Jan Kvasil; R. G. Nazmitdinov

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Microscopical Description of the Collective States in Rotating Nuclei – I.

JAN KVASIL
Faculty of Mathematics and Physics, Charles University, Prague*)

R. G. NAZMITDINOV
Tashkent, USSR**)

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The method combining the cranking model with the random phase approximation is suggested for the description of the collective excitations in rotating nuclei. The symmetries of rotating nucleus Hamiltonian are discussed. The results of the analysis of the properties of the collective excitations in the framework of several models are presented.

V práci je objasněna metoda kombinující cranking model (model vynucené rotace) s aproximací náhodné fáze (Random Phase Approximation — RPA) pro popis kolektivních excitací rotujících jader. Diskutují se různé symetrie hamiltoniánu rotujícího jádra. Výsledky analýzy vlastností kolektivních i nekolektivních excitací v rámci různých přístupů dané metody jsou uvedeny v závěru práce.

Изложен метод, комбинирующий модель принудительного вращения и приближения случайных фаз, для описания коллективных возбуждений вращающихся ядер. Обсуждаются симметрии, сохраняющиеся для вращающихся ядер, их связь с физическими состояниями. Представлены результаты анализа свойств коллективных возбуждений в рамках различных моделей.

1. Introduction

The investigation of rotational states represents the important source of information about the nuclear structure [1]. One can say that the interest in nuclear rotation has been grown from the time of discovery of rotational spectrum in nucleus [2]. This fact is caused by progress in experimental technique which allows to study the nuclei in the region of limiting angular momenta (see e.g. [3]) when the nuclei still exist as the whole object. Several regularities in spectra and electromagnetic transitions have been succesfully cleared up in terms of nuclear unified model [4]. In the base of this model there is a supposition about the adiabatic distribution of degrees of freedom into the intrinsic and rotational ones. However nuclear rota-

*) Department of Nuclear Physics, V Holešovičkách 2, 189 00 Praha 8, Czechoslovakia.
**) Institute of Nuclear Physics of Tashkent State University, Tashkent, USSR.

tion shows a considerable influence on the intrinsic degrees of freedom with increasing angular momenta when Coriolis and centrifugal interaction grow up [3,5–9].

Studying of nuclear structure in low spin region confirms the fact that the nucleon intrinsic motion is mainly determined by nuclear average field and by pairing correlation of superconducting type [10]. This approximation of nuclear average field is usually assumed to be valid also in extremal conditions when the angular moment is so high that the channel of nuclear fission begins to dominate. Such assumption about using the same model approaches in different angular moment region can be explained by means of nucleon energy balance. Simple estimation for rare earth nuclei (see [6, 11]) shows that rotational energy per nucleon for $A \sim 150$ doesn't exceed 5% of nucleus binding energy in maximal spins $J \sim 60\hbar$ which can be reached in rare earth nucleus rotation. Therefore one can expect that the similar or the same model approaches can be applied in broad region of angular moments [12, 13].

One of these approaches is represented by cranking model introduced for the first time by Inglis [14]. This model suppose the nuclear average field to rotate with constant rotational frequence round the stable axis. The cranking model was succeed in analyses of many rotating nucleus properties. Its variant with inclusion of pairing interactions was able to clear up the properties of yrast line states or the properties of one-, two-quasiparticle states [15]. However last experimental information obtained from the analyses of γ-spectra of fast rotating nuclei approve the collective character of the states not only near the yrast line but also of highly excited states ($\sim 15-20$ MeV above the yrast line at $J \sim 60\hbar$ [16]).

In [17] the attempts were done to describe the collective states near the yrast line as a small oscillations of transverse axis of triaxial rotor [1]. It is evident that this approach has the phenomenological character. Therefore the next effort has been devoted to form the microscopical model for description of the states near the yrast line. The example of such model is the cranking model (CM) added by random phase approximation (RPA) [18, 19]. One has to mention also the other approaches [20–22] based on the method of generalized density matrix, but the CM + RPA has been used most often in literature.

Our aim is to give the explanation of the CM + RPA method and its application for calculation of particular characteristics of rotating nuclei. This theme is divided into two papers.

The first paper is devoted to the explanation of basic theoretical ideas of CM + RPA approach. Here the symmetries of cranking Hamiltonian, the Hartree-Fock-Bogolubov problem for rotating systems, the solution of RPA equations of motion, selfconsistency of residual interaction with average nuclear field and other questions are discussed.

In the second paper the concrete properties of rotating nuclei are studied in the framework of CM + RPA approach such as quadrupole, octupole excitations, giant dipole resonance, transition probabilities.

2. The cranking model and the random phase approximation

2.1. The cranking Hamiltonian and its symmetry

As was mentioned in preceeding section the cranking model (CM) suppose the nucleus to rotate round the stable axis. This axis is usually chosen to be simultaneously the x-axis of the lab system and x-axis of the intrinsic system rotating together with nucleus. The Hamiltonian corresponding to such rotation expressed in the intrinsic system can be written as [23]

$$H' = H - \sum_{\tau} \lambda_{\tau} \hat{N}_{\tau} - \Omega \hat{J}_x \tag{1}$$

where H is the total Hamiltonian of nucleus*), $\hat{N}_{\tau}$ is the operator of proton $(\tau = Z)$ or neutron $(\tau = N)$ number, λ_{τ} is the corresponding chemical potential, Ω stands for the angular velocity of rotation, $\hat{J}_x$ represents the x-component of total angular momentum. Hamiltonian $\hat{H}$ contains the intrinsic and collective degrees of freedom (connected with the whole nucleus rotation and with translation of the nucleus center of mass). Their separation represents generally complex nontrivial problem (see e.g. [20, 25]) which usually requires some approximations in their solving. In this paper the random phase approximation is used for this aim.

It is natural to suppose the total Hamiltonian to be rotational and translational invariant. Besides that we assume the number of particle in processes studied in this paper doen't change. Therefore

$$[H, \hat{J}_i] = [H, \hat{P}_i] = [H, \hat{N}_{\tau}] = 0 \tag{2}$$

where $\hat{J}_i$ and $\hat{P}_i$ are components of the total angular moment and linear moment in lab system, respectively. Since

$$\begin{aligned} &[\hat{J}_i, P_j] = i\varepsilon_{ijk}\hat{P}_k \quad [\hat{J}_i, \hat{J}_j] = i\varepsilon_{ijk}\hat{J}_k \\ &[\hat{N}_{\tau}, \hat{J}_i] = [\hat{N}_{\tau}, \hat{P}_i] = [\hat{P}_i, \hat{P}_j] = 0 \end{aligned} \tag{3}$$

the symmetry conditions (2) yields for cranking Hamiltonian the following expressions

$$\begin{aligned} &[H', \hat{J}_x] = 0 \qquad [H', \hat{P}_x] = 0 \\ &[H', \hat{J}_y] = -i\Omega\hat{J}_z \quad [H', \hat{P}_y] = -i\Omega\hat{P}_z \quad [H', N_{\tau}] = 0 \\ &[H', \hat{J}_z] = i\Omega\hat{J}_y \quad [H', \hat{P}_z] = i\Omega\hat{P}_y \end{aligned} \tag{4}$$

The total nucleus Hamiltonian H contains the average field and the residual interactions. In semimicroscopical approaches the nuclear average field is approxi-

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* The total nucleus Hamiltonian H in (1) is the Hamiltonian in lab system but expressed trrough the variables of the intrinsic rotating system. In the same way the component of linear moment $\hat{P}i$ in further expressions is assumed to be expressed in the intrinsic variables [24], i.e. $\hat{P} \to e^{iJ_x\Omega} \cdot \hat{P} e^{-iJ_x\Omega t}$.

mated by phenomenological deformed potential which violates the conditions (2). Therefore one has to choose the residual interactions in the form restoring the symmetries (2) of total Hamiltonian violated by deformed average field. Otherwise the eigen vectors of total Hamiltonian describing the intrinsic nucleus excitations are mixed with unphysical (from the point of view of intrinsic degrees of freedom) components of collective type (see e.g. [26, −29]). In the case of violation of total Hamiltonian symmetries (2) the collective and intrinsic modes are mutually mixed. Since in next considerations the eigen modes of nucleus Hamiltonian are searched for in framework of RPA the same approximation is used for restoration of symmetries (2).

For simplicity of explanation of basic ideas of used CM + RPA model we will start with spherical symmetric average field. The residual interactions will be taken in the form of separable multipole-multipole forces (long-range part) and monopole pairing (short-range part). However it must be noted that the particular form of residual interactions doesn't influence the generality of explanation of CM + RPA method. In further parts of this paper, where the applications of this method are discussed, the particular form of average field and residual interactions is given, but the calculations are performed in the framework of scheme described in this part. So our starting total Hamiltonian has the form

$$H' = \sum_k e_k c_k^+ c_k - \tfrac{1}{4}\sum_\tau G_\tau \hat{P}_\tau^+ \hat{P}_\tau - \tfrac{1}{2}\sum_{\lambda=1,2,3,\ldots}\sum_{\mu=-\lambda}^{\lambda} \varkappa_\lambda \hat{Q}_{\lambda\mu}^+ Q_{\lambda\mu} \tag{5}$$

where

$$\hat{P}_\tau^+ = \sum_{k\in\tau} c_k^+ c_{\bar{k}} \quad \hat{Q}_{\lambda\mu} = \sum_{kl} q_{kl}^{\lambda\mu} c_k^+ c_l \quad q_{kl}^{\lambda\mu} = \langle k|r^\lambda Y_{\lambda\mu}|l\rangle \tag{6}$$

c_k^+ and c_k are the creation and annihilation single-particle operators in the state $|k\rangle$ of spherical nuclear field, respectively. $c_{\bar{k}}^+ = Tc_k^+ T^{-1}$ (T is the time reversal operator [31]), $\varkappa_\lambda$ and G_τ are the strength constants of multipole-multipole interaction and pairing ($\tau = N, Z$), respectively. Summing in rel. (6) goes not only over indices k, l but also over $\bar{k}$, $\bar{l}$.

Experiment confirms the fact, that the most of nuclei in the ground state and in the yrast line states has the P, $R_k(\pi)$, $S_k = P R_k^{-1}(\pi)$ symmetries (P is the intrinsic parity operator, $R_k(\pi) = e^{i\pi J_k}$ is the operator of rotation by angle π round the axis k of intrinsic fixed-body system). That means the nucleus deformed field posses deformation of even multipolarity. Because of the term $-\Omega J_x$ the cranking Hamiltonian (1) has only P, $R_1(\pi)$, S_1-symmetries. Therefore it is convenient to choose the single-particle states to be the eigen vectors of $R_1(\pi) = R_x(\pi)$ operator [32, 33], so

$$e^{i\pi \hat{J}_x}\begin{pmatrix} c_k^+ \\ c_{\bar{k}}^+ \end{pmatrix} e^{-i\pi \hat{J}_x} = \mp i \begin{pmatrix} c_k^+ \\ c_{\bar{k}}^+ \end{pmatrix} \tag{7}$$

In [30] some aspects of CM + RPA approach are discussed with use of nonseparable forces.

In last years a lot of experimental and theoretical papers (see e.g. [34, –, 39]) have been published where the possibility of stable octupole nucleus deformation have been discussed. Octupole deformation violates the P- and $R_x(\pi)$ symmetry of cranking Hamiltonian, however S_x symmetry remains. Generalisation of CM + + RPA method for the case of stable octupole deformation is presented in [40].

RPA calculations for nonrotating nuclei show importance of quadrupole and octupole excitations in description of observed spectrum. Therefore we restrict ourselves in (5) only for $\lambda = 2, 3$. Because of the intrinsic R_1-symmetry of nucleus we introduce the following combinations of multipole operators (see appendix)

$$\hat{Q}_0^{(+)} = \hat{Q}_{20} \quad \hat{Q}_1^{(+)} = \frac{i}{\sqrt{2}}(\hat{Q}_{21} + \hat{Q}_{2-1}) \qquad \hat{Q}_1^{(-)} = \frac{1}{\sqrt{2}}(\hat{Q}_{21} - \hat{Q}_{2-1})$$

$$\hat{Q}_2^{(+)} = \frac{1}{\sqrt{2}}(\hat{Q}_{22} + \hat{Q}_{2-2}) \qquad \hat{Q}_2^{(-)} = \frac{i}{\sqrt{2}}(\hat{Q}_{22} - \hat{Q}_{2-2})$$

$$\hat{F}_0^{(-)} = \hat{Q}_{30} \quad \hat{F}_1^{(+)} = \frac{1}{\sqrt{2}}(\hat{Q}_{31} - \hat{Q}_{3-1}) \qquad \hat{F}_1^{(-)} = \frac{i}{\sqrt{2}}(\hat{Q}_{31} + \hat{Q}_{3-1}) \tag{8}$$

$$\hat{F}_2^{(+)} = \frac{i}{\sqrt{2}}(\hat{Q}_{32} - \hat{Q}_{3-2}) \qquad \hat{F}_2^{(-)} = \frac{1}{\sqrt{2}}(\hat{Q}_{32} + \hat{Q}_{3-2})$$

$$\hat{F}_3^{(+)} = \frac{1}{\sqrt{2}}(\hat{Q}_{33} - \hat{Q}_{3-3}) \qquad \hat{F}_3^{(-)} = \frac{i}{\sqrt{2}}(\hat{Q}_{33} + \hat{Q}_{3-3})$$

with symmetries

$$e^{i\pi J_x}\begin{pmatrix}\hat{Q}_{\bar{m}}^{(\pm)}\\ \hat{F}_{\bar{m}}^{(\pm)}\end{pmatrix} e^{-i\pi J_x} = \pm\begin{pmatrix}\hat{Q}_{\bar{m}}^{(\pm)}\\ \hat{F}_{\bar{m}}^{(\pm)}\end{pmatrix} \tag{8a}$$

These operators allow to rewrite the Hamiltonian (5) as

$$H = \sum_k e_k c_k^+ c_k - \tfrac{1}{4}\sum_\tau G_\tau \hat{P}_\tau^+ \hat{P}_\tau - \frac{\varkappa_2}{2}\Big[\sum_{\bar{m}=0}^{2}\hat{Q}_{\bar{m}}^{(+)}\hat{Q}_{\bar{m}}^{(+)} + \sum_{\bar{m}=1}^{2}\hat{Q}_{\bar{m}}^{(-)}\hat{Q}_{\bar{m}}^{(-)}\Big] - \tag{9}$$

$$- \frac{\varkappa_3}{2}\Big[\sum_{m=1}^{3}\hat{F}_m^{(+)}\hat{F}_m^{(+)} + \sum_{m=0}^{3}\hat{F}_m^{(-)}\hat{F}_m^{(-)}\Big]$$

The CM + RPA model involves two separate steps

i) In the first step, the Cranked Hartree-Fock-Bogolubov problem is solved. As a result we obtain the quasiparticle spectrum for a given rotational frequency Ω. The quasiparticle vacuum $|\Omega\rangle$ characterizes the nucleus state in the yrast line with a spin J corresponding to mean value of the operator $\hat{J}_x$ in the state $|\Omega\rangle$ with given frequency Ω.

All operators are defined in the system with quantum axis. z.

ii) In the second step, vibrations around the quasiparticle vacuum (i.e. around the yrast-line states) are described by the RPA method.

Later we are going to discuss both steps in more detail. It must be also noted that all discussions in this paper deal with the case of even-even nuclei.

2.2. The method of Hartree-Fock-Bogolubov (HFB) for rotating nuclei (description of yrast line)

The HFB method is used for description of many characteristics of rotating nuclei [53–59] and is described in detail in [42, 60]. Therefore only the basic ideas, necessary for further considerations are given in this part.

Using the Bogolubov transformation the quasiparticle operators α_i, α_i^+ are introduced

$$\alpha_i^+ = \sum_k (A_k^i c_k^+ + B_k^i c_k) \tag{10}$$

$$\alpha_{\bar{i}}^+ = \sum_k (A_k^{\bar{i}} c_k^+ + B_k^{\bar{i}} c_k)$$

where as a consequence of (7) the quasiparticle states are also the eigen states of the operator $\hat{R}_1(\pi)$

$$e^{i\pi \hat{J}_x} \begin{pmatrix} \alpha_i^+ \\ \alpha_{\bar{i}}^+ \end{pmatrix} e^{-i\pi \hat{J}_x} = \pm i \begin{pmatrix} \alpha_i^+ \\ \alpha_{\bar{i}}^+ \end{pmatrix} \tag{11}$$

Quasiparticle energies E_i, $E_{\bar{i}}$ and coefficients A_k^i, $A_k^{\bar{i}}$, B_k^i, $B_k^{\bar{i}}$ can be determined from the equations

$$\mathcal{M} \begin{pmatrix} A_k^i \\ B_k^i \end{pmatrix} = E_i \begin{pmatrix} A_k^i \\ B_k^i \end{pmatrix} \quad \mathcal{M} \begin{pmatrix} B_k^{\bar{i}} \\ A_k^{\bar{i}} \end{pmatrix} = E_{\bar{i}} \begin{pmatrix} B_k^{\bar{i}} \\ A_k^{\bar{i}} \end{pmatrix} \tag{12}$$

with*)

$$\mathcal{M} \begin{pmatrix} h^{(1)} & \Delta \\ \Delta^+ & h^{(2)} \end{pmatrix}$$

where the corresponding matrix elements have the form

$$\begin{aligned} h_{kl}^{(1)} &= \delta_{kl}(e_k - \tau_\tau) - \Omega \langle k| J_x |l\rangle - \varkappa_2 \langle \Omega| Q_{20} |\Omega\rangle \langle k| Q_{20} |l\rangle - \\ &\quad - \varkappa_2 \langle \Omega| Q_2^{(+)} |\Omega\rangle \langle k| Q_2^{(+)} |l\rangle \\ h_{kl}^{(2)} &= -\delta_{kl}(e_{\bar{k}} - \lambda_\tau) - \Omega \langle k| J_x |l\rangle + \varkappa_2 \langle \Omega| Q_{20} |\Omega\rangle \langle k| Q_{20} |l\rangle - \\ &\quad - \varkappa_2 \langle \Omega| Q_2^{(+)} |\Omega\rangle \langle k| Q_2^{(+)} |l\rangle \\ \Delta_{kl} &= -G_\tau / 4 \langle \Omega| P_\tau^+ |\Omega\rangle \delta_{kl} \end{aligned} \tag{13}$$

Here $\langle k| \hat{S} |l\rangle$ is the single-particle matrix element of given operator, $\langle \Omega| \hat{S} |\Omega\rangle$ stands for mean value of the corresponding operator in the quasi-particle vacuum

$|\Omega\rangle$, which is represented in CHFB model by the yrast state with given rotational frequency Ω. The way of determination of expectation values of single-particle operators is given in appendix. The selfconsistent equations (12), (13) have to be added by the following conditions

$$\langle\Omega|\,\hat{N}_{\tau=N}|\Omega\rangle = N_0 \quad \langle\Omega|\,\hat{N}_{\tau=Z}|\Omega\rangle = Z_0 \quad \langle\Omega|\,\hat{J}_x|\Omega\rangle = J_0 \tag{14}$$

for the nucleus with N_0 neutrons, Z_0 protons and with the rotational frequency Ω corresponding to moment J_0 in the yrast line.

The solution of the HFB problem allows us to rewrite the Hamiltonian (1) in the form

$$\begin{aligned} H' = {} & \langle\Omega|\,H|\Omega\rangle + \sum_i E_i(\alpha_i^+\alpha_i + \alpha_i^+\alpha_i) - \sum_\tau \frac{G_\tau}{4:}\,(\hat{P}_\tau - \langle\Omega|\,\hat{P}_\tau|\Omega\rangle) \\ & (\hat{P}_\tau - \langle\Omega|\,\hat{P}_\tau|\Omega\rangle))\,: \\ & - \frac{\varkappa_2}{2}\sum_{m=0}^{2} :(\hat{Q}_m^{(+)} - \langle\Omega|\,\hat{Q}_m|\Omega\rangle)\,(\hat{Q}_m^{(+)} - \langle\Omega|\,\hat{Q}_m|\Omega\rangle) - \\ & - \frac{\varkappa_2}{2}\sum_{m=1}^{2} :\hat{Q}_m^{(-)}\hat{Q}_m^{(-)}: - \frac{\varkappa_3}{2}\,[\sum_{m=1}^{3} :\hat{F}_m^{(-)}\hat{F}_m^{(-)}: + \sum_{m=0}^{3} :\hat{F}_m^{(+)}\hat{F}_m^{(+)}:] \end{aligned} \tag{15}$$

where the symbol :: denotes the normal ordering with respect to quasiparticle vacuum $|\Omega\rangle$. Since the Hamiltonian (1) is invariant to the transformation $R_x(\pi)$ and since $[H', \hat{P}] = 0$ ($\hat{P}$ is the space axis inversion operator), the quasiparticle vacuum can be searched for with this symmetries

$$e^{i\pi J_x}|\Omega\rangle = P|\Omega\rangle = |\Omega\rangle \tag{16}$$

On the base of (16) one can conclude that the mean values is nonzero only for the operators $\hat{A}$ not changing the parity and with positive signature.**)

The problem of solving of Eq. (12), (13) with the conditions (14) by precise self-consistent way is quite complicated. Therefore the nuclear average field is usually approximated by phenomenological potential of Nilson (see e.g. [53, 57]) or Saxon-Woods [56, 61] type. The parameters of this phenomenological field are determined from the requirement of reproduction of experimental single-particle characteristics in the ground state of given nucleus. Then the minimum of the total energy of deformed nucleus is searched for given rotational frequency Ω using the method

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* Equations (12) can be for instance obtained by variation method [42, 44]

$$\delta\,\{\langle\Omega|\,H'|\Omega\rangle + \sum_{ij} E_i\,[\sum_l (A_l^i A_l^j + B_l^i B_l^j + A_l^i A_l^j + B_l^i B_l^j) - 2]\}$$

with the total Hamiltonian H given in (5).

** Operator $\hat{A}$ has the positive signature if $R_x(\pi)\,\hat{A}_x^{-1}(\pi) = \hat{A}$, and negative if $R_x(\pi)\,.\,\hat{A}R_x^{-1}(\pi) = -\hat{A}$ [18, 19]).

of Strutinski [12, 57]. So in the case of phenomenological deformed average field the selfconsistency is performed only through the pairing term and cranking term in Hamiltonian (1).

As has been mentioned above the phenomebological deformed average field violates the symmetries (2) or (4). In order to restore these symmetries it is necessary to choose the corresponding residual interactions (see e.g. [26, −, 28, 62, −66]). The connection between the residual interactions and symmetries (4) is discussed in detail in part 3 of this paper.

2.3. The Random Phase Approximation (RPA) for rotating nuclei

2.3.1 Hamiltonian RPA and equation of motion

The method of RPA has been succesfully used for studying of nonrotational nuclei during last 30 years (see e.g. [1, 7, 10, 60] and references there). Firstly the RPA together with cranking model was suggested for desctiption of rotating nuclei properties in papers of Marshalek [67, 68, 18, 30, 43] and of Jansen and Mikhailov [19, 44]. There the basic ideas of HFB + RPA approach were formulated. The further progress of this method was connected with the construction of particular models for qualitative and quantitative investigation of nucleus properties [69, 45−52 70−76] and also with the improvement of theoretical scheme of restoration of total Hamiltonian symmetries in rotating nuclei [24, 29]. In [77] the method of strength function was used for description of solutions of RPA equations of motion. The generalization of the HFB + RPA approach for the case of stable octupole deformation is given in [40].

In the framework of HFB + RPA the states near the yrast line are described by phonons which are linear combinations of two-quasiparticle bosons (see e.g. [43])

$$b^+_{kl} = \alpha^+_k \alpha^+_l \quad b^+_{k\bar{l}} = i\alpha^+_k \alpha^+_l \quad b^+_{\bar{k}\bar{l}} = i\alpha^+_{\bar{k}} \alpha^+_l \tag{17a}$$

$$\alpha^+_k \alpha_l = \sum_m (b^+_{km} b_{lm} + b^+_{k\bar{m}} b_{l\bar{m}}) \tag{17b}$$

$$\alpha^+_k \alpha_l = i \sum_m (b^+_{k\bar{m}} b_{l\bar{m}} - b^+_{km} b_{lm})$$

These bosons are antisymmetric in indexes ($b_{ik} = -b_{ki}$) and fulfil the following commutation relations

$$[b_{kl}, b^+_{mn}] = \delta_{km}\delta_{ln} - \delta_{kn}\delta_{lm} \quad [b_{k\bar{l}}, b^+_{m\bar{n}}] = \delta_{km}\delta_{l\bar{n}} \tag{18}$$

$$[b_{kl}, b_{mn}] = [b_{kl}, b^+_{\bar{m}\bar{n}}] = 0$$

The RPA suppose the mean number of quasi-particles in vacuum to be zero

$$\langle \Omega | \alpha^+_k \alpha_l | \Omega \rangle \approx 0$$

The condition (11) implies

$$e^{i\pi J_x}\begin{bmatrix} b^+_{ik(i\bar{k})} \\ b^+_{i\bar{k}} \end{bmatrix} e^{-i\pi J_x} = \mp \begin{bmatrix} b^+_{ik(i\bar{k})} \\ b^+_{i\bar{k}} \end{bmatrix} \tag{19}$$

Every single-particle operator contained in Hamiltonian (15) can be expressed as an expansion in bosons (17). Particular form of this expansion depends on the symmetry of given operator regarding Hermitian conjugation, time reversal, transformation $R_x(\pi)$ (see appendix). Substituting these boson expansions of all operators involved in cranking Hamiltonian into (15) we obtain the boson representation of H'. $R_x(\pi)$-symmetry of Hamiltonian and commutation relations (18) allow to express the cranking Hamiltonian as a sum of four mutually commuting parts

$$H' = \langle\Omega|\, H'|\Omega\rangle + H^{(+)}_{(+)} + H^{(-)}_{(+)} + H^{(+)}_{(-)} + H^{(-)}_{(-)} \tag{20}$$

where

$$H^{(+)}_{(+)} = \sum_{ik} E_{i\bar{k}} b^+_{i\bar{k}} b_{i\bar{k}} - \tfrac{1}{4}\sum_{\tau} G_\tau P_\tau(1) P_\tau(1) - \frac{\varkappa_2}{2}\sum_{m=0}^{2} Q^{(+)}_m(1) Q^{(+)}_m(1) \tag{21a}$$

$$H^{(-)}_{(+)} = \tfrac{1}{2}\sum_{ik}\left(E_{ik} b^{(+)}_{ik} b_{ik} + E_{i\bar{k}} b^+_{i\bar{k}} b_{i\bar{k}}\right) - \frac{\varkappa_2}{2}\sum_{m=1}^{2} Q^{(-)}_m(1) Q^{(-)}_m(1) \tag{21b}$$

$$H^{(+)}_{(-)} = \sum_{ik} E_{i\bar{k}} b^+_{i\bar{k}} b_{i\bar{k}} - \frac{\varkappa_3}{2}\sum_{m=1}^{3} F^{(+)}_m(1) F^{(+)}_m(1) \tag{21c}$$

$$H^{(-)}_{(-)} = \tfrac{1}{2}\sum_{ik}\left(E_{ik} b^+_{ik} b_{ik} + E_{i\bar{k}} b^+_{i\bar{k}} b_{i\bar{k}}\right) - \frac{\varkappa_3}{2}\sum_{m=0}^{3} F^{(-)}_m(1)\, F^{(-)}_m(1) \tag{21d}$$

Here $E_{ik} = E_i + E_k$ and symbols $P_\tau(1)$, $Q^{(\pm)}_m(1)$, $F^{(\pm)}_m(1)$ represent the linear boson terms of expansions of corresponding operators (see appendix). Lower index in symbol $H^{(\sigma=\pm)}_{(\pi=\pm)}$ in (21) characterizes the parity of the operators involved in given part of Hamiltonian and upper index characterizes their signature.

Substituting the boson expansion (A5) into the symmetry conditions (4) one can write in the framework of RPA

$$\left[H^{(+)}_{(+)}, J_x(1)\right] = \left[H^{(+)}_{(+)}, N_\tau(1)\right] = 0 \tag{22}$$

$$\left[H^{(-)}_{(+)}, J_y(1)\right] = -i\Omega\, J_z(1) \qquad \left[H^{(-)}_{(+)}, J_z(1)\right] = i\Omega\, J_y(1) \tag{23}$$

$$\left[H^{(+)}_{(-)}, P_x(1)\right] = 0 \tag{24}$$

$$\left[H^{(-)}_{(-)}, P_y(1)\right] = -i\Omega\, P_z(1) \quad \left[H^{(-)}_{(-)}, P_z(1)\right] = i\Omega\, P_y(1) \tag{25}$$

$$\left[J_i(1), P_j(1)\right] = \left[J_i(1), N_\tau(1)\right] = \left[P_i(1), N_\tau(1)\right] = \left[P_i(1), P_j(1)\right] = \\ = \left[J_x(1), J_y(1)\right] = \left[J_x(1), J_z(1)\right] = 0 \tag{26}$$

$$\left[J_y(1), J_z(1)\right] = i\langle\Omega|\, \hat{J}_z|\Omega\rangle \tag{27}$$

Since all parts of cranking Hamiltonian (20) mutually commute the RPA equations of motion

$$[H', \mathscr{P}_\nu] = i\omega_\nu^2 \mathscr{X}_\nu \quad [H', \mathscr{X}_\nu] = -i\mathscr{P}_\nu \quad [\mathscr{X}_\nu, \mathscr{P}_{\nu'}] = i\delta_{\nu\nu'} . \tag{28}$$

can be solved independently for each of parts [79]. In (28) $\mathscr{X}_\nu$ and $\mathscr{P}_\nu$ represent the generalised coordinates and linear momentum, respectively, ω_ν is corresponding energies. The way of solving (28) is described in detail in the following section of this paper.

The Hamiltonian H' expressed in canonical conjugated variables $\mathscr{X}_\nu$ and $\mathscr{P}_\nu$ has the form

$$H' = \tfrac{1}{2}\sum_\nu (\mathscr{P}_\nu^2 + \omega_\nu^2 \mathscr{X}_\nu^2) = \tfrac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0}} (\mathscr{P}_\nu^2 + \omega_\nu^2 \mathscr{X}_\nu^2) + \tfrac{1}{2}\sum_{\substack{\nu_0 \\ \omega_{\nu 0}=0}} \mathscr{P}_{\nu_0}^2 = \tag{29}$$

$$= \sum_{\substack{\nu \\ \omega_\nu \neq 0}} \omega_\nu (\mathcal{O}_\nu^+ \mathcal{O}_\nu + \tfrac{1}{2}) + \tfrac{1}{2}\sum_{\substack{\nu_0 \\ \omega_{\nu 0}=0}} \mathscr{P}_{\nu_0}^2$$

where the creation operator of phonon

$$\mathcal{O}_\nu^+ = \frac{1}{\sqrt{2}}\left(\sqrt{(\omega_\nu)}\, \mathscr{X}_\nu - \frac{i}{\sqrt{(\omega_\nu)}}\, \mathscr{P}_\nu\right) \tag{30}$$

with $\omega_\nu \neq 0$ is introduced. The equation of motion (28) for $\omega_\nu \neq 0$ can be rewritten as

$$[H', \mathcal{O}_\nu^+] = \omega_\nu \mathcal{O}_\nu^+ \quad [H', \mathcal{O}_\nu] = -\omega_\nu \mathcal{O}_\nu \quad [\mathcal{O}_\nu, \mathcal{O}_{\nu'}] = \delta_{\nu\nu'} \tag{31}$$

Comparison of RPA equation of motion (28) or (31) with the symmetry conditions (22)–(25) allows to determine all "unphysical" (spurious) modes of Goldstone type (see 79) for all four parts of Hamiltonian (20).

From comparison of (22) with (28) it follows that there is one solution of RPA equation with Hamiltonian $H_{(+)}^{(+)}$ which is connected with the operator $J_x(1)$ and two solutions connected with $N_\tau(1)$ ($\tau = N, Z$). Therefore

(32)

$$\left.\begin{aligned} \mathscr{P}_{\nu_0 = J_x} &= \sqrt{(g_{J_x})}\, J_x(1) \\ \mathscr{P}_{\nu_0 = N_\tau} &= \sqrt{(g_{N_\tau})}\, N_\tau(1) \end{aligned}\right\} \Rightarrow H_{(+)}^{(+)} = \tfrac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0, \Omega}} (\mathscr{P}_\nu^2 + \omega_\nu^2 \mathscr{X}_\nu^2) + \tfrac{1}{2} g_{J_x} J_x^2(1) + \tfrac{1}{2}\sum_\tau g_{N\tau} N_\tau^2(1)$$

The way of determination of mass parameters g_{J_x} and g_{N_τ} will be discussed later. From (32) one can see that the quantity $\Phi_x = 1/g_{J_x}$ represents the moment of inertia regarding the rotating axis x.

The comparison of (23) with (28) allows to conclude that it is possible to construct the mode of Hamiltonian $H_{(+)}^{(-)}$ with energy $\omega = \Omega$ from the operators $J_y(1)$ and $J_z(1)$

$$
\left.\begin{aligned}
&[H^{(-)}_{(+)}, \Gamma^+] = \Omega\Gamma^+ \\
&[H^{(-)}_{(+)}, \Gamma] = -\Omega\Gamma \\
&[\Gamma, \Gamma^+] = 1
\end{aligned}\right\} \Rightarrow H^{(-)}_{(+)} = \tfrac{1}{2} \sum_{\substack{\nu \\ \omega_\nu \neq 0, \Omega}} (\mathscr{P}^2_\nu + \omega^2_\nu \mathscr{X}^2_\nu) + \Omega(\Gamma^+\Gamma + \tfrac{1}{2}) \tag{33}
$$

where the operators Γ, Γ^+ are defined as

$$
\Gamma^+ = \frac{J_y(1) - i\, J_z(1)}{\sqrt{\langle\Omega|\, \hat{J}_x |\Omega\rangle}}
$$

Comparing (24) with (28) one can write

$$
\mathscr{P}_{\nu_0 = P_x} = \sqrt{(g_{P_x})}\, P_x(1) \Rightarrow H^{(+)}_{(-)} = \tfrac{1}{2} \sum_{\substack{\nu \\ \omega_\nu \neq 0}} (\mathscr{P}^2_\nu + \omega^2_\nu \mathscr{X}^2_\nu) + \tfrac{1}{2} g_{P_x} P_x(1)^2 \tag{34}
$$

where the quantity $M = 1/g_{P_x}$ represents the effective mass nucleus.

The comparison of (25) with (28) makes it possible to expect for the mode $(P_y(1), P_z(1))$ to be the solution of RPA eqauation with Hamiltonian $H^{(-)}_{(-)}$ with energy $\omega = \Omega$. However in consequence of commutation $[P_y(1), P_z(1)] = 0$ such a mode would be nonnormalizable and therefore $(P_y(1), P_z(1))$ doesn't form the solution of $H^{(-)}_{(-)}$. Nevertheless it can be shown that the modes based on the operators $P_y(1)$, $P_z(1)$ and corresponding coordinates $X_y(1)$ and $X_z(1)$ are orthogonal to all solutions of RPA equations and therefore don't mix with them. Therefore the Hamiltonian can be expressed in terms of its RPA modes as follows

$$
H^{(-)}_{(-)} = \tfrac{1}{2} \sum_{\substack{\nu \\ \omega_\nu \neq 0}} (\mathscr{P}^2_\nu + \omega^2_\nu \mathscr{X}^2_\nu) + \Omega(X_y(1)\, P_z(1) - X_z(1)\, P_y(1)) \tag{35}
$$

where the last term assures the validity of conditions (25).

One can see from (32)–(35) that the comparison of symmetry conditions (22)–(25) with the RPA equations gives the possibility to extract the Goldstone modes connected with the rotation of whole nucleus and with the motion of center of mass of nucleus (or with number of particle conservation) from the solutions of RPA equations.

2.3.2. Diagonalization of RPA Hamiltonian

In this section the general scheme of diagonalization of RPA Hamiltonian and searching for the mass parameters g_{J_x}, g_{N_τ} and g_{P_x} is given (see [29, 77]).

From (20) and (21) it follows that each of four parts of Hamiltonian H' has the following structure

$$
H_B = \sum_\mu E_\mu b^+_\mu b_\mu + \sum_{s_1=1}^{n_1} \varkappa_{s_1} V_{s_1} V_{s_1} + \sum_{s_2=1}^{n_2} \varkappa_{s_2} W_{s_2} W_{s_2} \tag{36}
$$

where V_{s_1} and W_{s_2} are the linear boson parts of corresponding operators

(37)
$$V_{s_1} = \sum_\mu V_\mu^{s_1}(b_\mu^+ + b_\mu) \quad s_1 = 1, \ldots, n_1$$
$$W_{s_2} = \sum_\mu W_\mu^{(s_2)}(b_\mu^+ - b_\mu) \quad s_2 = 1, \ldots, n_2$$

Therefore the diagonalization procedure described further for the case of Hamiltonian H_B (36) is applicable for arbitrary part of Hamiltonian (20).

Hamiltonian H_B is characteristic by definite symmetries (see (22)–(25)) what is formally expressed by zero commutators of Hamiltonian H_B with corresponding operators (e.g. $J_x(1)$ and $N_\tau(1)$ in the case of Hamiltonian $H_{(+)}^{(+)}$). Let us suppose that for Hamiltonian H_B there are L_1 operators of type $(b_\mu^+ + b_\mu)$ and L_2 operators of type $(b_\mu^+ - b_\mu)$

(38)
$$\mathscr{D}_{l_1} = \sum_\mu \mathscr{D}_\mu^{(l_1)}(b_\mu^+ + b_\mu) \quad K_{l_2} = \sum_\mu K_\mu^{(l_2)}(b_\mu^+ - b_\mu)$$
$$l_1 = 1, \ldots, L_1 \qquad l_2 = 1, \ldots, L_2$$

which for all l_1 and l_2 fulfil the following commutation relations

(39)
$$[H_B, \mathscr{D}_{l_1}] = [H_B, K_{l_2}] = 0$$

(40)
$$[\mathscr{D}_{l_1}, \mathscr{D}_{l_1'}] = [K_{l_2}, K_{l_2'}] = [\mathscr{D}_{l_1}, K_{l_2}] = 0$$

Substituting (36), (37), (38) into (39) and using (18) one can obtain

(41)
$$\mathscr{D}_\mu^{(l_1)} E_\mu = \sum_{s_2=1}^{n_2} \varkappa_{s_2} a_{s_2}^{l_1} W_\mu^{(s_2)} \quad l = 1, \ldots, L_1$$

$$K_\mu^{(l_2)} E_\mu = \sum_{s_1=1}^{n_1} \varkappa_{s_1} b_{s_1}^{l_2} V_\mu^{s_1} \qquad l_2 = 1, \ldots, L_2$$

where

(42)
$$a_{s_2}^{l_1} = 2[W_{s_2}, \mathscr{D}_{l_1}] = 4 \sum_\mu \mathscr{D}_\mu^{(l_1)} W_\mu^{s_2}$$
$$b_{s_1}^{l_2} = 2[V_{s_1}, K_{l_2}] = 4 \sum_\mu K_\mu^{(l_2)} V_\mu^{s_1}$$

It must be noted that the quantities $a_{s_2}^{l_1}$ and $b_{s_1}^{l_2}$ in particular approaches can be expressed in terms of the expectation values of type $\langle\Omega| \hat{Q}_{\tau=0,2}^{(+)} |\Omega\rangle$, $\langle\Omega| \hat{P}_\tau |\Omega\rangle$ and others (see [19, 40, 43, 44]). This means the relations (41) represents the conditions of selfconsistency between the quasi-particle HFB field and residual interactions of cranking Hamiltonian in the framework of RPA.

The boson representation of canonical conjugated coordinates $\mathscr{X}_\nu$ and linear momenta $\mathscr{P}_\nu$ can be searched for in the form

(43)
$$\mathscr{X}_\nu = \sum_\mu X_\mu^\nu (b_\mu^+ + b_\mu)$$
$$\mathscr{P}_\nu = i \sum_\mu P_\mu^\nu (b_\mu^+ - b_\mu) \qquad [\mathscr{X}_\nu, \mathscr{P}_{\nu'}] = 2i \sum_\mu X_\mu^\nu \mathscr{P}_\mu^{\nu'} = i\delta_{\nu\nu'}$$

Substituting these expressions and relation for H_B (36) into the RPA equation of motion (28) and using (18) one obtains the matrix representation of RPA equations

$$
\begin{aligned}
X_\mu^\nu &= \sum_{s_2=1}^{n_2} \varkappa_{s_2} A_{s_2}^\nu \frac{E_\mu W_\mu^{s_2}}{E_\mu^2 - \omega_\nu^2} - \sum_{s_1=1}^{n_1} \varkappa_{s_1} B_{s_1}^\mu \frac{V_\mu^{s_1}}{E_\mu^2 - \omega_\nu^2} \\
\mathscr{P}_\mu^\nu &= \omega_\nu^2 \sum_{s_1=1}^{n_2} \varkappa_{s_2} A_{s_2}^\nu \frac{W_\mu^{s_2}}{E_\mu^2 - \omega_\nu^2} - \sum_{s_1=1}^{n_1} \varkappa_{s_1} B_{s_1}^\nu \frac{E_\mu V_\nu^{s_1}}{E_\mu^2 - \omega_\nu^2}
\end{aligned} \tag{44}
$$

where

$$
A_{s_2}^\nu = 4 \sum_\mu X_\mu^\nu W_\mu^{s_2} \quad B_{s_1}^\nu = 4 \sum_\mu \mathscr{P}_\mu^\nu V_\mu^{s_1} \tag{45}
$$

Resubstitution of (44) into (45) yields the system of algebraic equations for $(n_1 + n_2)$ unknowns $A_{s_2}^\nu$ and $B_{s_1}^\nu$

$$
\sum_{s_2=1}^{n_2} \varkappa_{s_2} A_{s_2}^\nu \left(S_{W_{s(2)} W_{s'(2)}} - \frac{\delta_{s_2 s_2'}}{4\varkappa_{s_2}} \right) - \sum_{s_1=1}^{n_1} \varkappa_{s_1} B_{s_1}^\nu \mathscr{U}_{V_{s(1)} W_{s'(2)}} = 0 \quad s_2' = 1, \ldots, n_2 \tag{46a}
$$

$$
\sum_{s_2=1}^{n_2} \varkappa_{s_2} A_{s_2}^\nu \omega_\nu^2 \mathscr{U}_{V_{s'(1)} W_{s(2)}} - \sum_{s_1=1}^{n_1} \varkappa_{s_1} B_{s_1}^\nu \left(S_{V_{s(1)} V_{s'(1)}} + \frac{\delta_{s_1 s_1'}}{4\varkappa_{s_1}} \right) = 0 \quad s_1' = 1, \ldots, n_1 \tag{46b}
$$

where

$$
S_{RT} = \sum_\mu \frac{E_\mu R_\mu T_\mu}{E_\mu^2 - \omega^2} \qquad U_{RT} = \sum_\mu \frac{R_\mu T_\mu}{E_\mu^2 - \omega^2} \tag{47}
$$

which fulfil the following useful relations

$$
S_{RT} = S_{RT}(0) + \omega^2 W_{RT} \quad U_{RT} = U_{RT}(0) + \omega^2 \mathscr{L}_{RT} \tag{48}
$$

$$
\mathscr{W}_{RT} = \sum_\mu \frac{R_\mu T_\mu}{E_\mu (E_\mu^2 - \omega^2)} \qquad \mathscr{L}_{RT} = \sum_\mu \frac{R_\mu T_\mu}{E_\mu^2 (E_\mu^2 - \omega^2)} \tag{49}
$$

Among the solutions of (46) there are ones with zero energy $\omega_\nu = 0$ which can be connected with the symmetries of cracking Hamiltonian (see sec. 2.3.1.). These zero solutions have to be extracted because the physical interest is caused only by the solutions with $\omega_\nu \neq 0$. For $\omega_\nu = 0$ the system of Eqs. (46) can be rewritten as

$$
\begin{aligned}
&\sum_{s_1=1}^{n_2} \varkappa_{s_2} A_{s_2}^{l_1}(0) \left(S_{W_{s(2)} W_{s'(2)}}(0) - \frac{\delta_{s_2 s_2'}}{4\varkappa_{s_2}} \right) = 0 \quad s_2' = 1, \ldots, n_2 \\
&\sum_{s_1=1}^{n_1} \varkappa_{s_1} B_{s_1}^{l_2}(0) \left(S_{V_{s(1)} V_{s'(1)}}(0) + \frac{\delta_{s_1 s_1'}}{4\varkappa_{s_1}} \right) = 0 \quad s_1' = 1, \ldots, n_1
\end{aligned} \tag{50}
$$

The condition of solvability of (50) is

$$\det \left| S_{W_{s(2)}W_{s'(2)}}(0) - \frac{\delta_{s_2 s_2'}}{4\varkappa_{s_2}} \right| = 0$$

(51)

$$\det \left| S_{V_{s(1)}V_{s'(1)}}(0) + \frac{\delta_{s_1 s_1'}}{4\varkappa_{s_1}} \right| = 0$$

It is evident that if we put

(52) $$A^{l_1}_{s_2}(0) = \sqrt{(g_{l_1})}\, a^{l_1}_{s_2} \quad B^{l_2}_{s_1}(0) = \sqrt{(g_{l_2})}\, b^{l_2}_{s_1}$$

the selfconsistent conditions (41) and Eqs. (50) become equivalent. Besides that using (50) from equation (51a) it follows

(53) $$\sum_{s_1=1}^{n_1} \varkappa_{s_1} B^{l_1}_{s_1}(0)\, U_{V_{s(1)}W_{s(2)}}(0) = 0$$

Multiplying each of the equations (46a) by $\varkappa_{s_2} a^{l_1}_{s_2}$ and summing through index s_2 and similarly multiplying each of the equations (46b) by $\varkappa_{s_1} b^{l_2}_{s_1}$ and summing through index s_1 with using of (50) and (52) one can obtain the following system of equations

$$\sum_{s_2'=1}^{n_2} \sum_{s_2=1}^{n_2} \varkappa_{s_2}\varkappa_{s_2}\, a^{l_1}_{s_1} A^{\nu}_{s_2'} (S_{W_{s(2)}W_{s'(2)}} - S_{W_{s(2)}W_{s'(1)}}(0)) - \sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} \varkappa_{s_1}\varkappa_{s_2} a^{l_1}_{s_2}\, U_{V_{s(1)}W_{s(2)}} = 0$$

(54)

$$\omega_\nu^2 \sum_{s_1=1}^{n_1} \sum_{s_2=1}^{n_2} \varkappa_{s_1}\varkappa_{s_2} b^{l_2}_{s_1} A^{\nu}_{s_2}\, U_{V_{s(1)}W_{s(2)}} -$$

$$- \sum_{s_1'=1}^{n_1} \sum_{s_1=1}^{n_1} \varkappa_{s_1}\varkappa_{s_1'}\, b^{l_2}_{s_1} B^{\nu}_{s_1'} (S_{V_{s(1)}V_{s'(1)}} - S_{V_{s(1)}V_{s'(1)}}(0) = 0$$

With help of (48) and (53) the system of equations (54) can be rewritten in the form

$$\omega_\nu^2 \sum_{s_2} \sum_{s_2'} \varkappa_{s_2}\varkappa_{s_2'}\, a^{l_1}_{s_2} A^{\nu}_{s_2'}\, \mathscr{W}_{W_{s_2}W_{s_2'}} - \omega_\nu^2 \sum_{s_1} \sum_{s_2} \varkappa_{s_1}\varkappa_{s_2}\, a^{l_1}_{s_2} B^{\nu}_{s_1}\, \mathscr{L}_{V_{s(1)}W_{s(2)}} = 0$$

(55a) $$l_1 = 1, \ldots, L_1$$

(55b)

$$\omega_\nu^2 \sum_{s_1} \sum_{s_2} \varkappa_{s_1}\varkappa_{s_2}\, b^{l_2}_{s_1} A^{\nu}_{s_2} \mathscr{L}_{V_{s_1}W_{s_2}} - \omega_\nu^2 \sum_{s_1} \sum_{s_1'} \varkappa_{s_1}\varkappa_{s_1'}\, b^{l_2}_{s_1} B^{\nu}_{s_1'} \mathscr{W}_{V_{s_1}V_{s_1'}} = 0 \quad l_2 = 1, \ldots, L_2$$

In such a way the complete system of equations for $(n_1 + n_2)$ unknowns $A^{\nu}_{s_2}$ and $B^{\nu}_{s_1}$ consists of L_1 equations (55a), L_2 equations (55b) $(n_1 - L_1)$ arbitrary chosen equations (46a) and $(n_2 - L_2)$ arbitrary chosen equations (46b). From corresponding secular equation

(56) $$\omega_\nu^{2(L_1+L_2)} |\mathscr{D}(\omega_\nu)| = 0$$

(here $|\mathscr{D}(\omega_\nu)|$ is determinant of obtained system of Eqs.) it follows the $2(L_1 + L_2)$ order degeneracy of the solution with $\omega_\nu = 0$. The energy of nonzero solutions can be obtained from equation

(57) $$|\mathscr{D}(\omega_\nu)| = 0$$

Then the structure of corresponding phonon (i.e. amplitudes X_μ^ν an $\mathscr{P}_\mu^\nu$ in (43) is determined from equation (44) using the normalization conditions in (43) and the solution of (57).

The only unknowns to be found are the mass parameters g_{l_1} and g_{l_2}. The way of determination of these parameters differs in two following cases.

i) the simultaneous existence of spurious states caused by the operators K_{l_2} and $\mathscr{D}_{l_1}$;

ii) existence of spurious states created only by operators K_{l_2} or only by operators $\mathscr{D}_{l_1}$.

If the spurious states of both the types simultaneously exist, the relations (50) and (53) hold. From the Eqs. (44) for $\omega_\nu = 0$ with respect to (52) it follows

$$\mathscr{P}_\mu^{l_2}(0) = -\sqrt{g_{l_2}} \sum_{s_1=1}^{n_1} \varkappa_{s_1} b_{s_1}^{l_2} \frac{V_\mu^{s_1}}{E_\mu}$$

(58)

$$X_\mu^{l_2}(0) = \sum_{s_2=1}^{n_2} \varkappa_{s_2} A_{s_2}^{l_2}(0) \frac{W_\mu^{s_2}}{E_\mu} - \sqrt{(g_{l_2})} \sum_{s_1} \varkappa_{s_1} b_{s_1}^{l_2} \frac{V_\mu^{s_1}}{E_\mu^2}$$

Substituting these expressions into normalization condition (43)

$$[X_{l_2}(0), \mathscr{P}_{l_2}(0)] = 2i \sum_\mu X_\mu^{l_2}(0)\, \mathscr{P}_\mu^{l_2}(0) = i \tag{59}$$

one can get the following equation

$$-\sqrt{(g_{l_2})} \sum_{s_1} \sum_{s_2} \varkappa_{s_1} \varkappa_{s_2} A_{s_2}^{l}(0)\, b_{s_1}^{l_2}\, U_{V_{s(1)} W_{s(2)}}(0) + g_{l_2} \sum_{s_1} \sum_{s_1'} \varkappa_{s_1} \varkappa_{s_1'}\, b_{s_1}^{l_2} b_{s_1'}^{l_2} \sum_\mu \frac{V_\mu^{s_1} V_\mu^{s_1'}}{E_\mu^3} = \tfrac{1}{2}$$

From (53) and (52) one can see that the first term in the right-hand side of Eq. (59) is equal to zero. Using (59) and (42) one obtains eventually the expressions for generalised inertial coefficients

$$\frac{1}{g_{l_2}} = 2 \sum_\mu \frac{K_\mu^{(l_2)} K_\mu^{(l_2)}}{E_\mu} \tag{60}$$

A similar expression can be obtained for coefficients g_{l_1}

$$\frac{1}{g_{l_1}} = 2 \sum_\mu \frac{\mathscr{D}_\mu^{(l_1)} \mathscr{D}_\mu^{(l_1)}}{E_\mu} \tag{61}$$

Let us investigate the case ii), when only the spurious states of one type exist (e.g. created by operator of type K_{l_2}). In this case the relation (53) doesn't hold and therefore we have to make an other way of determination of inertial coefficients. From (46) with regard to (52) it follows

$$\sum_{s_2'=1}^{n_2} \varkappa_{s_2'} A_{s_2'}^{l_2}(0) \left(S_{W_{s(2)} W_{s'(2)}}(0) - \frac{\delta_{s_2 s_2'}}{4\varkappa_{s_2}} \right) - \sqrt{(g_{l_2})} \sum_{s_1=1}^{n_1} \varkappa_{s_1} b_{s_1}^{l_2}\, \mathscr{U}_{V_{s(1)} W_{s(2)}}(0) = 0 \tag{62a}$$

(62b) $$\sqrt{(g_{l_2})}\sum_{s_1'=1}^{n_1}\varkappa_{s_1'}b_{s_1'}^{l_2}\left(S_{V_{s(1)}V_{s'(1)}}(0)+\frac{\delta_{s_1s_1'}}{4\varkappa_{s_1}}\right)=0$$

Using (41) Eq. (62a) can be rewritten as

(63) $$\sum_{s_2'=1}^{n_2}\varkappa_{s_2'}A_{s_2'}^{l_2}(0)\left(S_{W_{s(2)}W_{s'(2)}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right)-\sqrt{(g_{l_2})}\,S_{K_{l(2)}W_{s(2)}}(0)=0$$

$$s_2=1,\ldots,n_2$$

Relations (58) and (59) hold in this case too. Substituting (41) into (59) we obtain

(64) $$-\sqrt{(g_{l_2})}\sum_{s_2=1}^{n_2}\varkappa_{s_2}A_{s_2}^{l_2}(0)\,S_{K_{l(2)}W_{s(2)}}(0)+g_{l_2}\,S_{K_{l(2)}K_{l(2)}}(0)=\tfrac{1}{2}$$

The inhomogeneous system of equations (63) and (64) allows us to determine the unknowns $A_{s_2}^{l_2}(0)$ and g_{l_2} using Kramer's method

(65) $$g_{l_2}=\frac{1}{2}\frac{\left|S_{W_{s(2)}W_{s'(2)}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right|}{|\mathscr{D}_{l_2}|}$$

where $|\mathscr{D}_{l_2}|$ is the determinant of the equation system (63) and (64).

In the end of this section one has to mention that the representation (43) can be understood as a transformation from the space of two-quasiparticle bosons into the space of RPA modes $(X_\nu, \mathscr{P}_\nu)$ (or $(\mathcal{O}_\nu, \mathcal{O}_\nu^+)$). This transformation is given by unitary Hermitian matrix with elements X_μ^ν and $\mathscr{P}_\mu^\nu$. Number of two-quasiparticle bosons b_μ, b_μ^+ must correspond to number of RPA modes (X_ν, P_ν). This completeness of both spaces can be expressed by relation (see e.g. [40])

(66) $$b_\mu^+ = i\sum_\nu\{[b_\mu^+, X_\nu]\,\mathscr{P}_\nu+[\mathscr{P}_\nu, b_\mu^+]\,X_\nu\}$$

where the summation goes through the spurious as well as through nonspurious modes.

2.3.3. Eigen vectors of cranking Hamiltonian

All solutions of the RPA equations are known to be orthogonal to each other. Therefore, all normal modes $(\mathscr{X}_\nu, \mathscr{P}_\nu)$ of Hamiltonian (20) with $\omega_\nu \neq 0$ are orthogonal to spurious Goldstone $(\Theta_x(1), J_x(1))$ mode (symbol $\Theta_x(1)$ stands for the linear boson part of angle Θ_x canonical conjugated to operator J_x). Therefore the mean value of the J_x component of the total angular momentum in one-phonon states

(67) $$\langle\Omega|\,\mathcal{O}_\nu\hat{J}_x\mathcal{O}_\nu^+|\Omega\rangle=\langle\Omega|\,[\mathcal{O}_\nu,\hat{J}_x]\,\mathcal{O}_\nu^+|\Omega\rangle+\langle\Omega|\,\hat{J}_x\mathcal{O}_\nu\mathcal{O}_\nu^+|\Omega\rangle\approx\langle\Omega|\,\hat{J}_x|\Omega\rangle$$

corresponds to mean value of the operator $\hat{J}_x$ in the yrast line state $|\Omega\rangle$ (rel. (67) valids in RPA order, that means up to the second order in boson expansion).

The state $|\Omega\rangle$ of yrast line represents the vacuum for quasi-particle and phonon operators. Therefore

$$\mathcal{O}_\nu|\Omega\rangle = N_\tau(1)|\Omega\rangle = P_x(1)|\Omega\rangle = \Gamma|\Omega\rangle = J_x(1)|\Omega\rangle = 0 \tag{68}$$

Since $J_x(1) \approx J_x - \langle\Omega|J_x|\Omega\rangle$, the relation (68) corresponds to the fact that the angular momentum in the state $|\Omega\rangle$ is aligned along the rotational axis x. This is expressed by cranking condition

$$\langle\Omega|\hat{J}_x|\Omega\rangle \approx \sqrt{J(J+1)} \tag{69}$$

From (67) and (69) it follows that the phonon creation operators $\mathcal{O}_\nu^+$ acting on the yrast line state $|\Omega\rangle$ do not change the angular momentum J.

From (68) and from condition $\langle\Omega|P_x|\Omega\rangle = 0$ one can also make conclusion that there are no vibrations of nucleus center, of mass along the axis x. Generally such a vibrations can arise as a consequence of nuclear excitation but the extraction of spurious mode (X, P_x) from the RPA solutions guarantees the orthogonality of normal modes to (X, P_x) mode. As was mentioned above the vibrations of center of mass along the y,z-axes in lab system also don't mix with the normal RPA modes.

In accordance with the papers of Marshalek [30, 43] (see also [81, 40]) the eigen vectors of cranking Hamiltonian can be written as

$$|\alpha JM\rangle = |\alpha J\rangle \times |JM\rangle \tag{70}$$

$$|JM\rangle = \frac{e^{i(J-J_0)\Theta_x}}{\sqrt{2\pi}} \frac{(\Gamma^+)^{J-M}}{\sqrt{(J-M)!}} |J_0 M = J_0\rangle \tag{71a}$$

(71b)

$$\left|\alpha J\right\rangle = \frac{e^{i(N-N_0)\Theta_N}}{\sqrt{2\pi}} \frac{e^{i(Z-Z_0)\Theta_z}}{\sqrt{2\pi}} \frac{e^{iP_x X}}{2\pi} \prod_{\substack{n_{\nu(-)} \\ n_{\nu(+)}}} \frac{(\mathcal{O}_{\nu_-}^+)^{n_{\nu(-)}}}{\sqrt{n_{\nu_-}!}} \frac{(\mathcal{O}_{\nu_+}^{n_{\nu(+)}})^{n_{\nu(+)}}}{\sqrt{n_{\nu_+}!}} \left| 0, 0, N_0, Z_0, 0 \right\rangle$$

where n_{ν_-} and n_{ν_+} are numbers of the phonons of negative and positive signature, respectively, $\Theta_\tau(\tau = N, Z)$ and Θ_x are the angles conjugated to N_τ and J_x and X is a coordinate x-component. The ket $|n_{\nu_-} = 0, n_{\nu_+} = 0, N = N_0, Z = Z_0, P_x = 0\rangle = |\Omega\rangle$ describes the yrast line state with angular momentum J_0 (with projection $M = J_0$ onto x-axis) for a nucleus with N_0 neutrons and Z_0 protons. Index α in (71b) substitutes the quantum number n_{ν_+}, n_ν, N, Z, P_x.

Since the intrinsic part $|\alpha J\rangle$ is supposed to be symmetric with respect to the $R_x(\pi)$-transformation and since the yrast line state symmetry (16) is valid it is possible to write the following condition

$$(-1)^J (-1) \exp \sum_{\nu A} n_{\nu_-} = 1$$

According to this relation the single-phonon states with positive signature correspond to even values of angular momentum J and vice versa.

2.3.4. Electric Transition Probabilities

The analysis of E2-transitions in high angular momentum limit was performed in papers of Mikhailov [83] and Marshalek [43]. The E1- and E3- transitions were discussed in [40]. Qualitative estimations of E1-transition probabilities for rotating nuclei can be found in papers [70–73, 82].

Reduced electric transition probability is given by (see e.g. [21])

$$B(E\lambda;\, \alpha_1 J_1 \to \alpha_2 J_2) = \frac{|\langle \alpha_2 J_2 \,\|\, \hat{\mathcal{M}}(E\lambda) \|\, \alpha_1 J_1 \rangle|^2}{2J_1 + 1} \tag{72}$$

where $\hat{\mathcal{M}}(E\lambda, \mu)$ is the electric transition operator of multipolarity ν with the projection μ. Tensor operator $\hat{\mathcal{M}}_{\lambda\mu}$ defined in lab system can be expressed in terms of its components $\mathcal{M}'_{\lambda\nu}$ in intrinsic system as follows

$$\hat{\mathcal{M}}_{\lambda\mu} = \sum_{\nu} \hat{\mathcal{D}}^{\lambda}_{\mu\nu} \hat{\mathcal{M}}'_{\lambda\nu} = \tfrac{1}{2} \sum_{\nu} \{\hat{\mathcal{M}}'_{\lambda\nu}, \hat{\mathcal{D}}^{\lambda}_{\nu\mu}\} \tag{73}$$

where the Wigner function $\mathcal{D}^{\lambda}_{\nu\mu}$ is understood to be in operator form (see [30, 85]). In the case when the quantum axis coincides with the rotational one in $J \gg 1$ the reduced matrix element has the form [83, 30]

$$\langle \alpha_2 J + \nu \| \mathcal{M}_{\lambda} \| \alpha_1 J \rangle = \sqrt{(2J+1)}\, (J\lambda J\nu \mid J + \nu J + \nu)\, \langle \alpha_2 J + \nu | \mathcal{M}'_{\lambda\nu} | \alpha_1 J \rangle \tag{74}$$

Since all projections of angular moments in wave functions (70), (71) are defined in the system with axis x as a quantum axis, all multipole operators involved in Hamiltonian H' have to be rewritten using transformation

$$\hat{Q}_{\lambda\mu_x} = \sum_{\mu_z} \mathcal{D}^{\lambda}_{\mu_x\mu_z}(\tfrac{1}{2}\pi)\, Q_{\lambda\mu_z} \tag{75}$$

Substituting the boson expansions of the operators (see appendix) and explicite form of Wigner functions (see [84]) into (75) one can obtain the following expressions (up to the second order in boson expansion)

$$\begin{aligned} \hat{Q}^{(+)}_{\lambda\mu_x=0,\pm 2} &= \langle \Omega | \hat{Q}^{(+)}_{\lambda\mu_x=0,2} | \Omega \rangle\, \delta_{\lambda,2} + \sum_{kl} \{\mathcal{M}^{\lambda \pm \mu_x}_{kl} b^{+}_{kl} + (-1)^{\lambda}\, \mathcal{M}^{\lambda \mp \mu_x}_{kl} b_{kl}\} \\ \hat{Q}^{(-)}_{\lambda\mu_x=\pm 1,\pm 3} &= c \sum_{kl} \{\mathcal{M}^{\lambda \pm \mu_x}_{kl}\, b^{+}_{kl} + \mathcal{M}^{\lambda \mp \mu_x}_{kl} b_{kl} + \mathcal{M}^{\lambda \pm \mu_x}_{\bar{k}\bar{l}} b^{+}_{\bar{k}\bar{l}} + \mathcal{M}^{\lambda \mp \mu_x}_{\bar{k}\bar{l}} b_{\bar{k}\bar{l}}\} \end{aligned} \tag{76}$$

where $-\lambda \leqq \mu_x \leqq \lambda$ and $c = \begin{cases} +1 & \lambda = 2 \\ -1 & \lambda = 1,3 \end{cases}$, $\mathcal{M}^{\lambda\mu}_{kl}$, $\mathcal{M}^{\lambda\mu}_{\bar{k}\bar{l}}$, $\mathcal{M}^{\lambda\mu}_{k\bar{l}}$ are quasi-particle matrix elements. Their explicite forms are given in appendix. The expressions (76) have be to added by condition

$$\hat{Q}^{(\pm)+}_{\lambda\mu_x} = (-1)^{\mu_x}\, \hat{Q}^{(\pm)}_{\lambda-\mu_x} \tag{77}$$

Here the symbols $(\pm)$ above the operator indicates its signature. Substituting space completeness condition (66) for all parts (21) of Hamiltonian (20) into (77) the operators $\hat{Q}^{(\pm)}_{\lambda\mu_x}$ can be expressed by the normal RPA modes $(X_{\nu_\pm}, \mathscr{P}_{\nu_\pm})$, spurious modes $(\Theta_x(1), J_x(1))$, $(\Theta_\tau(1), N_\tau(1))$, $(J_y(1), J_z(1))$, $(X(1), P_x(1))$ and the modes $(Y^{(1)}, P^{(1)}_y)$, $(Z(1), P_z(1))$. In the expressions thus obtained there are the commutators $[\hat{Q}^{(\pm)}_{\lambda\mu_x}, J_x(1)]$, $[\hat{Q}^{(\pm)}_{\lambda\mu_x}, J^{(1)}_{y.z}]$, $[\hat{Q}^{(\pm)}_{\lambda\mu_x}, P_x(1)]$ and $[\hat{Q}^{(\pm)}_{\lambda\mu_x}, P^{(1)}_{y.z}]$ which can be substituted by the corresponding linear combinations of multipole operator mean values in the yrast states (see [19, 40, 43. 44]). Further, according to prescription of Marshalek (see [30]), in these expressions one has to replace $\Theta_x(1) \to \hat{\Theta}_x$, $\Theta_\tau(1) \to \hat{\Theta}_\tau, X(1) \to \hat{X}, N_{\tau=N}(1) \to \hat{N}_N - N_0$, $N_{\tau=z}(1) \to \hat{N}_z - Z_0$, $J_x(1) \to \hat{J}_x - J_0$, $P_x(1) \to \hat{P}_x$ in order to get the relations for $\hat{Q}_{\lambda\mu_x}(\pm)$ in the lab system in which the wave function (70) acts. The obtained expressions can be used in (73) for determination of $\hat{Q}'_{\lambda\mu_x}$ acting in the intrinsic fixed-body system (i.e. in the space of the function $|\alpha J\rangle$, see (71b)).

For this purpose, one has to express the Wigner function $\mathscr{D}^{\lambda}_{\mu\nu}$ in operator form in terms of the operators $J_x(1)$, $\Theta_x(1)$, Γ, Γ^+ and to make the replacement $\Theta_x(1) \to$ $\to \hat{\Theta}_x$, $J_x(1) \to \hat{J}_x - J_0$ (see [30]). As a result, we obtain

$$\text{(78a)} \qquad \hat{Q}^{(+)'}_{\lambda\mu_x=0} = \sum_\nu A^{(\lambda 0)}_\nu (\mathscr{O}^+_\nu + (-1)^\lambda \mathscr{O}_\nu) + \delta_{\lambda,2}\{\langle\Omega|\, \hat{Q}^{(+)}_{2\mu_x=0}|\Omega\rangle +$$

$$+ [\hat{Q}^{(+)}_{2\mu_x=0}, i\hat{\Theta}_N](N - N_0) + [\hat{Q}^{(+)}_{2\mu_x=0}, i\Theta_z](Z - Z_0) + [\hat{Q}^{(+)}_{2\mu_x=0}, i\Theta_x](J - J_0)\}$$

$$\text{(78b)} \qquad \hat{Q}^{(\sigma)'}_{\lambda\mu_x=1,2,3} = \sum_\nu (A^{(\lambda\mu_x)}_\nu \mathscr{O}^+_\nu + A^{(\lambda-\mu_x)}_\nu \mathscr{O}_\nu) + \delta_{\lambda,2}\{\langle\Omega|\, \hat{Q}^{(+)}_{2\mu_x=2}|\Omega\rangle +$$

$$+ [\hat{Q}^{(+)}_{2\mu_x=2'}, i\hat{\Theta}_N](N - N_0) + [\hat{Q}^{(+)}_{2\mu_x=2'}, i\hat{\Theta}_z](Z - Z_0) + [\hat{Q}^{(+)}_{2\mu_x=2}, i\Theta_x](J - J_0)\}$$

where $\sigma = +$ for $\mu_x = 2$, $\sigma = -$ for $\mu_x = 1, 3$. The matrices $A^{\lambda\mu_x}_\nu$ have the following form

$$A^{\lambda\mu_x=0}_\nu = \sum_{ik} \mathscr{M}^{\lambda\mu=0}_{i\bar{k}} (\varphi^\nu_{i\bar{k}}) + \psi^\nu_{i\bar{k}})$$

$$\text{(79)} \qquad A^{\lambda\mu_x=\pm1,\pm2,\pm3}_\nu = 2\sum_{ik} (\mathscr{M}^{\lambda\mu_x}_{ik}\varphi^\nu_{ik} - \mathscr{M}^{\lambda-\mu_x}_{ik}\psi^\nu_{ik} + \mathscr{M}^{\lambda\mu_x}_{i\bar{k}}\varphi^\nu_{i\bar{k}} - \mathscr{M}^{\lambda-\mu_x}_{i\bar{k}}\psi^\nu_{i\bar{k}})$$

$$(A^{\lambda\mu}_\nu)^* = (-1)^{\lambda+\mu} A^{\lambda\mu}_\nu, \quad -\lambda \leqq \mu \leqq \lambda$$

and the meaning of $\psi^\nu_{ik(i\bar{k})}$, $\varphi^\nu_{ik(i\bar{k})}$, $\psi^\nu_{i\bar{k}}$, $\varphi^\nu_{i\bar{k}}$ follows from the phonon relations

$$\text{(80)} \qquad \mathscr{O}^+_{\nu_+} = \sum_{ik} (\psi^\nu_{i\bar{k}} b^+_{i\bar{k}} + \varphi^\nu_{i\bar{k}} b_{i\bar{k}})$$

$$\mathscr{O}^+_{\nu_-} = \sum_{ik} (\psi^\nu_{ik} b^+_{ik} + \varphi^\nu_{ik} b_{ik} + \psi^\nu_{i\bar{k}} b^+_{i\bar{k}} + \varphi^\nu_{i\bar{k}} b_{i\bar{k}})$$

Static nuclear moments are defined by mean values of the operators $\hat{Q}^{(\pm)'}_{\lambda\mu_x}$ in the state $|\alpha J\rangle$. Using (71b) and (78) nuclear moments can be written in the form of Taylor expansion

(81) $$eQ_{2\mu_x=0,2} = \left\langle \alpha J \left| \sqrt{\frac{16\pi}{S}}\, \hat{Q}^{(+)}_{2\mu_x=0,2} \right| \alpha J \right\rangle =$$

$$= \left\langle \Omega \left| \sqrt{\frac{16\pi}{S}}\, \hat{Q}^{(+)'}_{2\mu_x=0,2} \right| \Omega \right\rangle + \left(\frac{\partial Q_{2\mu_x}}{\partial N_{N=N_0}}\right)(N - N_0) + \left(\frac{\partial Q_{2\mu_x}}{\partial Z_{Z=Z_0}}\right)(Z - Z_0) +$$

$$+ \left(\frac{\partial Q_{2\mu_x}}{\partial J_{J=J_0}}\right)(J - J_0)$$

where the following assignments are introduced

(82) $$\left(\frac{\partial Q_{2\mu_x}}{\partial N_{N=N_0}}\right) = \sqrt{\frac{16\pi}{S}}\,[\hat{Q}^{(+)}_{2\mu_x}, i\hat{Q}_N] \qquad \left(\frac{\partial Q_{2\mu_x}}{\partial Z_{Z=Z_0}}\right) = \sqrt{\frac{16\pi}{S}}\,[\hat{Q}^{(+)}_{2\mu_x}, i\hat{Q}_Z]$$

$$\left(\frac{\partial Q_{2\mu_x}}{\partial J_{J=J_0}}\right) = \sqrt{\frac{16\pi}{S}}\,[\hat{Q}^{(+)}_{2\mu_x}, i\hat{Q}_x]$$

It must be noted that the moments in (81) are determined with respect to axis x. The expressions for nuclear moments in the system with quantum axis z can be obtained by means of the transformation (75).

Substitution of (74) into (72) and using of the asymptotic expressions for Clebsch-Gordan coefficients ($J_1, J_2 \gg \lambda$) yields ($\hat{\mathcal{M}}(E\lambda, \mu) = e^{\lambda}_{\text{eff}}\hat{Q}^{(\pm)}_{2\mu_x}$)

(83) $$B(E\lambda; \alpha_1 J \to \alpha_2 J - \nu) = \left|\langle J - \nu, \alpha_2 \right| e^{\nu}_{\text{eff}} \hat{Q}^{(\pm)'}_{\lambda\mu_x=-\nu} \left| J\alpha_1 \right\rangle\right|^2$$

where depending on whether the signature in the given transition changes or not, we have to take a $(-)$ or $(+)$ index in (83), respectively. Symbol e^{λ}_{eff} in (83) means the multiplying of single additive components of moment $\hat{Q}^{(+)'}_{\lambda\mu_x}$ corresponding to given type of particle by effective charge. The relations (78) can be substituted into (83), and we obtain all searched for E1, E2, E3 reduced probabilities.

For transitions not changing the number of phonons (i.e. transition along the rotational bands including the yrast band), one can write

(84) $$B(E\,2; \alpha J \to \alpha J - \nu) = \left|\langle \Omega \right| \tilde{Q}^{(+)}_{2\mu_x=2} \left|\Omega\right\rangle + \left[\tilde{Q}^{(+)}_{2\mu_x=2}, i\Theta_N\right](N - N_0) +$$

$$+ \left[\tilde{Q}^{(+)}_{2\mu_x=2}, i\Theta_z\right](Z - Z_0) + \left[\tilde{Q}^{(+)}_{2\mu_x=2}, i\Theta_x\right](J - J_0)\Big|^2$$

where $\tilde{Q}^{(+)}_{2\mu_x} = e^{(2)}_{\text{eff}}\hat{Q}^{(+)'}_{2\mu_x}$. It is evident that in the normal case, when the HFB and RPA problems are solved in the framework of one nuclei and when the intrinsic structure of the states $|\alpha J\rangle$ does not drastically depend on J, one can put $N = N_0$, $Z = Z_0$, $J = J_0$ (see 30).

The transition changing the number of phonons by 1 (extraband transitions) can be divided according to the angular momentum change

i) for transitions with $\Delta J = 0$

(85) $$B(E\lambda; n_\lambda J_0 \to n_\lambda \pm 1 J_0) = \left|\tilde{\Lambda}^{\lambda 0}_{\nu_+}\right|^2 \times \begin{Bmatrix} n_{\nu_+} \\ n_{\nu_+} \end{Bmatrix} + 1$$

where $\lambda = 1, 2, 3$, $\tilde{\Lambda} = e_{\text{eff}}\Lambda$

ii) for transitions with $\Delta J = 1$

$$B(E\lambda, n_\nu J_0 \to n_\nu \pm 1 J_0 - 1) = |\tilde{A}^{\lambda-1}_{\nu\sigma}|^2 \times \begin{cases} n_{\nu\sigma} + 1 \\ n_{\nu\sigma} \end{cases} \tag{86}$$

where $\lambda = 1, 2, 3$, $\sigma = +$ for $\lambda = 3$, $\sigma = -$ for $\lambda = 1, 2$

iii) for transitions with $\Delta J = 2$

$$B(E\lambda, n_\lambda J_0 \to n_\nu \pm 1 J_0 - 2) = |\tilde{A}^{\lambda\mu_\varkappa = \mp 2}_{\nu+}|^2 \times \begin{cases} n_{\nu+} + 1 \\ n_{\nu+} \end{cases} \tag{87}$$

iv) for transitions with $\Delta J = 3$

$$B(E\lambda, n_\lambda J_0 \to n_\nu \pm J_0 - 2) = |\tilde{A}^{\lambda\mu_\varkappa = \mp 3}_{\nu-}|^2 \times \begin{cases} n_\nu + 1 \\ n_\nu \end{cases} \tag{88}$$

In (85)–(88) the index $\nu\sigma$ characterises the signature of phonon by which the initial state differs from the final state in the given transition. It is possible to expect that all transitions with a change of the number of phonons (extraband transitions) are weaker than the transitions along one rotational band, since the reduced probabilities of one-phonon transitions are proportional to the amplitudes $|A^{\lambda\mu_\varkappa}_{\nu\sigma}|^2$ (see (85)–(88)) which are less by a factor of boson decomposition than the mean values of multipole operators $\langle\Omega|\,\hat{Q}_{\nu\mu_\varkappa}|\Omega\rangle$ characterizing the intraband transitions.

In the end of this section one has to note that the expressions (85)–(89) have been obtained in high spin limit $J_1, J_2 \gg \lambda$. However in many papers (see e.g. [9, 86]) the arguments are given for possibility of applying the CHFB approaches for description of low-lying and low-spin states. In this case the relation (76) is used without asymptotic form of Clebsch-Gordan coefficients and corresponding expressions for reduced transition probabilities can be found in [87].

2.3.5. Strength function method

In order to obtain the information on transition probabilities between discrete levels in the framework of SCCM + RPA approach it is necessary to know the structure of corresponding initial and final states. For it one has to solve the corresponding secular equation (56) and system of equations (46). However the investigation of single solutions of secular equations in the region of high excitations (f.i. giant resonances) loses its sense because the density of levels in this region of spectrum grows up very hardly and experimental information has the statistical character. Therefore it is used to apply the method of strength function [31] for description of electromagnetic transition in the high excited states region. Strength function method has been used with succes in analysis of giant resonances of nonrotating nuclei [88, 89, 41]. Further we give the basic ideas of this method in the case of rotating systems (see [77]).

Let us examine the quantity $b(\omega_i)$ which has physical meaning in the points ω_i satisfying the following equation

(89) $$\mathscr{F}(\omega_i) = 0$$

The function $b(\omega_i)$ can be for instance the probability of transition of given type and multipolarity from one-phonon states into yrast line. In this case ω_i represent energies of phonons and equation (89) coincides with (57). The strength function of quantity $b(\omega)$ is defined as an expectation value of this quantity averaged in some energetical interval Δ in neighbourhood of point ω_i (see [31])

(90) $$b_\Delta(\omega) = \sum_i b(\omega_i)\, \varrho_\Delta(\omega - \omega_i)$$

where $\varrho_\Delta(\omega - \omega_i)$ is weight function normalised to unity with maximum in points $\omega - \omega_i = 0$. Weight function is usual taken in the form

(91) $$\varrho_\Delta(\omega - \omega_i) = \frac{1}{2\pi} \frac{\Delta}{(\omega - \omega_i)^2 + \Delta^2/4}$$

Considering all solutions of (89) nondegenerate, i.e. $(\partial\mathscr{F}/\partial\omega_{\omega=\omega_i}) \neq 0$, one can introduce the function $P(\omega)$ as follows

(92) $$b(\omega_i) = \left(\frac{P(\omega)}{\dfrac{\partial\mathscr{F}(\omega)}{\partial\omega_{\omega=\omega_i}}} \right)$$

If function $P(z)/\mathscr{F}(z)$ as a function of complex variable z hasn't any peculiarities except of simple poles in zeroes of $\mathscr{F}(z)$ and if

$$\lim_{z\to\infty} P(z)/\mathscr{F}(z) = 0$$

then using the theorem of Cauchy it is possible to write

(93) $$b_\Delta(\omega) = \frac{1}{\pi} \operatorname{Im} \frac{P(\omega + i\frac{1}{2}\Delta)}{\mathscr{F}(\omega + i\frac{1}{2}\Delta)}$$

Let us exam the question of construction of strength function for transitions from one-phonon states into yrast line states. With this aim we write the motion equations (44) in more symmetric form. Instead of unknowns (45) we introduce the following ones (we restrict ourselves to the case $\omega_\nu \neq 0$)

(94) $$\mathscr{R}^{(+)}_{s_2}(\nu) = \sqrt{(\omega_\nu)}\, \varkappa_{s_2} A^\nu_{s_2} \qquad \mathscr{R}^{(-)}_{s_1}(\nu) = -\frac{1}{\sqrt{(\omega_\nu)}} \varkappa_{s_1} B^\nu_{s_1}$$

Then (44) has the form

(95) $$X^\nu_\mu = \frac{E_\mu}{E^2_\mu - \omega^2_\nu} \frac{1}{\sqrt{\omega_\nu}} \sum_{s_2} \mathscr{R}^{(+)}_{s_2}(\nu)\, W^{s_2}_\mu + \frac{1}{E^2_\mu - \omega^2_\nu} \sqrt{\omega_\nu} \sum_{s_1} \mathscr{R}^{(-)}_{s_1}(\nu)\, V^{s_1}_\mu$$

$$\mathscr{P}^\nu_\mu = \frac{\omega^2_\nu}{E^2_\mu - \omega^2_\nu} \frac{1}{\sqrt{\omega_\nu}} \sum_{s_2} \mathscr{R}^{(+)}_{s_2}(\nu)\, W^{s_2}_\mu + \frac{E_\mu}{E^2_\mu - \omega^2_\nu} \sqrt{\omega_\nu} \sum_{s_1} \mathscr{R}^{(-)}_{s_1}(\nu)\, V^{s_1}_\mu$$

The system of equations (46) can be then written as

$$(96)\qquad \sum_{s_2}\mathscr{R}^{(+)}_{s_2}(\nu)\left(S_{W_{s(2)}W_{s'(2)}}-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right)+\omega_\nu\sum_{s_1}\mathscr{R}^{(-)}_{s_1}(\nu)\,\mathscr{U}_{V_{s(1)}W_{s'(2)}}=0$$

$$\omega_\nu\sum_{s_2}\mathscr{R}^{(+)}_{s_2}(\nu)\mathscr{U}_{V_{s'(1)}W_{s(2)}}+\sum_{s_1}\mathscr{R}^{(-)}_{s_1}(\nu)\left(S_{V_{s(1)}V_{s'(1)}}+\frac{\delta_{s_1s_1'}}{4\varkappa_{s_1}}\right)=0$$

So we have obtained the system of homogeneous equations for the vectors $\mathscr{R}^{(\mp)}$

$$(97)\qquad \sum_{s'}\mathscr{S}_{ss'}\mathscr{R}_{s'}=0$$

where

$$\mathscr{S}=\begin{pmatrix} S_{W_{s(2)}W_{s'(2)}}-\dfrac{\delta_{s_2s_2'}}{4\varkappa_{s_2}} & \omega\mathscr{U}_{V_{s(1)}W_{s(2)}} \\ \omega\mathscr{U}_{V_{s(1)}W_{s(2)}} & S_{V_{s(1)}V_{s'(1)}}+\dfrac{\delta_{s_1s_1'}}{4\varkappa_{s_1}}\end{pmatrix},\qquad \mathscr{R}=\begin{pmatrix}\mathscr{R}^{(+)}_{s_2}\\ \mathscr{R}^{(-)}_{s_1}\end{pmatrix}$$

The normalization condition from (43) with inclusion of (95) yields

$$\sum_\mu X^\nu_\mu\mathscr{P}^\nu_\mu=\sum_\mu\frac{1}{(E^2_\mu-\omega^2_\nu)}\{E_\mu\omega_\nu(\sum_{s_2s_2'}\mathscr{R}^{(+)}_{s_2}(\nu)\,\mathscr{R}^{(+)}_{s_2'}(\nu)\,W^{s_2}_\mu\,W^{s_2'}_\mu+$$

$$+\sum_{s_1s_1'}\mathscr{R}^{(-)}_{s_1}(\nu)\,\mathscr{R}^{(-)}_{s_1'}(\nu)\,V^{s_1}_\mu\,V^{s_1'}_\mu)+(E^2_\mu+\omega^2_\nu)\sum_{s_1s_1'}\mathscr{R}^{(+)}_{s_2}(\nu)\,\mathscr{R}^{(-)}_{s_2}(\nu)\,V^{s_1}_\mu V^{s_2}_\mu\}=$$

$$(98)\qquad =\tfrac{1}{2}\sum_{s_1s_1'=1}^{n=n_1+n_2}\mathscr{R}_s(\nu)\,\mathscr{R}_s(\nu)\,\frac{\partial\mathscr{S}_{ss'}}{\partial\omega_{\omega=\omega_\nu}}\Bigg/=\frac{1}{2}$$

We take over one term to right side of Eq. (97), say e.g. term containing $\mathscr{R}_n(\nu)$, then one obtains

$$(99)\qquad \sum_{s'=1}^{n-1}\mathscr{S}_{ss'}(\nu)\,\mathscr{R}_{s'}(\nu)=-\mathscr{S}_{sn}\mathscr{R}_n(\nu)\Rightarrow\sum_{s'=1}^{n-1}\mathscr{S}_{ss'}a_{s'}=-\mathscr{S}_{sn}$$

where

$$(100)\qquad a_{s'}=\frac{\mathscr{R}_{s'}}{\mathscr{R}_n}\Rightarrow\mathscr{R}_{s'}=a_{s'}\mathscr{R}_n$$

Substitution of (100) into normalization condition (98) gives

$$(101)\qquad \mathscr{R}^2_n\sum_{ss'}a_sa_{s'}\,\frac{\mathscr{S}\partial_{ss'}}{\partial\omega_{\omega=\omega_\nu}}\Bigg/=1$$

Secular equation corresponding to system of equations (97) can be expressed in the form of expansion of determinant $|\mathscr{S}|=0$ into its algebraic complements

$$(102)\qquad |\mathscr{S}|=\sum_{s,s'=1}^{n}\mathscr{S}_{ss'}A_{ss'}=0\Rightarrow\sum_{s'=1}^{n-1}\mathscr{S}_{ss'}\frac{A_{ss'}}{A_{sn}}=-\mathscr{S}_{sn}$$

where s is arbitrary index from 1, ..., n. Comparing (102) with (99) one obtains

$$a_{s'} = \frac{A_{ss'}}{A_{sn}} \tag{103}$$

Since s is arbitrary, $s = n$ can be chosen

$$a_{s'} = \frac{A_{ns'}}{A_{nn}} \tag{104}$$

Substitution of (104) into (101) yields

$$\mathscr{R}_n^2 = \frac{A_{nn}}{\sum\limits_{ss'} A_{ss'} \dfrac{\mathscr{S}\partial_{ss'}}{\partial\omega_{\omega=\omega_\nu}}\Big/} \tag{105}$$

The total determinant $|\mathscr{S}|$ is defined as

$$|\mathscr{S}| = \sum_P (-1)^P \mathscr{S}_{1p_1} \dots \mathscr{S}_{sp_s} \dots \mathscr{S}_{np_\mu}$$

$$\Rightarrow \frac{\partial|\mathscr{S}|}{\partial\omega} = \sum_{sp} (-1)^P \mathscr{S}_{1p_1} \dots \frac{\partial\mathscr{S}_{sp_s}}{\partial\omega} \dots \mathscr{S}_{np_n} =$$

$$= \sum_s \begin{vmatrix} \mathscr{S}_{11} & \dots & \mathscr{S}_{1n} \\ \dfrac{\partial\mathscr{S}_{s1}}{\partial\omega} & \dots & \dfrac{\mathscr{S}\partial_{3n}}{\partial\omega} \\ \vdots & & \vdots \\ \mathscr{S}_{n1} & \dots & \mathscr{S}_{nn} \end{vmatrix} = \sum_{ss'} A_{ss'} \frac{\partial\mathscr{S}_{ss'}}{\partial\omega}, \tag{107}$$

Then eventually one obtains

$$\mathscr{R}_n^2 = \frac{A_{nn}}{\left.\dfrac{\partial\mathscr{S}}{\partial\omega}\right|_{\omega=\omega_\nu}} \tag{108}$$

The remained unknowns $\mathscr{R}_s$ can be determined from (100) and (104). The generalized coordinates X_μ^ν and moments $\mathscr{P}_\mu^\nu$ are able to expressed in terms of $\mathscr{R}_n$ (see (95))

$$\begin{aligned} X_\mu^\nu &= \mathscr{R}_n \left\{ \frac{E_\mu}{E_\mu^2 - \omega_\nu^2} \frac{1}{\sqrt{\omega_\nu}} \sum_{s_2} a_{s_2}(\nu)\, W_\mu^{s_2} + \frac{\sqrt{\omega_\nu}}{E_\mu^2 - \omega_\nu^2} \sum_{s_1} a_{s_1}(\nu)\, V_\mu^{s_1} \right\} \\ \mathscr{P}_\mu^\nu &= \mathscr{R}_n \left\{ \frac{\omega_\nu^2}{E_\mu^2 - \omega_\nu^2} \frac{1}{\sqrt{\omega_\nu}} \sum_{s_2} a_{s_2}(\nu)\, W_\mu^{s_2} + \frac{E_\mu \sqrt{\omega_\nu}}{E_\mu^2 - \omega_\nu^2} \sum_{s_1} a_{s_1}(\nu)\, V_\mu^{s_1} \right\} \end{aligned} \tag{109}$$

and corresponding phonon amplitudes ψ and φ (see (80))

$$\begin{aligned} \psi_\mu^\nu &= \frac{1}{\sqrt{2}} \left((\sqrt{(\omega_\nu)}\, X_\mu^\nu + \frac{1}{\sqrt{\omega_\nu}}\, \mathscr{P}_\mu^\nu \right) = \frac{1}{\sqrt{2}} \frac{\mathscr{R}_n}{E_\mu - \omega_\nu} \left[\sum_{s_2} a_{s_2}(\nu)\, W_\mu^{s_2} + \sum_{s_1} a_{s_1}(\nu)\, V_\mu^{s_1} \right] \\ \varphi_\mu^\nu &= \frac{1}{\sqrt{2}} \left(\sqrt{\omega_\nu}\, X_\mu^\nu - \frac{1}{\sqrt{\omega_\nu}}\, \mathscr{P}_\mu^\nu \right) = \frac{1}{\sqrt{2}} \frac{\mathscr{R}_n}{E_\mu + \omega_\nu} \left[\sum_{s_2} a_{s_2}(\nu)\, W_\mu^{s_2} - \sum_{s_1} a_{s_1}(\nu)\, V_\mu^{s_1} \right] \end{aligned} \tag{110}$$

Since the amplitudes ψ and φ of one-phonon states are proportional to $\mathscr{R}_n$ the arbitrary quantity quadratic in these amplitudes (e.g. the probabilities of electric transitions (85)–(88) from one-phonon states into yrast line) contains the factor (108) making possible to use the strength function [77] (compare (108) with (92)).

As an example we can give the strength function of reduced probability of electric dipole transition from one-phonon state into the state of yrast line without change of spin $\Delta J = 0$. Reduced probability of such transitions is determined by (85)

$$B(E1; n_\nu = 1, J \to n_\nu = 0, J) = \left|\sum_\mu \mathscr{M}_\mu^{10}(\psi_\mu^\nu + \varphi_\mu^\nu)\right|^2 \tag{111}$$

Substitution of (110) into (111) gives

$$B(E1; n_\nu = 1, J \to n_\nu = 0, J) = \tag{112}$$

$$= 2A_{nn}\left|\sum_\mu \mathscr{M}_\mu^{10}\left(\frac{E_\mu}{E_\mu^2 - \omega_\nu^2}\sum_{s_2} a_{s_2} W_\mu^{s_2} + \frac{\omega_\nu}{E_\mu^2 - \omega_\nu^2}\sum_{s_1} a_{s_1} V_\mu^{s_1}\right)\right|^2 \Big/ \left(\frac{\partial \mathscr{S}}{\partial \omega}\right)_{\omega=\omega_\nu}$$

Comparison of (112) with (92) yields the expression for the function $P(\omega)$

$$P_{E1,\Delta J=0}(\omega) = 2A_{nn}\left|\sum_\mu \mathscr{M}_\mu^{(10)}\left(\frac{E_\mu}{E_\mu^2 - \omega_\nu^2}\sum a_{s_2} W_\mu^{s_2} + \frac{\omega_\nu}{E_\mu^2 - \omega_\nu^2}\sum a_{s_1} V_\mu^{s_1}\right)\right|^2 \tag{113}$$

where a_{s_2} and a_{s_1} are determined by (103). Corresponding expression for strength function is then given by definition (93) and (113). The relations for strength function of reduced probabilities of arbitrary multipolarity with arbitrary change of spin can be obtained by analogous way.

3. Selfconsistency of residual interactions with average field

As was mentioned in preceeding sections in practical calculations of particular properties of rotating nuclei it is usual to start with phenomenologically chosen deformed average field which violates the symmetries (2) of the total Hamilotnian. In order to restore these symmetries one has to choose the appropriate form of residual interactions.

In this section the problem of restoration of symmetries of total Hamiltonian is discussed and the prescription for construction of residual interactions is proposed [131]. This prescription is based on the ideas of Pyatov [26, 27, 91–94]. The restoration symmetry of Hamiltonian of nonrotating nucleus is discussed in [28], the restoration of translational symmetry of rotating nucleus Hamiltonian is analysed in [24].

So the starting Hamiltonian is supposed to have the following form (compare with (5))

$$H = H_{AV} - \tfrac{1}{4}\sum_\tau G_\tau \hat{P}_\tau^+ \hat{P}_\tau + V_{\text{res}}^{(+)} + V_{\text{res}}^{(-)} \tag{144}$$

Here the separable residual interaction is written in the form of summ of two parts according to parity of single-particle operators involved in each of these parts. Since in the framework of RPA the components of linear momentum commutes with the components of angular momentum and with the pairlng operator $\hat{P}_\tau$ the part $V_{\text{res}}^{(+)}$ can be obtained from the restoration of rotational symmetry while $V_{\text{res}}^{(-)}$ is constructed from requirement of translational symmetry of total Hamiltonian.

3.1. The restoration of rotational symmetry of total Hamiltonian

The nuclear average field H_{AV} is supposed to be of the following isotopic structute (see e.g. [61])

$$H_{AV} = T_{\text{kin}} + \sum_{i=1}^{z} U^{(p)}(\vec{r}_i, \beta_\Omega) + \sum_{i=1}^{N} U^{(n)}(\vec{r}_i, \beta_\Omega) \tag{115}$$

where T_{kin} is the kinetic energy operator. The proton and neutron potentials can be written as

$$U^{(p)}(\vec{r}, \beta_\Omega) = U^{[0]}(\vec{r}, \beta_\Omega) + U^{[1]}(\vec{r}, \beta_\Omega)\,\tau_z + U_{ls}^{(p)}(\vec{r}, \beta_\Omega)\,U_{\text{Coul}}, \vec{r}, \beta_\Omega) \tag{116}$$

$$U^{(n)}(\vec{r}, \beta_\Omega) = U^{[0]}(\vec{r}, \beta_\Omega) + U^{(1]}(\vec{r}, \beta_\Omega)\,\tau_z + U_{ls}^{(n)}(\vec{r}, \beta_\Omega)$$

Here $U^{[0]}(\vec{r}, \beta_\Omega)$ and $U^{[1]}(\vec{r}, \beta_\Omega)$ represent the isoscalar and isovector potential, respectively. $U_{ls}(\vec{r}, \beta_\Omega)$ is the spin-orbital interaction potential and U_{Coul} is a Coulomb term. The symbol β_Ω denotes the set of deformation parameters and τ_z is the z component of isospin for a given nucleon. Since the deformation β_Ω depends on rotational frequency Ω, the index Ω is prescribed to β. In what follows, we restrict ourselves to the restoration of rotational and translational symmetries violated by the isoscalar and isovector parts of (116). The Coulomb term can be treated in the same manner. The restoration of translational and rotational symmetries violated by the spin-orbital interaction is quite a complicated problem. Nevertheless, one may expect that spin-orbital term is not substantional for investigation of properties of the state near the yrast line [26]. Therefore, the spin-orbital term will be omitted.

The isoscalar and isovector potentials are usually taken in the form [61]

$$U^{[0]}(\vec{r}, \beta_\Omega) = -V_0 f^{[0]}(\vec{r}, \beta_\Omega) \quad U^{[1]}(\vec{r}, \beta_\Omega) = V_1 f^{[1]}(\vec{r}, \beta_\Omega) \tag{117}$$

where V_0 and V_1 are the depths of corresponding potential wells, and $f^{[1]}(\vec{r}, \beta_\Omega)$ are the functions depending on the nucleus form. For instance, in the case of a deformed Saxon-Woods potential $f^{[\tau]}(\vec{r}, \beta_\Omega)$ is represented by the Fermi distribution [61]. Since the components of angular momentum J_i commute with the pairing term H_{pair} in the Hamiltonian H, the rotational symmetry (2) is violated only by a deformed average field H_{AV}, and H_{pair} needn't be taken into account in restoring rotational symmetry (2).

For further consideration it is convenient to express the single-particle potential in the form of the expansion in spherical harmonics

$$f^{[\tau]}(\vec{r}, \beta_\Omega) = \sum_{\substack{L=0 \\ \text{even}}}^{\infty} \sum_{M=-L}^{L} f^{[\tau]}_{LM}(\vec{r}, \beta_\Omega)\, Y_{LM}(\hat{r}) \quad \tau = 0, 1 \tag{118}$$

where $f^{[\tau]}_{LM}(r, \beta_\Omega) = f^{[\tau]}_{L-M}(r, \beta_\Omega)$. As a consequence of the cranking Hamiltonian invariance with respect to rotation $R_x(\pi)$ and space-axis inversion, only the even values of L remain in (118). Therefore, the nuclear average potential can be written as sum of the spherical symmetric part $U^{[\tau]}_0(\beta_\Omega)$, corresponding to the term with $L = 0$ in (118), and the deformed nonspherical part $\delta U^{[\tau]}(\beta_\Omega)$

$$U^{[\tau]}(\vec{r}, \beta_\Omega) = U^{[\tau]}_0(\beta_\Omega) + \delta U^{[\tau]}(\beta_\Omega) \tag{119}$$

$$\delta U^{[\tau]}(\beta_\Omega) = \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{M=-L}^{L} Y_{LM}(\hat{r}) \cdot \begin{cases} [-V_0 f^{[0]}_{LM}(r, \beta_\Omega)] \\ [V_1 f^{[1]}_{LM}(r, \beta_\Omega)\, \tau_z] \end{cases} \tag{120}$$

It is evident that the Hamiltonian rotational symmetry is violated by the nonspherical part of (119)

$$[U^{[\tau]}(\vec{r}, \beta_\Omega), J_\nu] = [\delta U^{[\tau]}(\beta_\Omega), J_\nu] \neq 0 \tag{121}$$

For every value of L and M of expansion (118) we can construct the system of operators

$$f^{[\tau]}_{LM}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r}) \quad \mu = -L, \ldots, L \tag{122}$$

which obeys the commutation relation with the angular momentum

$$[f^{[\tau]}_{LM}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r}), J_\nu] = \sqrt{[L(L+1)]}\, (L\mu\, 1\nu \mid L\mu + \nu) f^{[\tau]}_{LM}(r, \beta_\Omega)\, Y_{L\mu+\nu}(\hat{r}) \tag{123}$$

Instead of the system of operators (122) one can take a wider class of operators

$$g^{[\tau]}_{LM}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r}) \tag{124}$$

where $g^{[\tau]}_{LM}(r, \beta_\Omega)$ is an arbitrary linear combination of the radial function

$$g^{[\tau]}_{LM}(r, \beta_\Omega) = \sum_{\substack{M'=0 \\ \text{even}}}^{L} a^{[\tau]}_{MM'}(L) f^{[\tau]}_{LM'}(r, \beta_\Omega) \tag{125}$$

The ensemble of the operators (125) is used for construction of the residual interaction $V^{(+)}_{\text{Res}}$. Introducing the operators

$$F^{[0]}_{L\mu}(M) = g^{[0]}_{LM}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r}) \quad \vec{F}^{[1]}_{L\mu}(M) = q^{[1]}_{LM}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r})\, \vec{\tau} \tag{126}$$

the residual interaction $V^{(+)}_{\text{Res}}$ can be searched for in the form

$$V^{(+)}_{\text{Res}} = V^{[0]}_{\text{Res}}(\beta_\Omega) + V^{[1]}_{\text{Res}}(\beta_\Omega) \tag{127}$$

where

$$V_{\text{Res}}^{[0]}(\beta_\Omega) = -\delta U^{[0]}(\beta_\Omega) - \frac{V_0}{2} \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{\substack{M=0 \\ \text{even}}}^{L} \sum_{\mu=-L}^{L} F_{L\mu}^{[0]+}(M)\, F_{L\mu}^{[0]}(M)$$

(128)

$$V_{\text{Res}}^{[1]}(\beta_\Omega) = -\delta U^{[0]}(\beta_\Omega) + \frac{V_1}{2} \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{\substack{M=0 \\ \text{even}}}^{L} \sum_{\mu=-L}^{L} \vec{F}_{L\mu}^{[1]}(M)\, \vec{F}_{L\mu}^{[1]}(M)$$

Now the total Hamiltonian H (114) can be written as

$$H = T_{\text{kin}} + H_{\text{Pair}} + U^{[0]}(\vec{r}, \beta_\Omega) + U^{[1]}(\vec{r}, \beta_\Omega) + V_{\text{Res}}^{[0]}(\beta_\Omega) + V_{\text{Res}}^{[1]}(\beta_\Omega) + V_{\text{Res}}^{+} =$$

$$= T_{\text{kin}} + H_{\text{Pair}} + U_0^{[0]}(\beta_\Omega) + U_0^{[1]}(\beta_\Omega) - \frac{V_0}{2} \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{\substack{M=0 \\ \text{even}}}^{L} \sum_{\mu=-L}^{L} F_{L\mu}^{[0]}(M)\, F_{L\mu}^{[0]+}(M) +$$

$$+ \frac{V_1}{2} \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{\substack{M=0 \\ \text{even}}}^{L} \sum_{\mu=-L}^{L} \vec{F}_{L\mu}^{[1[}(M)\, \vec{F}_{L\mu}^{[1]}(M) + V_{\text{Res}}^{(-)}$$

As a consequence of the concrete form (128) of residual interaction, the nonspherical parts $\delta U^{[\tau]}(\beta_\Omega)$ cancel in (129), and therefore H is rotational-invariant, $[H, J_\nu] = 0$, for arbitrary constants $a_{MM'}^{[\tau]}(L)$ (because of $[V_{\text{Res}}^{(-)}, J_\nu]_{RPA} = 0$). The constants $a_{MM'}^{[\tau]}(L)$ will be determined from the conditions of self consistency of the residual interaction with the average field. The Hartree-Fock-Bogolubov averaging of (1) with H given in (129) must yield $\langle\Omega| H' |\Omega\rangle + \sum_j (E_j \alpha_j^+ \alpha_j + E_{\bar{j}} \alpha_{\bar{j}}^+ \alpha_{\bar{j}})$. This leads to the condition

$$\delta U^{[0]} = -\frac{V_0}{2} \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{\substack{M=0 \\ \text{even}}}^{L} \sum_{\mu=-L}^{L} \left[\langle\Omega| F_{L\mu}^{[0]+}(M) |\Omega\rangle F_{L\mu}^{[0]}(M) + \langle\Omega| F_{L\mu}^{[0]}(M) |\Omega\rangle F_{L\mu}^{[1]+}(M)\right]$$

(130)

$$\delta U^{[1]} = \frac{V_1}{2} \sum_{\substack{L=2 \\ \text{even}}}^{\infty} \sum_{\substack{M=0 \\ \text{even}}}^{L} \sum_{\mu=-L}^{L} \left[\langle\Omega| \vec{F}_{L\mu}^{[1]}(M) |\Omega\rangle \vec{F}_{L\mu}^{[1]}(M) + \langle\Omega| \vec{F}_{L\mu}^{[1]}(M) |\Omega\rangle \vec{F}_{L\mu}^{[1]}(M)\right]$$

where $\langle\Omega| F |\Omega\rangle$ is the mean value in the states of yrast line with a given rotational frequency Ω. It is reasonable to suppose that the single-particle density matrix $\varrho^{[\tau]}(\vec{r}, \beta_\Omega)$ has the same symmetry properties as the single-particle potentional of a deformed average field of s rotating nucleus. In that case one can write, in analogy with (118)

$$\varrho_\Omega^{[\tau]}(\vec{r}, \beta_\Omega) = \sum_{\substack{L=0 \\ \text{even}}}^{\infty} \sum_{M=-L}^{L} \varrho_{LM}^{[\tau]}(r, \beta_\Omega)\, Y_{LM}(\hat{r}) \tag{131}$$

where $\varrho_{LM}^{[\tau]}(r, \beta_\Omega) = \varrho_{L-M}^{[\tau]}(r, \beta_\Omega)$. Using (131) the mean values of the operators $F_{L\mu}^{[0]}(M)$ and $F_{L\mu}^{[1]}(M)$ have the form

(132) $$\langle\Omega|\, F^{[0]}_{L\mu}(M)\,|\Omega\rangle = \sum_{\substack{M'=L \\ \text{even}}}^{L} a^{[0]}_{MM'}(L)\, b^{[0]}_{M'M}(L)$$

$$\langle\Omega|\, \vec{F}^{[1]}_{L\mu}(M)\,|\Omega\rangle = \vec{\tau} \sum_{\substack{M'=0 \\ \text{even}}}^{L} a^{[1]}_{MM'}(L)\, b^{[1]}_{M'M}(L)$$

where

(133) $$b^{[\tau]}_{MM'} = \int f^{[\tau]}_{LM'}(r, \beta_\Omega)\, \varrho^{[\tau]}_{LM}(r, \beta_\Omega)\, r^2\, \mathrm{d}r$$

So, we have two expressions (130) and (120) for the nonspherical part of average potential (119). Comparison of these two expressions with the use of (125), (126), (132) and (133) gives the equations for unknown constants

(134) $$\tfrac{1}{2} \sum_{\substack{M'M'' \\ \text{even}}} \varkappa^{[\tau]}_{M'M''}(L)\left[b^{[\tau]}_{M'\mu}(L) f^{[\tau]}_{M'\mu}(r, \beta_\Omega) + b^{[\tau]}_{M''\mu}(L) f^{[\tau]}_{LM'}(r, \beta_\Omega)\right] = f^{[\tau]}_{L\mu}(r, \beta_\Omega)$$

Here $\varkappa^{[\tau]}_{MM'}(L)$ represents the real symmetric matrix

(135) $$\varkappa^{[\tau]}(L) = [a^{[\tau]}(L)]^T\, a^{[\tau]}(L)$$

One can suppose that the radial dependence of the average potential has the same character as the radial dependence of the mass distribution. Then, the radial function $\varrho^{[\tau]}_{LM}(r, \beta_\Omega)$ of the density matrix in (133) can be substituted by the corresponding radial function $f^{[\tau]}_{LM}(r, \beta_\Omega)$ of the average single-particle potential. In this case $b^{[\tau]}_{MM'}$ defined in (133) takes the form

(136) $$b^{[\tau]}_{MM'}(L) \to \tilde{b}^{[\tau]}_{MM'}(L) = \int f^{[\tau]}_{LM}(r, \beta_\Omega)\, f^{[\tau]}_{LM'}(r, \beta_\Omega)\, r^2\, \mathrm{d}r$$

and the self consistent equation (134) can be rewritten as

(137) $$\tfrac{1}{2} \sum_{\substack{M'M'' \\ \text{even}}} \tilde{\varkappa}^{[\tau]}_{M'M''}\left[\tilde{b}^{[\tau]}_{M'\mu}(L) f^{[\tau]}_{LM''}(r, \beta_\Omega) + \tilde{b}^{[\tau]}_{M''\mu}(L) f^{[\tau]}_{LM'}(r, \beta_\Omega)\right] = f^{[\tau]}_{L\mu}(r, \beta_\Omega)$$

The solution of (137) is

(138) $$\sum_{\substack{M'=0 \\ \text{even}}}^{L} \tilde{\varkappa}^{[\tau]}_{M'M''}(L)\, \tilde{b}^{[\tau]}_{M'\mu}(L) = \delta_{\mu M''}$$

hence

(139) $$\varkappa^{[\tau]}(L) \approx \tilde{\varkappa}^{[\tau]}(L) = [\tilde{b}^{[\tau]}(L)]^{-1}$$

So, the final expression for residual interaction $V^{(+)}_{\mathrm{Res}}$ restoring the rotational symmetry of the total Hamiltonian is

(140) $$V^{(+)}_{\mathrm{Res}} = V^{[0]}_{\mathrm{Res}}(\beta_\Omega) + V^{[1]}_{\mathrm{Res}}(\beta_\Omega)$$

where

$$V_{\mathrm{Res}}^{[0]}(\beta_\Omega) = -\delta U^{[0]}(\beta_\Omega) -$$

$$- \frac{V_0}{2} \sum_{L=0}^{\infty} \sum_{\mu=-L}^{L} \sum_{\substack{M'M''=0 \\ \text{even}}}^{L} [\tilde{b}^{[0]}(L)]^{-1}_{M'M''} (f^{[0]}_{LM'}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r}))^{+} (f^{[0]}_{LM''}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r}))$$

(141)

$$V_{\mathrm{Res}}^{[1]}(\beta_\Omega) = -\delta U^{[1]}(\beta_\Omega) +$$

$$+ \frac{V_1}{2} \sum_{L=0}^{\infty} \sum_{\mu=-L}^{L} \sum_{\substack{M'M''=0 \\ \text{even}}}^{L} [\tilde{b}^{[1]}(L)]^{-1}_{M'M''} (f^{[1]}_{LM'}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r})\vec{\tau})^{+} (f^{[1]}_{LM''}(r, \beta_\Omega)\, Y_{L\mu}(\hat{r})\vec{\tau})$$

In the case of axial symmetry of the average field ($\varrho^{[\tau]}_{LM}(r, \beta_\Omega) = \delta_{M0}\, \varrho^{[\tau]}_{L0}(r, \beta_\Omega)$) only one element of the matrix $b^{[\tau]}_{MM'}(L)$ is nonzero

(142) $$b^{[\tau]}_{MM'}(L) = b^{[\tau]}(L)\, \delta_{M0}\delta_{M'0}$$

where

(143) $$b^{[\tau]}(L) = \int f^{[\tau]}_{L0}(r, \beta_\Omega)\, \varrho^{[\tau]}_{L0}(r, \beta_\Omega)\, r^2\, \mathrm{d}r$$

If in this axial-symmetry case we restrict ourselves to the isoscalar part of residual interactions and to multipolarity $L = 2$, the Hamiltonian (129) can be rewritten as

(144) $$H = T_{\mathrm{kin}} + U_0^{[0]}(\beta) + H_{\mathrm{pair}} - \tfrac{1}{2} \sum_{\mu=-2}^{2} \varkappa_2\, F^{[0]}_{2\mu}(M=0)\, F^{[0]+}_{2\mu}(M=0) + V^{(-)}_{\mathrm{Res}}$$

where

(145) $$\varkappa_2 = \frac{V_0}{\int f^{[0]}_{20}(r, \beta_\Omega)\, f^{[0]}_{20}(r, \beta_\Omega)\, r^2\, \mathrm{d}r}$$

and

(146) $$F^{[0]}_{2\mu}(M=0) = f^{[0]}_{20}(r, \beta_\Omega)\, Y_{2\mu}(\hat{r})$$

The form (144) of Hamiltonian is similar to the one usually used in practical calculations (see e.g. [10]) except for the radial dependence $f^{[0]}_{20}(r, \beta_\Omega)$ of the residual interaction which is generally different from r^2 used in paper [10].

3.2. The restoration of translational symmetry of the total Hamiltonian (in the framework of RPA)

The restoration of translational symmetry of the total Hamiltonian in a rotating nucleus violated by the deformed average nuclear field is described in [24]. Therefore, here only the basic formulae and ideas are given for the purpose of completeness and mutual relation of translational and rotational symmetries.

Since the components of linear momenta P_μ ($\mu = x, y, z$) commute with the H_{pair} and with angular-momentum components in the framework of RPA, one can construct the negative-parity part $V_{\text{Res}}^{(-)}$ of residual interaction, according to the method of Pyatov [26], in separable form

$$V_{\text{Res}}^{(-)} = -\tfrac{1}{2} \sum_{\mu=x,y,z} \varkappa_\mu [\tilde{H}_{AV}, \tilde{P}_\mu]_{RPA}^{+} [\tilde{H}_{AV}, \tilde{P}_\mu]_{RPA} \tag{147}$$

where the strength constant $\varkappa_\mu$ will be determined from the requirement of translational symmetry $[\tilde{H}, \tilde{P}_\mu] = 0$ for all μ. Using (147) we have

(148)

$$[\tilde{H}, \tilde{P}_\mu] = [\tilde{H}_{AV}, \tilde{P}_\mu]_{RPA} - \tfrac{1}{2} \sum_{\nu=x,y,z} \varkappa_\mu \{[\tilde{H}_{AV}, \tilde{P}_\nu]_{RPA}, [\tilde{P}_\mu, [\tilde{H}_{AV}, \tilde{P}_\nu]]_{RPA}\} = 0$$

where the symbol $\{\ ,\ \}$ denotes the anticommutator. In the RPA order (i.e. $\tilde{P}_\mu$ is linear in bosons and $\tilde{H}_{AV}$ is quadratic in bosons) the double commutator in (148) is a c-number, which means in the RPA order
If we choose the double commutator as

$$[\tilde{P}_\mu, [\tilde{H}_{AV}, \tilde{P}_\nu]]_{RPA} = \langle \Omega | [\tilde{P}_\mu, [\tilde{H}_{AV}, \tilde{P}_\nu]] | \Omega \rangle \tag{149}$$

$$[\tilde{P}_\mu, [\tilde{H}_{AV}, P_\nu]]_{RPA} \approx c_\mu \delta_{\mu\nu} \tag{150}$$

and assume

$$\varkappa_\nu = \frac{1}{c_\nu}$$

the translation invariance condition $[\tilde{H}, P_\nu] = 0$ is fulfilled automatically.

As in the case of restoration of the rotational symmetry, only the isoscalar and isovector term is considered in the average potential (115) (Coulomb term can be treated in the same way, and the spin-orbital term is supposed to be unimportant for the states near the yrast line). Substituting of (115) into (147) gives

$$V_{\text{Res}} = -\tfrac{1}{2} \sum_{\substack{\tau=0,1 \\ \nu=x,y,z}} \varkappa_\nu^{[\tau]} \hat{R}_\nu^{+}[\tau]\, \hat{R}_\nu[\tau] \tag{152}$$

where

$$\hat{R}_\nu[0] = [U^{[0]}(r, \beta_\Omega), \hat{P}_\nu]_{RPA} \tag{153}$$

$$\hat{R}_\nu[1] = [U^{[1]}(r, \beta_\Omega)\, \tau_Z, \tilde{P}_\nu]_{RPA}$$

The strength constant $\varkappa_\nu^{[\tau]}$ is given by (151)

$$\varkappa_\nu^{[\tau]} = \frac{1}{\langle \Omega | [\tilde{P}_\nu, R_\nu^{[\tau]}] | \Omega \rangle} \tag{154}$$

From the symmetry properties of the operators $\hat{P}_\nu$ (see appendix) and from the symmetry of potential it follows (up to the second order in boson expansions)

$$R_1^{[\tau]} \sim R_1^{[\tau]}(1) = \sum_{\mu} r_{\mu}^{(1)}[\tau]\,(b_{\mu}^{+} - b_{\mu}), \quad \mu = (i\bar{k})$$

$$R_2^{[\tau]} \sim R_2^{[\tau]}(1) = \sum_{\mu} \tfrac{1}{2}[r_{\mu}^{(2)}[\tau]\,(b_{\mu}^{+} - b_{\mu}) - r_{\bar{\mu}}^{(2)}[\tau]\,(b_{\bar{\mu}}^{+} - b_{\bar{\mu}})], \quad \mu = (ik)$$

$$R_3^{[\tau]} \sim R_3^{[\tau]}(1) = -\frac{i}{2}\sum_{\mu} [r_{\mu}^{(3)}[\tau]\,(b_{\mu}^{+} + b_{\mu}) + r_{\bar{\mu}}^{(3)}[\tau]\,(b_{\bar{\mu}}^{+} + b_{\bar{\mu}}]$$

where $r_{\mu}^{(k)}[\tau]$ are the quasiparticle matrix elements of corresponding operators (see (A5) in appendix).

4. Conclusions

The SCCM + RPA method, described above, makes possible to obtain the quantitative informations about the structure of states of rotating nuclei in the framework of microscopical theory. However practical realization of this method is connected with the tedious numerical calculations and with introduction of number of approximations. In spite of this problem the SCCM + RPA approach have been succeeded in description of many characteristics of nucleus caused by rotation, e.g. isovector dipole excitations [70, 98], isoscalar quadrupole low-lying states [48, 49, 51], giant dipole resonance in rotating system [121, 122. 123] and others. The particular results of numerical calculations with the SCCM + RPA method will be discussed in the following paper.

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Appendix: Boson representation of single-particle operators

Every single-particle operator can be expressed in the form

$$\text{(A1)} \quad \hat{G} = \sum_{kl} \{ \langle k| G|l\rangle c_k^+ c_l + \langle k| \hat{G}|l\rangle c_k^+ c_l + \langle \bar{k}| \hat{G}|l\rangle c_{\bar{k}}^+ c_l + \langle \bar{k}| \hat{G}|l\rangle c_{\bar{k}}^+ c_l \}$$

Further the operator $\hat{G}$ is supposed to have the following symmetries

$$\text{(A2)} \qquad \begin{aligned} & T\hat{G}T^{-1} = \gamma_T \hat{G} \qquad && T = \mathcal{U}_T K \\ & \langle k| \hat{G}|l\rangle^* = r\langle k| \hat{G}|l\rangle && G^+ = hG \quad T^2 = (-1)^{2J} \\ & R_x(\pi)\, \hat{G} R_x^{-1}(\pi) = \gamma_x \hat{G} \end{aligned}$$

where T is the operator of time reversal, $\mathcal{U}_T$ is the unitary operator, K stands for the operator of complex conjugation. Numbers $\gamma_\tau = \pm 1$, $r = \pm 1$, $\gamma_x = \pm 1$, $h = \pm 1$ characterize the symetries of given operator with respect to corresponding transformation. In consequence of $R_x(\pi)$-symmetry of single-particle and quasiparticle vacuum combining (7) and (A2) we have

(A3)
$$\langle k|\ \hat{G}|l\rangle = \gamma_\tau r\langle k|\ \hat{G}|l\rangle$$
$$\langle \bar{k}|\ \hat{G}|l\rangle = -\gamma_\tau r\langle k|\ \hat{G}|l\rangle$$
$$\langle k|\ \hat{G}|\bar{l}\rangle = -\gamma_x\langle k|\ \hat{G}|l\rangle$$
$$\langle \bar{k}|\ \hat{G}|l\rangle = \gamma_x\langle k|\ \hat{G}|l\rangle$$

From (A3) it follows

(A4)
$$\hat{G} = \sum_{kl}\langle k|\ \hat{G}|l\rangle\,(c_k^+ c_l + \gamma_\tau r c_{\bar{k}}^+ c_l) \quad \text{for operators with} \quad \gamma_x = +1$$
$$\hat{G} = \sum_{kl}\langle k|\ \hat{G}|\bar{l}\rangle\,(c_k^+ c_l - \gamma_\tau r c_{\bar{k}}^+ c_l) \quad \text{for operators with} \quad \gamma_x = -1$$

In next considerations the assignment $\hat{G}^{(\pm)}$ will be used for the operator with $\gamma_x = \pm 1$ Using the Bogolubov transformation (10) and introducing two-particle bosons b_{kl}^+, $b_{\bar{k}\bar{l}}^+$, $b_{k\bar{l}}^+$ (see (17)) one can write for operators $\hat{G}^{(+)}(\gamma_x = +1)$ the following expression

(A5a)
$$\hat{G}^{(+)} = \langle\Omega|\ \hat{G}^{(+)}|\Omega\rangle + G_{(1)}^{(+)} + G_{(2)}^{(+)}$$
$$G_{(1)}^{(+)} = \sum_{ij} g_{ij}^{(+)}(b_{ij}^+ + hrb_{ij}), \quad g_{ij}^{(+)} = \sum_{kl}\langle k|\ \hat{G}^{(+)}|l\rangle\,(A_k^i B_l^{\bar{j}} - \gamma_\tau h B_k^i A_l^{\bar{j}})$$
$$G_{(2)}^{(+)} = \sum_{ijkl}\langle k|\ \hat{G}^{(+)}|l\rangle\,\{(A_k^i A_l^j - \gamma_\tau h B_k^i B_l^j)\sum_{\bar{m}}(b_{i\bar{m}}^+ j_{jm} + b_{i\bar{m}}^+ b_{j\bar{m}}) +$$
$$+ (\gamma_\tau r A_k^{\bar{i}} A_l^{\bar{j}} - hrB_k^{\bar{i}} B_l^{\bar{j}})\sum_m (b_{i\bar{m}}^+ b_{\bar{j}\bar{m}} + b_{im}^+ b_{\bar{j}m})\}$$

and for operators $\hat{G}^{(-)}$ $(\gamma_x = -1)$ the following one

(A5b)
$$\hat{G}^{(-)} = G_{(1)}^{(-)} + G_{(2)}^{(-)}$$
$$G_{(1)}^{(-)} = -\frac{i}{2}\sum_{ij}\{g_{ij}^{(-)}(b_{ij}^+ - rhb_{ij}) - \gamma_\tau r g_{i\bar{j}}^{(-)}(b_{i\bar{j}}^+ - rhb_{i\bar{j}})\}$$
$$g_{ij}^{(-)} = \sum_{kl}\langle k|\ \hat{G}^{(-)}|l\rangle\,(A_k^i B_l^j + \gamma_\tau h A_l^j B_k^i)$$
$$g_{i\bar{j}}^{(-)} = \sum_{kl}\langle k|\ \hat{G}^{(-)}|\bar{l}\rangle\,(\gamma_\tau h A_l^{\bar{j}} B_k^i + A_k^{\bar{i}} B_l^{\bar{j}})$$
$$G_{(2)}^{(-)} = i\sum_{ijkl}\langle k|\ \hat{G}^{(-)}|l\rangle\,\{(A_k^i A_l^{\bar{j}} + \gamma_\tau h B_l^{\bar{j}} B_k^i)\sum_{\bar{m}}(b_{i\bar{m}}^+ b_{\bar{j}\bar{m}}^+ - b_{im}^+ b_{\bar{j}m}) +$$
$$+ ihr(\gamma_\tau h A_k^{\bar{i}} A_l^j + B_k^{\bar{i}} B_l^j)\sum_m (b_{i\bar{m}}^+ b_{j\bar{m}} - b_{im}^+ b_{jm})$$

In (A5) the symbol $G_{(1)}^{(+)}$ and $G_{(2)}^{(+)}$ denote the linear and quadratic term in boson expansion of given operator $\hat{G}^{(\pm)}$. The quantities $g_{ij}^{(\pm)}$ and $g_{i\bar{j}}^{(\pm)}$ represent the quasiparticle matrix elements of $\hat{G}^{(\pm)}$. From (A5) one can see that the operators $\hat{G}^{(-)}$ (with negative signature) have zero mean values in quasiparticle vacuum $|\Omega\rangle$. As a consequence of $R_k(\pi)$-symmetry $(k = 1, 2, 3)$ of average nuclear field it is con-

venient to choose the phase of wave function so that the time reversal operator has the form

(A6) $$T = R_y^{-1}(\pi) \, . \, K$$

i.e. $\mathcal{U}_T = R_y^{-1}(\pi)$. In this case the operators of coordinates $\vec{r}$, linear momentum $\vec{P}$ and angular momentum $\vec{J}$ fulfil

(A7) $$\left.\begin{array}{l} T\vec{r}T^{-1} = \vec{r} \\ T\vec{P}T^{-1} = -\vec{P} \\ T\vec{J}T^{-1} = -\vec{J} \end{array}\right\} \Rightarrow \begin{array}{llll} \langle x\rangle^* = -\langle x\rangle & \langle P_x\rangle^* = \langle P_x\rangle & \langle J_x\rangle^* = \langle J_x\rangle \\ \langle y\rangle^* = \langle y\rangle & \langle P_y\rangle^* = -\langle P_y\rangle & \langle J_y\rangle^* = -\langle J_y\rangle \\ \langle z\rangle^* = -\langle z\rangle & \langle P_z\rangle^* = \langle P_z\rangle & \langle J_y\rangle^* = \langle J_z\rangle \end{array}$$

where in (A7) the symbol $\langle\ \rangle$ denotes the matrix element of corresponding operator. Using (A7) and (A6) it is possible to obtain the following symmetries of multipole operators $\hat{Q}_{\lambda\mu} = r^\lambda Y_{\lambda\mu}$

(A8) $$\hat{Q}^+_{\lambda\mu} = (-1)^\mu \, Q_{\lambda-\mu} \quad R_x(\pi)\, \hat{Q}_{\lambda\mu}\, R_x^{-1}(\pi) = (-1)^\lambda \, \hat{Q}_{\lambda-\mu}$$

$$T\hat{Q}_{\lambda\mu}T^{-1} = (-1)^\mu \, Q_{\lambda-\mu}$$

In the cranking Hamiltonian H' (1) and in the transition operators (see sect. 2.3.4.) there are the combinations of multipole operators $\hat{Q}_{\lambda\mu}$ given in (8) and in table bellow. Using (A7) and (A8) one can derive the values of γ_x, γ_τ, r, h for all operators (8), operators of coordinates, linear and angular momentus. These values, allowing to one to determine the boson structure of corresponding operator in terms of (A5), are collected in table.

The zero-boson and linear-boson terms for the particle number operator $\hat{N}_\tau$ ($\tau = N, Z$) and for pairing operator $\hat{P}^+_\tau$ (see (6)) have the forms

(A9) $$\langle\Omega|\, \hat{N}_\tau|\Omega\rangle = \sum_{kl}[(B^l_k)^2 + (B^{\bar{l}}_k)^2]$$

$$N_\tau(1) = \sum_{ij} n^\tau_{ij}(b^+_{ij} + b_{ij}) \quad n^\tau_{ij} = \sum_{k\in\tau}(A^i_k B^{\bar{j}}_k - B^i_k A^{\bar{j}}_k)$$

Table: Symmetries of single-particle operators

Operator	γ_x	γ_τ	h	r	Operator	γ_x	γ_τ	h	r
$\hat{\mathcal{D}}^{(-)}_0 = \hat{Q}_{10}$	−	+	+	−	$\hat{F}^{(-)}_2 = \frac{1}{\sqrt{2}}(\hat{Q}_{32} + \hat{Q}_{3-2})$	−	+	+	−
$\hat{\mathcal{D}}^{(+)}_1 = \frac{1}{\sqrt{2}}(\hat{Q}_{11} - \hat{Q}_{1-1})$	+	+	+	−	$\hat{F}^{(+)}_3 = \frac{1}{\sqrt{2}}(\hat{Q}_{33} - \hat{Q}_{3-3})$	+	+	+	−
$\hat{\mathcal{D}}^{(-)}_1 = \frac{i}{\sqrt{2}}(\hat{Q}_{11} + \hat{Q}_{1-1})$	−	+	+	+	$\hat{F}^{(-)}_3 = \frac{1}{\sqrt{2}}(\hat{Q}_{33} + \hat{Q}_{3-3})$	−	+	+	+
$\hat{Q}^{(+)}_0 = \hat{Q}_{20}$	+	+	+	+	$\hat{J}_x$	+	−	+	+

$$\hat{Q}_1^{(+)} = \frac{i}{\sqrt{2}}(\hat{Q}_{21} + \hat{Q}_{2-1}) \quad + + + - \qquad \hat{J}_y \qquad - - + -$$

$$\hat{Q}_1^{(+)} = \frac{1}{\sqrt{2}}(\hat{Q}_{21} - \hat{Q}_{2-1}) \quad - + + + \qquad \hat{J}_z \qquad - - + +$$

$$\hat{Q}_2^{(+)} = \frac{1}{\sqrt{2}}(\hat{Q}_{22} + \hat{Q}_{2-2}) \quad + + + + \qquad \hat{P}_x \qquad + - + +$$

$$\hat{Q}_2^{(-)} = \frac{i}{\sqrt{2}}(\hat{Q}_{22} - \hat{Q}_{2-2}) \quad - + + - \qquad \hat{P}_y \qquad - - + -$$

$$\hat{F}_{30}^{(-)} = \hat{Q}_{30}^{(-)} \qquad - + + - \qquad \hat{P}_z \qquad - - + -$$

$$\hat{F}_1^{(+)} = \frac{1}{\sqrt{2}}(\hat{Q}_{31} - \hat{Q}_{3-1}) \quad + + + - \qquad \hat{X} \qquad + + + -$$

$$\hat{F}_1^{(-)} = \frac{i}{\sqrt{2}}(\hat{Q}_{31} + \hat{Q}_{3-1}) \quad - + + + \qquad \hat{Y} \qquad - + + +$$

$$\hat{F}_2^{(+)} = \frac{i}{\sqrt{2}}(\hat{Q}_{32} - \hat{Q}_{3-2}) \quad + + + + \qquad \hat{Z} \qquad - + + -$$

(A10)
$$\langle\Omega|\,\hat{P}_\tau^+\,|\Omega\rangle = \sum_{k,l\in\tau}\left[A_k^l B_k^l + A_k^l B_{\bar{k}}^l\right]$$

$$P_\tau^+(1) = P_\tau^{(+)} + P_\tau^{(-)}$$

$$P_\tau^{(+)} = \sum_{ij} p_{ij}^{(+)}(b_{ij}^+ + b_{ij}) \quad p_{ij}^{(\pm)} = \sum_{k\in\tau}\left(A_{\bar{k}}^{\bar{i}} A_k^i + B_{\bar{k}}^i B_k^j\right)$$

$$P_\tau^{(+)} = \sum_{ij} p_{ij}^{(-)}(b_{ij}^+ - b_{ij})$$

At the end of this appendix we present the expressions for quasiparticle matrix elements of the operators $\hat{Q}_{\lambda\mu_\varkappa}$ (see (76))

$$\mathcal{M}_{kl}^{(10)} = d_{kl}^{(1+)}$$

$$\mathcal{M}_{kl}^{(11)} = \frac{\sqrt{2}}{4}(-id_{kl}^{(0-)} - d_{kl}^{(1-)}) \qquad \mathcal{M}_{kl}^{(1-1)} = \frac{\sqrt{2}}{4}(-id_{kl}^{(0-)} + d_{kl}^{(1-)})$$

$$\mathcal{M}_{\bar{k}\bar{l}}^{(11)} = \frac{\sqrt{2}}{4}(-id_{\bar{k}\bar{l}}^{(0-)} + d_{\bar{k}\bar{l}}^{(1-)}) \qquad \mathcal{M}_{\bar{k}\bar{l}}^{(1-1)} = \frac{\sqrt{2}}{4}(-id_{\bar{k}\bar{l}}^{(0-)} - d_{\bar{k}\bar{l}}^{(1-)})$$

$$\mathcal{M}_{kl}^{(20)} = \frac{\sqrt{3}}{2} q_{kl}^{(2+)} - \frac{1}{2} q_{kl}^{(0+)}$$

$$\mathcal{M}_{kl}^{(21)} = -\frac{\sqrt{2}}{4}(q_{kl}^{(2-)} + iq_{kl}^{(1-)}) \qquad \mathcal{M}_{kl}^{(2-1)} = \frac{\sqrt{2}}{4}(-q_{kl}^{(2-)} + iq_{kl}^{(1-)})$$

$$\mathcal{M}_{kl}^{(21)} = \frac{\sqrt{2}}{4}\left(-q_{kl}^{(2-)} + iq_{kl}^{(1-)}\right) \qquad \mathcal{M}_{kl}^{(2-1)} = -\frac{\sqrt{2}}{4}\left(q_{kl}^{(2-)} + iq_{kl}^{(1-)}\right)$$

$$\mathcal{M}_{kl}^{(22)} = \frac{\sqrt{2}}{4} q_{kl}^{(2+)} + i\frac{\sqrt{2}}{2} q_{kl}^{(1+)} + \qquad \mathcal{M}_{kl}^{(2-2)} = \frac{\sqrt{2}}{4} q_{kl}^{(2+)} - i\frac{\sqrt{2}}{2} q_{kl}^{(1+)} +$$

$$+ \frac{\sqrt{3}}{2\sqrt{2}} q_{kl}^{(0+)} \qquad + \frac{\sqrt{3}}{2\sqrt{2}} q_{kl}^{(0+)}$$

$$\mathcal{M}_{kl}^{(30)} = \frac{\sqrt{10}}{4} f_{kl}^{(3+)} - \frac{\sqrt{6}}{4} f_{kl}^{(1+)}$$

$$\mathcal{M}_{kl}^{(31)} = -\frac{\sqrt{30}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{5}}{8} f_{kl}^{(2-)} + \qquad \mathcal{M}_{kl}^{(3-1)} = \frac{\sqrt{30}}{16} f_{kl}^{(3-1)} + i\frac{\sqrt{5}}{8} f_{kl}^{(2-)} -$$

$$+ \frac{\sqrt{2}}{16} f_{kl}^{(1-)} - i\frac{\sqrt{3}}{8} f_{kl}^{(0-)} \qquad - \frac{\sqrt{2}}{16} f_{kl}^{(1-)} - i\frac{\sqrt{3}}{8} f_{kl}^{(0-)}$$

$$\mathcal{M}_{kl}^{(31)} = \frac{\sqrt{30}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{5}}{8} f_{kl}^{(2-)} - \qquad \mathcal{M}_{kl}^{(3-1)} = -\frac{\sqrt{30}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{5}}{8} f_{kl}^{(2-)} +$$

$$- \frac{\sqrt{2}}{16} f_{kl}^{(1-)} - i\frac{\sqrt{3}}{8} f_{kl}^{(0-)} \qquad + \frac{\sqrt{2}}{16} f_{kl}^{(1-)} - i\frac{\sqrt{3}}{8} f_{kl}^{(0-)}$$

$$\mathcal{M}_{kl}^{(32)} = \frac{\sqrt{3}}{4} f_{kl}^{(3+)} + i\frac{\sqrt{2}}{2} f_{kl}^{(2+)} + \qquad \mathcal{M}_{kl}^{(3-2)} = -\frac{\sqrt{3}}{4} f_{kl}^{(3+)} + i\frac{\sqrt{2}}{2} f_{kl}^{(2+)} -$$

$$+ \frac{\sqrt{5}}{4} f_{kl}^{(1+)}, \qquad - \frac{\sqrt{5}}{4} f_{kl}^{(1+)}$$

$$\mathcal{M}_{kl}^{(33)} = -\frac{\sqrt{2}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{6}}{8} f_{kl}^{(2-)} - \qquad \mathcal{M}_{kl}^{(3-3)} = \frac{\sqrt{2}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{6}}{8} f_{kl}^{(2-)} +$$

$$- \frac{\sqrt{30}}{16} f_{kl}^{(1-)} + \frac{i\sqrt{5}}{4\sqrt{2}} f_{kl}^{(0-)} \qquad + \frac{\sqrt{30}}{16} f_{kl}^{(1-)} + \frac{i\sqrt{5}}{4\sqrt{2}} f_{kl}^{(0-)}$$

$$\mathcal{M}_{kl}^{(33)} = \frac{\sqrt{2}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{6}}{8} f_{kl}^{(2-)} + \qquad \mathcal{M}_{kl}^{(3-3)} = -\frac{\sqrt{2}}{16} f_{kl}^{(3-)} + i\frac{\sqrt{6}}{8} f_{kl}^{(2-)} -$$

$$+ \frac{\sqrt{30}}{16} f_{kl}^{(1-)} + \frac{i\sqrt{5}}{4\sqrt{2}} f_{kl}^{(0-)} \qquad - \frac{\sqrt{30}}{16} f_{kl}^{(1-)} + \frac{i\sqrt{5}}{4\sqrt{2}} f_{kl}^{(0-)}$$

where $d_{kl}^{(0-)}$, $d_{kl}^{(1-)}$, $d_{kl}^{(1+)}$, $q_{kl}^{(0+)}$, $q_{kl}^{(2+)}$, $q_{kl}^{(1+)}$, $q_{kl}^{(2-)}$, $q_{kl}^{(1-)}$, $f_{kl}^{(1+)}$, $f_{kl}^{(2+)}$, $f_{kl}^{(3+)}$, $f_{kl}^{(0-)}$, $f_{kl}^{(1-)}$, $f_{kl}^{(2-)}$, $f_{kl}^{(3-)}$ represent quasi-particle matrix elements of the operators $\hat{\mathcal{D}}_0^{(-)}$, $\hat{\mathcal{D}}_1^{(\pm)}$, $\hat{Q}_0^{(+)}$, $\hat{Q}_1^{(\pm)}$, $\hat{Q}_2^{(\pm)}$, $\hat{F}_0^{(-)}$, $\hat{F}_1^{(\pm)}$, $\hat{F}_2^{(\pm)}$, $\hat{F}_3^{(\pm)}$. which can be determined by means of (A5) with using of table given in this appendix.