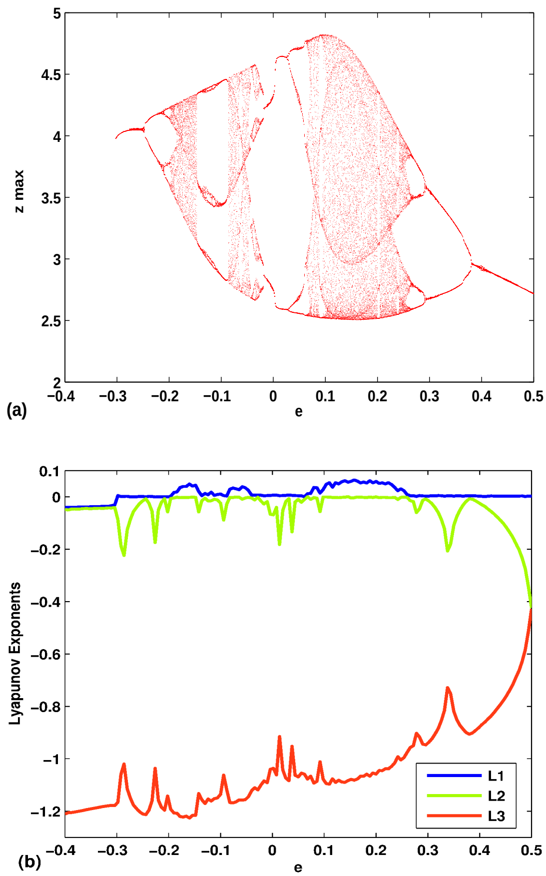


Fig. 5. Parameters values $(f, g) = (-0.1, 0.02)$ of system (1): (a) Lyapunov exponent spectrum with $e \in [-0.4, 0.5]$; (b) Bifurcation diagram of the variable z with $e \in [-0.4, 0.5]$.

$$\begin{cases} \dot{x_2} = y_2 z_2 + a + u_1 \\ \dot{y_2} = x_2^2 - y_2 + u_2 \\ \dot{z_2} = 1 - 4x_2 + u_3, \end{cases} \tag{12}$$

where u_1, u_2, u_3 are the nonlinear control laws such that two chaotic systems can be synchronized in the sense that

$$\lim_{t\to\infty} |x_2 - m_1 h(x) x_1| = 0, \tag{13}$$

$$\lim_{t\to\infty} |y_2 - m_2 h(x) y_1| = 0, \tag{14}$$

$$\lim_{t\to\infty} |z_2 - m_3 h(x) z_1| = 0. \tag{15}$$

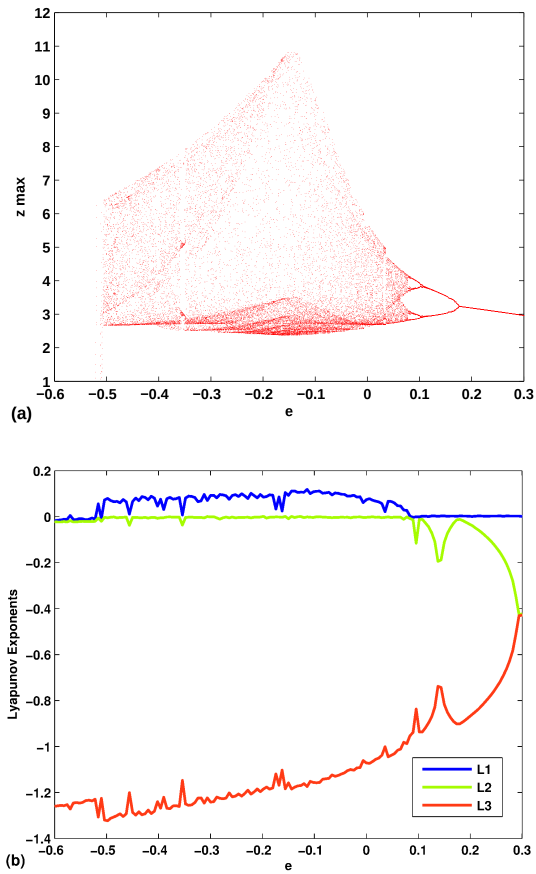


Fig. 6. Parameters values $(f, g) = (0, 0)$ of system (1): (a) Lyapunov exponent spectrum with $e \in [-0.6, 0.3]$;(b) Bifurcation diagram of the variable z with $e \in [-0.6, 0.3]$.

Now, define the error signals as

$$\begin{cases} \dot{e_x} = \dot{x_2} - m_1 h(t)\dot{x_1} - m_1 \dot{h(t)} x_1 \\ \dot{e_y} = \dot{y_2} - m_2 h(t)\dot{y_1} - m_2 \dot{h(t)} y_1 \\ \dot{e_z} = \dot{z_2} - m_3 h(t)\dot{y_1} - m_3 \dot{h(t)} z_1, \end{cases} \tag{16}$$

where $e_x = x_2 - m_1 h(x) x_1$, $e_y = y_2 - m_2 h(x) y_1$ and $e_z = z_2 - m_3 h(x) z_1$. Therefore, the error dynamical system

$$\begin{cases} \dot{e_x} = y_2 z_2 + a + u_1 - m_1 h(t)(y_1 z_1 + e x_1^2 + f x_1 + g) - m_1 \dot{h(t)} x_1 \\ \dot{e_y} = x_2^2 - y_2 + u_2 - m_2 h(t)(x_1^2 - y_1) - m_2 \dot{h(t)} y_1 \\ \dot{e_z} = 1 - 4x_2 + u_3 - m_3 h(t)(1 - 4x_1) - m_3 \dot{h(t)} z_1. \end{cases} \tag{17}$$

Our aim is to find control laws $u_i,\ i = 1, 2, 3$ for stabilizing the error variables of the system at the origin. For this end, we propose following control law:

$$\begin{cases} u_1 = -y_2 z_2 - a_1 + m_1 h(t)(y_1 z_1 + e_1 x_1^2 + f_1 x_1 + g_1) + m_1 \dot{h(t)} x_1 - k_1 e_x \\ u_2 = -x_2^2 + y_2 + u_2 + m_2 h(t)(x_1^2 - y_1) + m_2 \dot{h(t)} y_1 - k_2 e_y \\ u_3 = -1 + 4x_2 + m_3 h(t)(1 - 4x_1) + m_3 \dot{h(t)} z_1 - k_3 e_z, \end{cases} \tag{18}$$

and the update laws for the unknown parameters e_1, f_1, g_1 and a_1 are

$$\begin{cases} \dot{e_1} = -e_x m_1 h(t) x_1^2 - k_4(e_1 - e) \\ \dot{f_1} = -e_x m_1 h(t) x_1 - k_5(f_1 - f) \\ \dot{g_1} = -e_x m_1 h(t) - k_6(g_1 - g) \\ \dot{a_1} = e_x - k_7(a_1 - a), \end{cases} \tag{19}$$

where $k_i > 0,\ i = 1, 2, \dots, 7$.

Theorem 5.1. For given constant scaling matrix M and scaling function $h(t)$, the MFPS between two systems (11) and (12) will occur by the control law (18) and update law (19), and satisfy $\lim\limits_{t\to\infty} |e_1 - e| = 0$, $\lim\limits_{t\to\infty} |f_1 - f| = 0$, $\lim\limits_{t\to\infty} |g_1 - g| = 0$, $\lim\limits_{t\to\infty} |a_1 - a| = 0$.

P r o o f. Define a Lyapunov function,

$$V(e) = \frac{1}{2}(e_x^2 + e_y^2 + e_z^2 + e_e^2 + e_f^2 + e_g^2 + e_a^2),$$

where

$$e_e = e_1 - e,\ e_f = f_1 - f,\ e_g = g_1 - g,\ e_a = a_1 - a.$$

Therefore, the time derivative of the Lyapunov function

$$\dot{V(e)} = -k_1 e_x^2 - k_2 e_y^2 - k_3 e_z^2 - k_4 e_e^2 - k_5 e_f^2 - k_6 e_g^2 - k_7 e_a^2. \tag{20}$$

It is clear that $V(e)$ is positive definite and $\dot{V(e)}$ is negative definite. According to the Lyapunov stability theorem, the error system can converge to the origin asymptotically. Therefore, the drive system (10) and the response system (10) can be synchronized in the sense of MFPS. This completes the proof. □

We assume that the parameters $(e, f, g) = (-0, 14, 0, 0)$ and $a = 0.006$ (see [19]), the initial conditions of the drive system are $(x_1(0), y_1(0), z_1(0)) = (0.6, 0.9, -1.7)$, and the initial conditions of the response system are $(x_2(0), y_2(0), z_2(0)) = (1, 1, 1)$ (see [19]). Moreover, the initial conditions of the estimated parameters are chosen as $(e_1(0), f_1(0), g_1(0), a_1(0)) = (-0.3, 0.2, -0.2, 0)$. Let the scaling function be $h(t) = \sin(0.2\pi t)$ and the scaling factors are chosen as $m_1 = 1, m_2 = 3, m_3 = 5$. Furthermore, the control gains are chosen as $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k_7 = 2$. Figure 7 (a) displays the MFPS between systems (11) and (12). Figure 7 (b) show that the estimates e_1, f_1, g_1, a_1 of the unknown parameters converge to e, f, g, a as $t \to \infty$.

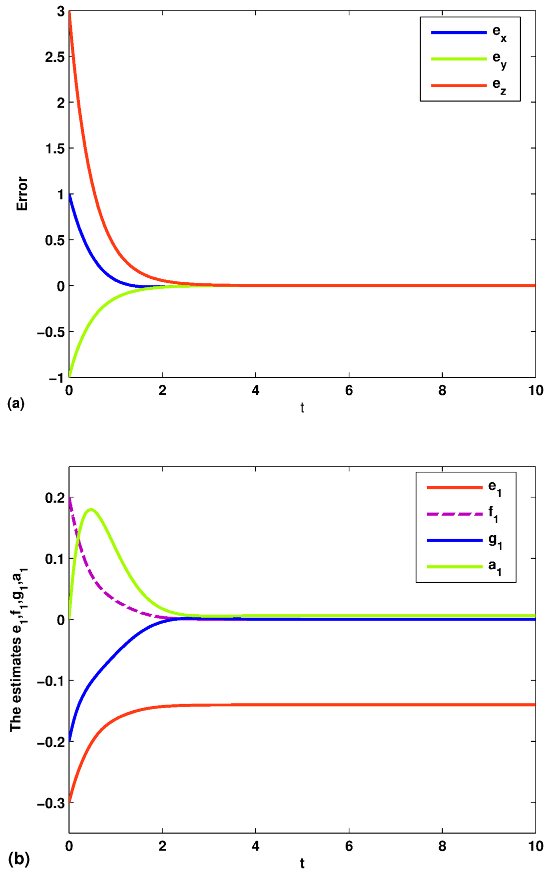


Fig. 7. (a) The behavior of the trajectories e_x, e_y, e_z of the error system between chaotic system (1) and Sprott E system for MFPS; (b) the estimates e_1, f_1, g_1, a_1 of the unknown parameters converge to $-0.14, 0, 0, 0.006$ as $t \to 10$.

6. CONCLUSION

In this paper, a changed Sprott E system is constructed. The chaotic attractors coexisting with only one stable equilibrium is different from the other existing homoclinic chaos or heteroclinic chaos. Some basic properties of the system have been investigated in terms of chaotic attractors, equilibria, Lyapunov exponent spectrum, bifurcation diagram and associated Poincaré map. However, the generation mechanism of chaos in the system still needs further studying and investigating, such as finding new criteria for the existence of chaos in some chaotic system with no homoclinic and heteroclinic orbits. Using numerical mathematical analysis and simulations, we detected the coexistence of limit cycles and chaotic attractors which verified rich dynamics of the extend Sprott E system. Moreover, we investigated modified function projective synchronization between the extended Sprott E system and original Sprott E system. Numerical simulations were provided to show the effectiveness of the proposed method. This special chaotic system has great potential for communication and electronics, it deserves further investigation in the near future.

In future works, we will use the proposed analysis method to investigate some complex chaotic systems, such as the typical multi-scroll chaotic systems by some effective design methods using piecewise-linear functions, cellular neural networks, nonlinear modulating functions, circuit component design, switching manifolds, etc. [5, 6, 25]. It is expected that more detailed theory analysis will be provided in a forthcoming paper.

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