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On a Class of Near-Rings Sum of Near-Fields

SILVIA PELLEGRINI MANARA*)
Institute of Mathematics, University of Parma**)

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We study the near-rings whose proper subnear-rings are near-fields and we call them s-near-fields. In this work we show that these structures are at most $E2$-generated and we characterize the general case, the zero-symmetric $E2$-generated and the constant $E2$-generated cases. near-rings are in fact a sum of near-fields. We shall deal with the remaining cases in other studies.

Studujeme skorookruhy, jejichž vlastní podskorookruhy jsou skorotělesy a nazýváme je s-skorotělesy. V této práci ukážeme, že tyto struktury jsou nejvýše E2-generovány a dáváme charakteristiku v obecném případě a ve dvou speciálních případech. Tyto skorookruhy jsou sumy skorotěles.

Мы изучаем почти-кольца, истинные подпочти-кольца которых являются почти-полями; они называются s-почти-полями. В этой работе мы покажем, что эти структуры по крайней мере E2-полождены и даем характеризацию в общем и в двух частных случаях. Эти почти-кольца являются суммами почти-полей.

1. Introduction

The near-fields have been studied in detail and even for their relations with various geometrical matters. We study in this work, the near-rings whose proper subnear-rings are near-fields: such structures will be called s-near-fields. This study can be also interpreted as dual of the one dealt with in [8]. We shall limit ourselves to the algebraic study of the s-near-fields that result as a sum of near-fields, dealing with the algebraic considerations of other cases and geometrical considerations as in [1], in other studies. Particularly we show that a near-ring $N = N_0 + N_c$ is an s-near-field if and only if it is generated by each element $a + h$ with $0 \neq a \in N_c$, $0 \neq h \in N_0$ and it is the sum of a near-field isomorphic to $M_c(Z_2)$ and of N_0, near-field with characteristic p, whose subnear-rings are near-fields. Moreover if $\mathrm{char}(N_0) = 2$, N^+ is an elementary abelian 2-group and N_c is a left ideal of N; if $\mathrm{char}(N_0) = p \neq 2$, N_0 is an ideal of N and N^+ is either a generalized dihedral group, or the direct sum of N_0^+ and of Z_2^+ with N_2^+ elementary abelian p-group. As far as the zero-symmetric

*) Work carried out on behalf of GNSAGA-C.N.R.

**) Instituto di Matematica, Università, 43100 Parma, Italy.

case is concerned, we show that if N is a zero-symmetric $E2$-generated s-near-field, without zero-divisors, its additive group is an elementary abelian p-group and N is generated by the sum of two fields isomorphic to Z_p. Some examples prove that previous cases exist. We also recall that in the zero-symmetric integer, $E2$-generated case, the s-near-fields are near-rings of (p) type (see [6]) already studied by Ligh. Finally we characterize completely, in the $E2$-generated case, the constant s-near-fields, the non-integer zero-symmetric s-near-fields and, the s-fields.

2. Preliminaries

We indicate with N a left near-ring; for the definitions and the fundamental notations we refer to [9] without express recall.

Definition A. We call s-nearfield a near-ring whose proper subnear-rings are near-fields.

Later on, we will say a near-ring N is n-generated if it can be generated by n elements; we will call a near-ring exactly n-generated (and we will write En-generated) if it has a system of n generators, but it cannot be generated by a system of $n - 1$ elements. Moreover, for $M \subseteq N$ we will indicate with $\langle M \rangle$ the subnear-ring of N generated by M.

Proposition 1. An s-near-field is at most $E2$-generated.

Proof. We suppose that N is at least $E3$-generated; in this case, for $a, b \in N$ the subnear-rings $\langle a \rangle$, $\langle b \rangle$, $\langle \{a, b\} \rangle$ are proper subnear-rings and then are near-fields. It follows that the identity of $\langle a \rangle$ coincides with the identity of $\langle b \rangle$, $\forall a, b \ \ b \in N$ and N has identity. Furthermore, each non-zero element of N, belonging to near-field generated by it, has inverse: in this way N results as a near-field, contrary to the assumed.

Proposition 2. The N-subgroups and the left ideals of an s-near-field are maximal.

Proof: Let A, B with $A \supseteq B$ two proper N-subgroups of N and therefore near-fields. Of course BA is contained in B and if u is the identity of B, (and then of A) we have that $uA = A$ and therefore $A \subseteq B$. It follows $A = B$ and consequently the thesis. The same for the left ideals.

The Prop. 2 allows us to extend the results of [4, 5] to our case. Moreover:

Proposition 3. A constant s-near-field is abelian.

Proof. The subnear-rings of a constant s-near-field are constant near-fields and so isomorphic to $M_c(Z_2)$ *); furthermore N is at most $E2$-generated. If N is 1-generated,

*) We recall that $M_c(Z_2) = \{f: Z_2 \rightarrow Z_2 / f \text{ constant}\}$ (see [9] 1.4.a).

it is obviously abelian because N^+ coincides with the cyclic group generated by the generator of N. If N is $E2$-generated, let $\{a, b\}$ a system of generators. The subnear-rings $\langle a \rangle$, $\langle b \rangle$ and $\langle a + b \rangle$ are isomorphic to $M_c(Z_2)$, therefore $0 = 2a = 2b = 2(a + b)$. From $2(a + b) = 0$ it follows $a + b = b + a$ and therefore N is abelian.

Corollary 1. The subnear-rings of a constant s-near-field are ideals of N.

Proof. Easy.

3. General case

We start with the study of the s-near-fields which are the sum of their constant part N_c and of their zero-symmetric part N_0 (see [9], prop. 1.13).

Theorem 1. A near-ring $N = N_0 + N_c$ is an s-near-field if and only if it is generated by each element $a + h$, with $0 \neq a \in N_c$ and $0 \neq h \in N_0$ and it is the sum of a constant near-field and of a zero-symmetric near-field with characteristic p (prime), whose subnear-rings are near-fields.

Proof: If $N = N_c + N_0$ is an s-near-field, the element $g = a + h$ with $0 \neq a \in N_c$ and $0 \neq h \in N_0$ generates N because if it generated a proper subnear-ring of N, this would be a near-field containing both zero-symmetric elements $(Og + g = h)$, and constant elements $(Og = a)$ and this is absurd (see [9], prop. 8.1). Moreover, N_c is a proper invariant subnear-ring of N (see [9], prop. 1.32b) and therefore is a constant near-field that is isomorphic to $M_c(Z_2)$, N_0 is a right ideal of N (see [9], prop. 1.32a) hence a near-field (and therefore abelian) whose subnear-rings are near-fields; lastly, the identity of N_0 generates a near-field and therefore isomorphic to Z_p. Then $\mathrm{char}(N_0) = p$. To prove the other part of the theorem we suppose that N is generated by each element $a + h$ with $0 \neq a \in M_c(Z_2)$ and $0 \neq h \in N_0$, then N has the proper subnear-rings contained in N_0, that is, in a near-field whose subnear-rings are near-fields and consequently N is an s-near-field.

In order to characterize the additive group N^+ of the s-near-field $N = N_0 + N_c$, we shall first show the following:

Lemma 1. If $N = N_0 + N_c$ is a non abelian s-near-field, the centralizer of each element of N_0 coincides with N_0.

Proof: Let N be non abelian, x an element of N_0 and $C_x = \{y \in N : x + y = y + x\}$ the centralizer of x. Obviously $C_x \supseteq N_0$ as N_0 is abelian being a near-field (see Th. 1); moreover, C_x is a normal subgroup of N^+ because the derived group $N^{+\prime}$ is contained in N_0 and consequently in C_x. Hence $C_x = N_0$ or $C_x = N$ because $|N^+/N_0^+| = 2$.

If an element $x \in N_0$ exists, such that $C_x = N$, then x belongs to the center of N^+, $Z(N^+)$ and $Z(N^+) \cap N_0 = K \neq \{0\}$. Now K is obviously a normal subgroup of N_0^+ and here it is also a left ideal of N_0, in fact: $\forall n_0 \in N_0$, $\forall z \in K$ and $\forall \bar{n}_0 + a = h \in N$ is $n_0 z + h = n_0 z + \bar{n}_0 + a$. Since N_0 is a near-field, we have $n_0 N_0 = N_0$ $\forall n_0 \in \in N_0 \setminus \{0\}$ and then $\exists n_0'$ so that $n_0 n_0' = \bar{n}_0$; moreover, the product $n_0 a = a, \forall n_0 \in N_0$ because $N_c \simeq M_c(Z_2)$ is an invariant subnear-ring and if $n_0 a = 0$, consequently $0(n_0 a) = 00 = 0 = (0n_0)\, a = a$, that is $a = 0$ and this is absurd. Then $n_0 z + h = = n_0 z + n_0 n_0' + n_0 a = n_0(z + n_0' + a) = n_0(n_0' + a + z) = h + n_0 z$ and $N_0 K \subseteq \subseteq K$. It follows that K is a left ideal of N_0 but N_0 is a near-field, hence $K = N_0 \subseteq \subseteq Z(N)$. This is also absurd because N should be abelian. Hence, the centralizer of each element $z \in N_0$ coincides with N_0.

Theorem 2. If $N = N_0 + N_c$ is an s-near-field with $\mathrm{char}(N_0) = 2$, it follows:

1. N^+ is an elementary abelian 2-group;
2. N_c is a left ideal of N.

Proof 1: We recall that generally it is $N^+ = N_0^+ +_g N_c^+$, where $+_g$ indicates a semidirect sum of N_0^+ and of N_c^+ (see [9], prop. 1.22a). Let $\varphi_a = g(a)$ an automorphism of N_0^+ and let N^+ be non abelian. By Lemma 1, the centralizer of each element of N_0, coincides with N_0, hence φ_a is a fixed point-free automorphism*). On the other nnad, the element $x = -y + \varphi_a(y)$ $(y \in N_0 \setminus \{0\})$ is a non-zero element and such that $\varphi_a(x) = -x$; since $\mathrm{char}(N_0) = 2$, it follows that $\varphi_a(x) = x$ and hence this is absurd and N is abelian. According to Th. 1, N_0' is an elementary abelian 2-group, moreover N^+ is abelian and, as direct sum of N_0^+ and of Z_0^+, is an elementary abelian 2-group.

Proof 2: Easy because N is abelian and N_c is an invariant subnear-ring of N.

Theorem 3. If $N = N_0 + N_c$ is an s-near-field with $\mathrm{char}(N_0) = p \neq 2$, then:

1. N_0 is an ideal of N;
2. $N^+ = \mathrm{Dih}(N_0)$, where $\mathrm{Dih}(N_0)$ is the generalized dihedral group determinated by N_0**), or $N^+ = N_0 \dotplus Z_2^+$ with N_0^+ elementary abelian p-group.

Proof 1: We know, by Th. 1, that the elements of N_0 have order p: if $a + h$ with $a \in N_c$ and $h \in N_0$ is an element of order p, it must be $p(a + h) = 0$ and then $0(p(a + h)) = 0$, but $0(p(a + h)) = a$ for each $p \neq 2$ (prime) as it is odd, and this is absurd. We have shown that only the elements of N_0 have order p; hence N_0 is a left ideal of N: in fact $\forall n \in N$ and $\forall h \in N_0$ is $p(nh) = n(ph) = 0$. So N_0 is an ideal of N because it is always a right ideal.

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*) In fact if $\exists y \in N_0 \setminus \{0\}$ such that $\varphi_a(y) = y$, we have for $a \in N_c \setminus \{0\}$ $\langle y, 0\rangle + \langle y', a\rangle = = \langle y + y', a\rangle$ and $\langle y', a\rangle + \langle y, 0\rangle = \langle y' + \varphi_a(y), a\rangle = \langle y + y', a\rangle$ and the centralizer of $\langle y, 0\rangle$ is different from N_0.

**) For the definition of Dih (N_0) see for instance [12] pag. 10.

Proof 2: If N^+ is non abelian, it is again $N^+ = N_0^+ +_\theta N_c^+$ (see proof of the previous Th. 2). Let $\varphi_a = g(a)$ an automorphism of N_0^+: by Lemma 1 it immediately follows that φ_a is a fixed point-free automorphism. Now let $\bar{\varphi}_a : N_0^+ \to N_0^+$ the homomorphism thus defined: $\bar{\varphi}_a(b) = -b + \varphi_a(b)$. For ii of [10] pag. 278, if φ_a is a fixed point-free automorphism of a group, $\bar{\varphi}_a$ is a monomorphism. Let $H^+ = \bar{\varphi}_a(N_0^+)$; H^+ is normal in N_0^+ and moreover $\varphi_a(x) = -x \; \forall x \in H^+$. Then $\varphi_a(H^+) = H^+$. For each $x \in H^+$, $\bar{\varphi}_a(x) = -2x$ and therefore $\bar{\varphi}_a$ is an epimorphism of H^+ because H^+ has exponent p, with p prime. The conditions of (viii) [10] pag. 279 hold. It follows that φ_a induces in $\mathfrak{N}(H^+)/H^+ = N_0^+/H^+$ (now $\mathfrak{N}(H^+)$ is the normalizer of H^+ in N_0^+) a fixed point-free automorphism. This is obviously absurd if $N_0^+ \neq H^+$ *) and therefore $\bar{\varphi}_a$ is an epimorphism. Hence $\varphi_a : N_0^+ \to N_0^+$ is the automorphism defined by $x \mapsto -x \; \forall x \in N_0$, and N^+ is the generalized dihedral group. Lastly, if N is abelian, N^+ is direct sum of an elementary abelian p-group and of Z_2^+.

Examples:

a) As additive group we consider Klein's 4-group and we define the product as it follows:

	0	a	b	$c = a + b$
0	0	a	0	a
a	0	a	a	0
b	0	a	b	c
c	0	a	c	b

It is a (non-direct) sum of Z_2 and $M_c(Z_2)$ (see Th. 2).

b) As additive group we consider S_3, dihedral group of the permutations on three elements and we define the product as it follows:

	0	a	$a+2b$	$a+b$	b	$2b$
0	0	a	a	a	0	0
a	0	a	a	a	0	0
$a+2b$	0	a	a	a	0	0
$a+b$	0	a	a	a	b	0
b	0	a	$a+2b$	$a+b$	b	$2b$
$2b$	0	a	$a+b$	$a+2b$	$2b$	b

It is a (non-direct) sum of Z_3 and $M_c(Z_2)$ (see Th. 3).

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*) In fact if $\tilde{\varphi}_a : N_0^+/H^+ \to N_0^+/H^+$ is induced by φ_a we shall have for $y \in N_0^+$, $\tilde{\varphi}_a(y + H^+) = \varphi_a(y) + H^+$ and it is $\tilde{\varphi}_a(y + H^+) = y + H^+$ because $H^+ = \bar{\varphi}_a(N_0^+)$.

c) As additive group we consider Z_6^+, and we define the following products:

	0	1	2	3	4	5
0	0	3	0	3	0	3
1	0	5	4	3	2	1
2	0	5	4	3	2	1
3	0	3	0	3	0	3
4	0	1	2	3	4	5
5	0	1	2	3	4	5

	0	1	2	3	4	5
0	0	3	0	3	0	3
1	0	3	0	3	0	3
2	0	5	4	3	2	1
3	0	3	0	3	0	3
4	0	1	2	3	4	5
5	0	3	0	3	0	3

They are again examples concerning Th. 3.

4. $E2$-generated s-near-fields

Proposition 4. A zero-symmetric $E2$-generated s-near-field is without nilpotent elements.

Proof: If N is $E2$-generated, each of its non-zero elements will generate a proper subnear-ring and therefore a near-field: N is hence without nilpotent elements.

Proposition 5. A zero-symmetric s-near-field N, without zero divisors is:

1. N-simple, strongly monogenic, faithful and 2-primitive;
2. the semigroup $(N, \cdot)$ is a right group*).

Proof 1: Let N be a zero-symmetric s-near-field without zero-divisors. If nN is a proper N-subgroup, it is a near-field. If u is the identity of nN, hence $\forall z \in N$, $u(nz) = (nz)\,u$, and $z = zu$; moreover, $u(uz) = uz$ and $uz = z$ (in fact N is an integer near-ring and so the left cancellation law holds (see prop. 1.111 a. of [9])). Then N has identity and $uN = N \subseteq nN$, that is $nN = N$. It follows that N is strongly monogenic (see [9], def. 3.1); moreover, N is without right ideals because it is zero-symmetric, then N is simple and subdirectly irreducible (see [9], cor. 1.1). Lastly by the N-simplicity if follows that N is faithful and 2-primitive (see [9], def. 1.17 and 4.2).

Proof 2: Easy by th. 4.3 of [11].

Particularly, we can observe that according to Prop. 5 the integer and zero-symmetric s-near-fields are near-rings of type (p) already studied by Ligh [6]. In fact in this, $\forall x \in N \setminus \{0\}, A_d(x) = \{y \in N \mid xy = 0\}$ is zero (otherwise it should be a proper N-subgroup of N and it is absurd) and therefore it is an ideal of N.

Theorem 4. If N is an integer zero-symmetric $E2$-generated s-near-field, then N^+ is an elementary abelian p-group, each element of N generates a near-field and N is generated by the sum of two fields isomorphic to Z_p.

*) For the definition of right group see for instance [3].

Proof: We start by showing that N is abelian. If N is $E2$-generated each element $a \in N$ generates a proper subnear-ring and therefore a near-field with a prime number characteristic. So the equation $x + x = a$ has one and only one solution for each $a \in N$. Let e be the identity of a subnear-field M of N with char $M = p \neq 2$. We suppose that such identity exists, because if it doesn't exist, N has characteristic 2 and is abelian. We now define the map $f: N \to N$ such that $f(x) = (-e)\, x$, $\forall x \in N$. This map is an automorphism of N^+ because it is obviously a homomorphism, moreover, it is a monomorphism because N is integer and so the left cancellation law holds; lastly it's an epimorphism because by the Prop. 5. $nN = N\ \forall n \in N$ and then $\forall z \in N$ the equation $(-e)\, x = z$ has solution. We now show that such automorphism is fixed point-free: if it is $(-e)\, x = x$ for some x, it is $(-e)\, xy = xy\ \forall y \in \in N$. The product xy, while y varies in N, describes N, so $-e$ is a left identity. In particular $(-e)\, e = e$ but $(-e)\, e = -e$ because e is the identity of the near-field where $-e$ belongs and this is to be excluded. In this way the hypotheses of theorem of [7] hold and N is abelian. We now can show that char $N = p$. In fact, if N has, together with elements of order p, elements of order q, with $p \neq q$, the near-ring generated by the sum of an element of order p and of an element of order q, is a near-field with characteristic pq, and this is absurd (see [9], prop. 8.9c). We also prove that N can't have aperiodic elements, each of these having to generate a near-field M with char M a prime number. Nor, for the same reason, can elements of order p-power be in N. Therefore, char $N = p$ and N^+ is an elementary abelian p-group. In N each element generates a near-field of characteristic p, moreover N is $E2$-generated, therefore at least two proper subnear-rings whose identities generate two fields I and J both isomorphic to Z_p, exist in N *). The near-ring generated by $I + J$, can't be a proper subnear-ring of N, because it should be a near-field, so it coincides with N.

Example:

As additive group we consider $Z_3 \dotplus Z_3$ and we define the following product:

	(0, 0)	(0, 1)	(1,1)	(1, 0)	(1, 2)	(2, 1)	(2, 0)	(2, 2)	(0, 2)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
(0, 1)	(0, 0)	(0, 1)	(1, 1)	(1, 0)	(1, 2)	(2, 1)	(2, 0)	(2, 2)	(0, 2)
(1, 1)	(0, 0)	(0, 2)	(2, 2)	(2, 0)	(2, 1)	(1, 2)	(1, 0)	(1, 1)	(0, 1)
(1, 0)	(0, 0)	(0, 1)	(1, 1)	(1, 0)	(1, 2)	(2, 1)	(2, 0)	(2, 2)	(0, 2)
(1, 2)	(0, 0)	(0, 2)	(2, 2)	(2, 0)	(2, 1)	(1, 2)	(1, 0)	(1, 1)	(0, 1)
(2, 1)	(0, 0)	(0, 1)	(1, 1)	(1, 0)	(1, 2)	(2, 1)	(2, 0)	(2, 2)	(0, 2)
(2, 0)	(0, 0)	(0, 2)	(2, 2)	(2, 0)	(2, 1)	(1, 2)	(1, 0)	(1, 1)	(0, 1)
(2, 2)	(0, 0)	(0, 1)	(1, 1)	(1, 0)	(1, 2)	(2, 1)	(2, 0)	(2, 2)	(0, 2)
(0, 2)	(0, 0)	(0, 2)	(2, 2)	(2, 0)	(2, 1)	(1, 2)	(1, 0)	(1, 1)	(0, 1)

*) We observe that the subnear-rings of N can't intersect in the same Z_p, because otherwise N should be a near-field.

Theorem 5. A zero-symmetric near-ring N with zero-divisors and without nilpotent elements, is an s-near-field if and only if it is the direct sum of two fields isomorphic to Z_p and Z_q with prime numbers p and q.

Proof: In a zero-symmetric near-ring without nilpotent elements if $ab = 0$ then $ba = 0$ (see lemma 1 of [2] and th. 3 of [8]). If y is a zero-divisor of N, $A(y) = \{x \in N \mid xy = yx = 0\}$ is an ideal of N: we suppose ab absurdo that $A(y)$ is the only ideal of N and let $i \in A(y) \setminus \{0\}$; $A(t)$ is again a proper ideal of N and so is $A(t) = A(y)$ that is $i^2 = 0$ and this is to be excluded because N is without non-zero nilpotent elements. Therefore N is zero-symmetric, without nilpotent elements, non integer, and its ideals are near-fields. By th. 3 of [8] N results a near-ring with exactly two ideals, and if we call them X and Y, N can be expressed as direct sum of X and Y, (see [4] too). If $\langle u_x \rangle$ and $\langle u_y \rangle$ are subnear-rings of X and Y generated by the respective identities u_x and u_y, they obviously are isomorphic to Z_p and Z_q, for some p, and q prime. The direct sum $\langle u_x \rangle \dotplus \langle u_y \rangle$, if it is a proper subnear-ring, is a near-field and this is absurd. So $X = \langle u_x \rangle$ and $Y = \langle u_y \rangle$ and the theorem has been proved. The converse is trivial.

Corollary 3. A zero-symmetric near-ring N with zero-divisors, is an $E2$-generated s-near-field if and only if it is the direct sum of two fields isomorphic to Z_p.

Proof. If N is a non integer, zero-symmetric, $E2$-generated s-near-field, by Prop. 4 and Th. 5, N is the direct sum of two fields isomorphic to Z_p and Z_q. If $p \neq q$, the near-ring N is 1-generated, and generated by each element $a + b$ with $a \neq 0 \neq b$, $a \in Z_p$ and $b \in Z_q$, and this is to be excluded. Therefore the theorem has been proved. The converse is trivial.

Corollary 4. A zero-symmetric near-ring with zero-divisors, and without nilpotent elements, is a 1-generated s-near-field if and only if it is the direct sum of two fields isomorphic to Z_p and Z_q with $p \neq q$ prime numbers.

Proof: Easy by Th. 5 and Cor. 3.

Proposition 6. A constant near-ring is an $E2$-generated s-near-field if and only if it is the direct sum of two constant near-fields.

Proof. If N is a constant $E2$-generated s-near-field, each of its non-zero elements, generates a constant near-field which therefore is isomorphic to $M_c(Z_2)$ and by Cor. 1 ideal of N; therefore N is the direct sum of two of its ideals (see [4] and [8]). The converse is trivial.

5. S-fields

At last we consider the particular case of the rings.

Definition B. We call s-field a ring whose proper subrings are fields.

Lemma 2. If A is an s-field, it is without nilpotent elements and has zero-divisors.

Proof. An s-field A can't have nilpotent elements because a nilpotent element can't generate A which should be a zero-ring, neither can it generate a proper subring which, in our hypotheses, must be a field. Let's suppose that A is without zero-divisors. In this case A has identity because we know by the hypotheses that it has subfields. Moreover, A is simple because each ideal of A, as subfield, contains the identity of A. But a simple ring, without zero-divisors, is a field and this is to be excluded.

Theorem 6. A ring A is an s-field if and only if $A = Z_p \dotplus Z_q$ with p and q as prime numbers.

Proof. By Lemma 2 it follows that an s-field is without nilpotent elements and has zero-divisors, then Th. 5 holds and so does the thesis.

Corollary 5. An s-field $A = Z_p \dotplus Z_q$ is 1-generated if and only if $p \neq q$.

Proof. Easy.

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