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## Statistical Aspects of Ductile Fracture Propagation and the Conditions of the Conversion Ductile into Cleavage Fracture

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This paper describes a method for ascertaining how in spheroidized steels, the probability of cleavage initiation and ductile failure initiation in the transition temperature region varies with the distance from a pre-crack tip. These dependence have been utilized for estimating the distance at which stable crack propagation is converted into a cleavage process, and the lowest temperature at which ductile tearing can occur.

### 1. Introduction

Freezing carbon steels to the transition temperature region sharply reduce their fracture toughness, from its upper to its lower bound and change the fracture mechanism: ductile-fracture is replaced by a cleavage process. The mechanism by which voids are nucleated, grow and coalesce [1], is supplanted by cleavage. At very low temperatures, in the lower part of the transition temperature range, the ductile failure mechanism is suppressed altogether and fracture occurs by cleavage only. From the physical-metallurgy viewpoint, there can be no unstable cleavage crack propagation unless the cleavage trigger points lie sufficiently close to the starting pre-crack tip. When those points lie far from that tip, the results is a ductile crack, which grows until it reaches a trigger point and if enough strain energy is available, than turns into a low energy cleavage [2]. This raises two questions: how increasing temperature will affect the reduction of ductile and cleavage failure areas on the fracture surface; and what is the lowest temperature, in the transition region, at which we may expect a fracture surface to indicate stable ductile crack propagation. These questions are not easy to answer.

This paper therefore aims to establish only how the temperature and distance from the pre-crack tip affect the probability of initiation of the two processes and further, to estimate at what distance we may anticipate a conversion of the void coalescence mechanism into a cleavage process.

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## 2. Elementary probability of cleavage failure

The experimental researches of Rosenfield and Shetty [2] have conformed that cleavage failure process in steels may be initiated at second phase particles, especially at carbides. These microcracks will spread spontaneously into the matrix of the steel only if the acting tensile stress surpasses the local cleavage strength  $\sigma_f$  as defined by Curry and Knott [3]:

$$\sigma_f^2(2r) = B/(2r); \quad \text{where } B = \pi E \gamma_{eff} / (1 - \nu^2) \quad (1)$$

$\gamma_{eff}$  being the effective surface energy of the matrix,  $2r$  is the size of a penny-shaped microcrack,  $E$  is Young's modulus and  $\nu$  is Poisson's ratio. The tensile component of the stress field ahead of a pre-crack is known to vary with the distance from the pre-crack tip, as is described by following equation [4]:

$$\sigma(x) = Ax^{-1/(N+1)}; \quad \text{where } A = \sigma_y \left[ \frac{(1 - \nu^2) K_I^2}{I(N) \sigma_y^2} \right]^{1/(N+1)} \tilde{\sigma}(N, \Theta) \quad (2)$$

$N$  being the work hardening exponent;  $K_I$  the mode I stress intensity factor;  $I(N)$  a numerical constant dependent on work hardening exponent  $N$  and  $\tilde{\sigma}(N, \Theta)$  a function that varies with the work hardening exponent and with the relevant angle in a polar co-ordinate system which has its origin at the pre-crack tip. Consequently, for any given distance  $x$  there is a certain critical size of the microcrack, which according to equations (1) and (2) is given by:

$$2r_f(\sigma(x)) = BA^{-2} x^{2/(N+1)}. \quad (3)$$

As has been shown by Strnadel et al. [5], the distribution function for the precipitated carbides sizes can be closely approximated by a Weibull distribution,

$$\Phi(2r) = 1 - \exp \left[ - \left( \frac{2r}{R_0} \right)^{\alpha_0} \right] \quad (4)$$

where  $\alpha_0$  is a shape and  $R_0$  a size parameter. Godse and Gurland [6] state that at a stress level of  $\sigma(x)$  a microcrack will be initiated in a carbide of size  $2r$  if the following condition is met:

$$2r \geq [K_1 / (\sigma(x) - \sigma_i)]^2 \quad (5)$$

where  $\sigma_i$  is the friction stress given by the Hall-Petch relationship and  $K_1$  is a temperature-independent constant equal approximately to 2 MPa m<sup>1/2</sup>. On intersecting from equation (2) into equation (5), the Godse-Gurland [6] elementary probability of cleavage failure at a distance of  $x$  from the pre-crack tip works out as:

$$\Psi(x) = \exp \left[ \left( \frac{K_1^2}{R_0 (Ax^{-1/(N+1)} - \sigma_i)^2} \right)^{\alpha_0} - \left( \frac{Bx^{2/(N+1)}}{A^2 R_0} \right)^{\alpha_0} \right] \quad (6)$$

### 3. Elementary probability of ductile failure

A temperature increase can also cause the micromechanism of failure initiation ahead of the pre-crack tip to alter so that microcleavage is replaced by a process involving the growth and mutual coalescence of voids that were previously nucleated at carbides. Garrison and Thompson [7] observed that the local strain level given by equation:

$$\varepsilon_f = (1/S) \ln (\lambda_p/2r) \quad (7)$$

where  $S = 0.322 \exp (1.5 \sigma_m/\bar{\sigma})$ , governs the onset of the void coalescence process, grows with the mean interparticle spacing  $\lambda_p$ , but decreases as the secondary carbides grow coarser.

On the assumption that the spatial distribution of the carbides conforms to a Poisson distribution, the probability density of their distance is expressed by:

$$l(\lambda_p) = 2\pi\lambda_p N_A \exp (-\pi\lambda_p^2 N_A) \quad (8)$$

where  $N_A$  is the carbide density per unit area. The mutual coalescence of two voids, nucleated at adjacent particles (of usually different sizes) is subjected to the condition that  $\bar{R}_1 + \bar{R}_2 = \lambda_{p12}$ , where  $\bar{R}_1$  and  $\bar{R}_2$  are the sizes of the voids at the two particles. The local fracture strain according to equation (7) is related to the particle sizes  $r_1$  and  $r_2$  as follows:

$$\varepsilon_f(\lambda_{p12}, r_1 + r_2) = (1/S) \ln [\lambda_{p12}/(r_1 + r_2)]. \quad (9)$$

If we assume that the radii of two mutually adjacent carbides are statistically independent of one another, then the probability density of the sum of these radii will be:

$$g(r_1 + r_2) = \int_0^\infty \varphi(r) \varphi(r_1 + r_2 - r) dr \quad (10)$$

where  $\varphi(r) = 2 d\varphi(2r)/d(2r)$  is the probability density of the carbide particle radii. Transformation of equation (9) and

$$\xi(\lambda_{p12}, r_1 + r_2) = r_1 + r_2 \quad (11)$$

allows us to utilize equations (8) to (11) for establishing the probability density of the local fracture strain in the form of

$$h(\varepsilon_p) = \int_0^\infty J(\varepsilon_f, \xi) g(\xi) l [\xi \exp (\varepsilon_f S)] d\xi \quad (12)$$

where  $J(\varepsilon_f, \xi) = \xi \exp (\varepsilon_f S)$  is a Jacobian transformation of equations (9) and (11). The probability  $\zeta(x)$  of void coalescing at distance  $x$  from a blunted pre-crack tip clearly depends on the probability that plastic strain  $\varepsilon_p(x)$  will exceed the local fracture strain level, so that

$$\zeta(x) = Pr(\varepsilon_f \leq \varepsilon_f(x)) = \int_0^{\varepsilon_p(x)} h(\varepsilon_f) d\varepsilon_f = H(\varepsilon_p(x)) - H(0) \quad (13)$$

where  $H(\varepsilon_p(x))$  is the distribution function of local fracture strain for a plastic strain magnitude of  $\varepsilon_p(x)$ . As  $\varepsilon_p(x)$  is known to diminish as the distance from the blunted pre-crack tip increases [4], it is evident from equation (13) that the probability of void coalescence  $\zeta(x)$  increases as distance  $x$  grows smaller.

#### 4. Results and discussion

The experimentally ascertained temperature dependence of the yield point in investigated low carbon steel [5] was utilized in conjunction with equation (2) to determine how the tensile component of stress  $\sigma(x)$  varies with the value of parameter  $x\sigma_y^2/K_I^2$  at selected temperatures in the range between 57 and 162 K, assuming a parameter value of  $\sigma(N, \Theta) \approx 2.1$  [8]. The temperature dependence of fracture toughness [5] proves that the selected temperature interval covers the lower bound range and the lower ductile-brittle transition region. Equation (6) was used to establish how the elementary probability of cleavage failure varies in dependence on parameter  $x\sigma_y^2/K_I^2$ . A formula compiled by Mc Meeking [4]

$$\delta = 0.6K_I^2(1 - \nu^2)/(E\sigma) \quad (14)$$

was employed for evaluating the  $\Psi(x/\delta)$  functions plotted for nine selected temperatures within the examined temperature interval, in Fig. 1.

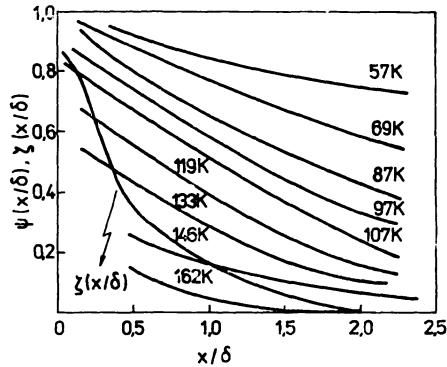


Fig. 1. How the probability of cleavage and ductile failure initiation in the spheroidized low carbon steel varies with parameter  $x/\delta$  which expresses the distance from a main crack tip.

The probability that voids nucleated at carbides would coalesce was assessed by a statistical interpretation of the coalescence criterion as formulated by Garrison and Thompson [7], i.e. with the aid of equation (7). The probability  $\zeta(x/\delta)$  that two voids will coalesce, which is governed by condition  $\epsilon_f(\lambda_{p12}, r_1 + r_2) \leq \epsilon_p(x)$ , is as follows from equation (13), determined by the difference between the distribution function  $H(\epsilon_f)$  for the value of  $\epsilon_p(x)$  and for zero. Figure 1 presents the numerically established dependence of the elementary probability of ductile failure initiation upon parameter  $x/\delta$ . If we accept that the stretched zone width equals approximately half the crack tip opening displacement,  $SZW \approx \delta/2$  [9], then Fig. 1 suggests that at temperatures up to about 140 K it is virtually impossible for the initial propagation of the main crack to take place by the coalescence of voids, since at  $x/\delta > 0.5$  the value of  $\Psi(x/\delta)$  exceeds that of  $\zeta(x/\delta)$ . Under such circumstances the growth of the

main crack must evidently be supported by the initiation of cleavage microcracks on carbides and their subsequent spreading into the matrix of the steel.

At temperatures higher than about 140 K given an  $x/\delta$  value between 0.5 and  $\Delta a/\delta$  ( $\Delta a$  being the SZW plus the stable crack length),  $\Psi(x/\delta)$  is less than  $\zeta(x/\delta)$  and consequently we may expect cleavage to be preceded by a ductile tearing zone. This

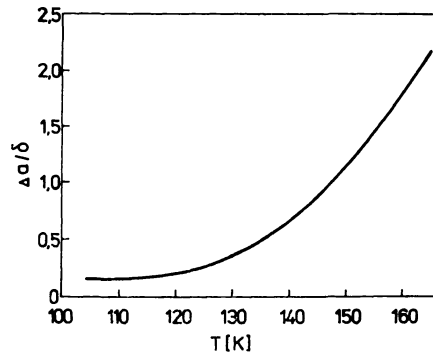


Fig. 2. Temperature dependence of parameter  $\Delta a/\delta$  in the spheroidized low carbon steel, where  $\Delta a$  is the SZW plus the stable crack length.

zone is unlikely to be detectable at 140 K but, as is evident in Fig. 2, it will grow fairly rapidly as the temperature increases, so that the failure initiation mechanism will convert from a ductile to a cleavage process at an ever greater distance from the main crack tip. It seems that in such spheroidized steels [5] a point around 140 K is not only the temperature at which the elastic stress field ahead of a crack tip becomes elastic-plastic [10], and at which there is a change of the micromechanism that controls cleavage [6], but also the highest temperature at which stable crack propagation cannot take place by the void coalescence micromechanism.

This model only examines changes in the elementary probability of failure initiation ahead of the crack tip. Changes in the integral probability in the whole active volume will be the object of a separate investigation.

### Conclusions

The object of this work was to establish how the elementary probability of cleavage initiation or ductile failure initiation depends on the distance from the crack tip, expressed by parameter  $x/\delta$ . A comparison of both these dependences produced a criterion for ascertaining the lowest temperature at which stable crack propagation may be converted into cleavage, and for estimating the distance of this conversion point from a starting pre-crack tip.

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