Feebly Canonical and 1-Hypergroups

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In the paper, two classes of hypergroups are introduced and investigated. The first one generalizes the classes of canonical hypergroups and complete commutative hypergroups. The second class is formed by hypergroups with the core of cardinality one.

V článku jsou zavedeny a studovány dvě třídy hypergrup. První zobecňuje třídy kompletních hypergrup a kanonických hypergrup. Druhá třída je tvořena hypergrupami, jejichž jádro má pouze jeden prvek.

В статье изученны два класса гипергрупп. Первый обобщает класс польных гипергрупп и второй состоит из гипергрупп, которые имеют ядро мощности 1.

Canonical hypergroups, studied both in the context of field theory and in that one of characters of finite groups, have many properties owned also by complete commutative hypergroups which are correlated with the theory of groupoids and quasigroups, for instance both of them are join spaces and regular reversible hypergroups. This paper concernes just a class (feebly canonical hypergroups), denoted by F.C., which contains strictly the union of the two classes and has a lot of their properties. Another class is considered here, which in the commutative case is near enough, as F.C., to that one of abelian groups. The core of a group has cardinality 1. The converse is not true. Just the class of 1-hypergroups, i.e. of those the core of which has cardinality 1, is introduced.

In the following, $E(H)$ is the set of identities of a hypergroup H, $i(P)$ is the set of the inverses of elements of P, a/b is the set $\{x; a \in x \,.\, b\}$, $\mathscr{C}(x)$ is the complete closure of $\{x\}$ and ω_H is the core of H.

Definition 1. A hypergroup H is called feebly canonical if it is commutative, regular, reversible and satisfies the following condition:

(i) $\forall x \in H$, if x', x'' are inverses of x, then $\forall a \in H$ we have $a \,.\, x' = a \,.\, x''$.

Examples. Canonical hypergroups and complete commutative hypergroups are feebly canonical. The following examples A and A' are feebly canonical but neither canonical nor complete.

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A. Let G be an abelian group and S a set. One supposes that $|G| > 1 < |S|$ and $G \cap S = \emptyset$. We define the hyperproduct as follows: $\forall (g, s) \in G \times S$, $g \cdot s = s \cdot g = S$; $\forall (g_1, g_2) \in G^2$, $g_1 \cdot g_2 = g_1 g_2$; $\forall (s_1, s_2) \in S^2$, $s_1 \cdot s_2 = G$. Let's pose $A = \langle G \cup S, \cdot \rangle$.

A'.	a	b	c	d
a	a	b	c, d	c, d
b	b	a	c, d	c, d
c	c, d	c, d	A'	A'
d	c, d	c, d	A'	A'

If P is a non-empty part of a regular hypergroup H, we denote by $i(P)$ the set of the elements of H which are inverses of some elements of P.

Lemma 1. Feebly canonical hypergroups are join spaces.

Let x be in the set $a/b \cap c/a$, that is, $a \in b \cdot x$, $c \in d \cdot x$, then there exists an element $b' \in i(b)$ such that $x \in b' \cdot a$, it follows that $c \in d \cdot b' \cdot a = a \cdot d \cdot b'$, therefore there exists an element $v \in a \cdot d$ such that $c \in v \cdot b'$, the reversibility of H implies that there exists an element $b'' \in i(b')$ such that $v \in b'' \cdot c$, then for (i), $b'' \cdot c = b \cdot c$, it follows $a \cdot c \cap b \cdot c \neq \emptyset$, and therefore H is a join space.

Lemma 2. Let H be a join space which has at least one identity and which satisfies the condition

(ii) $\forall (u, v) \in H^2$, $\forall e \in E(H)$, $\forall x \in H$, $u \cdot v \cap e \cdot x \neq \emptyset \Rightarrow x \in u \cdot v$.

Then H is reversible.

Suppose $a \in b \cdot x$, $e \in E(H)$, $b' \in i(b)$, $e \in b \cdot b'$; then $b \in a/x \cap e/b'$, it follows $a \cdot b' \cap x \cdot e \neq \emptyset$, thus, from (ii) we have $x \in a \cdot b'$, and hence H is reversible.

Lemma 3. If $H \in F.C.$ then H satisfies (ii).

Let e be an identity of H and suppose $a \in u \cdot v \cap x \cdot e$. Since $a \in x \cdot e$, there exists $x' \in i(x)$ such that $e \in x' \cdot a$, thus $\{x, a\} \subseteq i(x)$ and we have $z \cdot a = z \cdot x \ \forall z \in H$. From $a \in u \cdot v$ it follows that there exists an element $u' \in i(u)$ such that $v \in u' \cdot a$, then, since $u' \cdot a = u' \cdot x$, we have $v \in u' \cdot x$. Hence, there must be an element $u'' \in i(u')$ such that $x \in u'' \cdot v$. From (i) it follows $u'' \cdot v = u \cdot v$, thus $x \in u \cdot v$.

Lemma 4. If H is a join space which satisfies (ii) then H satisfies also (i).

Suppose $\{e, e_1\} \subseteq E(H)$, $e \in b'' \cdot b$, $e_1 \in b' \cdot b$, then we have $a/b'' \cap e_1/b' \neq \emptyset$, it follows $e \cdot b' \cap e_1 \cdot b'' \neq \emptyset$, hence $b'' \in e \cdot b'$. Now, if c is an element of H and $z \in c \cdot b'$, from $b'' \in e \cdot b'$ we have $b''/e \cap z/c \neq \emptyset$, therefore $b'' \cdot c \cap e \cdot z \neq \emptyset$, hence $z \in b'' \cdot c$ and $b' \cdot c \subseteq b'' \cdot c$. In like manner we find that $b'' \cdot c \subseteq b' \cdot c$, and thus (i) is satisfied.

Theorem 1. If H is a commutative hypergroup such that $E(H) \neq \emptyset$ then the following conditions are equivalent:

(1) H is a join space and satisfies (ii).

(2) H is reversible and satisfies (i).

It follows immediately from Lemmas 1, ..., 4.

Let's denote by RR the class of regular reversible hypergroups, by JS the class of join spaces, by $C(i)$ and by $C(ii)$ the classes of hypergroups which satisfy the conditions (i) and (ii), resp.

Theorem 2. 1)$M = RR \cap C(ii) \nsubseteq C(i)$ and $M \nsubseteq JS$.

2) $N = JS \cap C(i) \nsubseteq C(ii)$ and $N \nsubseteq RR$.

3) $P = R \cap JS \nsubseteq C(i)$ and $P \nsubseteq C(ii)$.

1) Let H be a set. Suppose $H = \bigcup_{i \in I} A_i \cup \{e\}$, where $|I| \geqq 2$, $e \notin \bigcup_{i \in I} A_i$, $A_i \cap A_j = \emptyset$ if $i \neq j$. Let's denote by A_i' the set $A_i \cup \{e\}$ for any $i \in I$. Let's set for any $(i, j) \in I^2$ and any $(x, y) \in A_i \times A_j$,

I) $e \,.\, e = e$, $x \,.\, e = e \,.\, x = A_i$,

II) $x \,.\, y = H$, if $i \neq j$; $x \,.\, y = A_i$, if $i = j$.

One can verify that $H \in M - C(i)$.

2) Taking into account the hypergroups, the tables of which follow, one sees that $U \in N - C(ii)$ and $V \in P - C(i)$.

$U = \{a, b, c\}$

	a	b	c
a	a	b	b, c
b	b	U	b, c
c	b, c	b, c	U

$V = \{p, q, s\}$

	p	q	s
p	V	V	V
q	V	p	p
s	V	p	p

Theorem 3. If $H \in DC$, then $\forall b \in H$, $\forall e \in E(H)$, $\forall b' \in i(b)$, we have $i(b) = e \,.\, b'$.

1) $e \,.\, b' \subseteq i(b)$.

If $z \in e \,.\, b'$, then $e \,.\, z \cap e \,.\, b' \neq \emptyset$, it follows $b' \in e \,.\, z$, thus there exists $e_1 \in E(H)$ such that $e_1 \in b' \,.\, b \subseteq e \,.\, z \,.\, b$. Then there exists $y \in z \,.\, b$ such that $e_1 \in e \,.\, y$, hence $y \in i(e)$. Since $e \in i(e)$, we have, for every $x \in H$, $x \in x \,.\, e = x \,.\, y$, i.e. $y \in E(H)$. Now, $z \,.\, b \in E(H) \neq \emptyset$, therefore $z \in i(b)$ and thus $e \,.\, b' \subseteq i(b)$.

2) $i(b) \subseteq e \,.\, b'$.

If $b'' \in i(b)$, there exists $e_1 \in E(H)$ such that $e_1 \in b'' \,.\, b$, hence $b \in e_1/b''$. Analogously, there exists $e_2 \in E(H)$ such that $b \in e_2/b'$. It follows $e_1 \,.\, b' \cap e_2 \,.\, b'' \neq \emptyset$, therefore $b'' \in e_1 \,.\, b'$ and $i(b) \subseteq e_1 \,.\, b'$. Since $\{e, e_1\} \subseteq i(e_1)$, we have $\forall s \in H$ $e \,.\, s = e_1 \,.\, s$ and thus $e_1 \,.\, b' = e \,.\, b'$.

Theorem 4. Let H be a regular join space such that $\forall x \in H$, $\forall x' \in i(x)$, $i(x) = e \,.\, x'$. Then H is reversible.

Suppose $a \in b \,.\, x$, $e \in E(H)$, $e \in x \,.\, x'$, then from $x \in a/b \cap e/x'$, it follows $a \,.\, x' \cap e \,.\, b' \neq \emptyset$. If $b_1 \in e \,.\, b \cap a \,.\, x'$, the hypothesis of the Theorem implies that $\forall b' \in i(b)$, we have $b_1 \in i(b')$, hence $e \,.\, b_1 = i(b')$. It follows $b \in b_1 \,.\, e \subseteq$ $\subseteq a \,.\, x' \,.\, e$, thus there exists $x'' \in x' \,.\, e = i(x)$ such that $b \in a \,.\, x''$, and so H is reversible.

Remarque. The hypergroup V which has been considered before, proves that the hypothesis of Th. 4 does not imply (i).

Theorem 5. Let H be a feebly canonical hypergroup. Then $\forall (x, y) \in H^2$, $\forall x' \in i(x)$, $\forall y' \in i(y)$, we have:

1) $x \,.\, y' = x/y$,
2) $x \,.\, x' = I_p(x) = I_p(x') = i(I_p(x)) = I_p(i(x))$,
3) $\forall e \in E(H)$, $\forall a \in I_p(x)$, $e \,.\, a \subseteq I_p(x)$,
4) if $\{a, b\} \subseteq I_p(x)$, there exists $q \in I_p(x)$ with $a \in b \,.\, q$.

To prove 1) and 2), it suffices to remember the reversibility of H and the condition (i). To prove 3), notice that for every $a' \in i(a)$, we have $e \,.\, a = i(a') \subseteq i(I_p(x))$ and apply 2). 4) From the hypothesis, if $x' \in i(x)$, then $b \in x \,.\, x' \subseteq a \,.\, x \,.\, x'$ and there exists $q \in x \,.\, x'$ such that $b \in a \,.\, q$.

Theorem 6. If $H \in F.C.$, $(x_1, \ldots, x_n) \in H^n$, $(x'_1, \ldots, X'_n) \in i(x_1) \times \ldots x\, i(x_n)$, then we have $x'_1 \,.\, x'_2 \ldots x'_n = i(x_1 \,.\, x_2 \ldots x_n)$.

Suppose $a \in x_1 \,.\, x_2$, $a' \in i(a)$; from the reversibility of H and from the condition (i), it follows $x_2 \in a \,.\, x'_1$. Then $x'_1 \in a' \,.\, x_2$, and afterwards $a' \in x'_1 \,.\, x'_2$. Suppose now that the result is true for any $k < n$; if $b \in x_1 \ldots x_n$ and $b' \in i(b)$, there exists $y \in x_1 \ldots x_{n-1}$ such that $b \in y \,.\, x_n$. By induction one obtains $y' \in x'_1 \ldots x'_{n-1}$ for any $y' \in i(y)$, and then, since $b' \in y' \,.\, x'_n$, it follows $i(x_1 \ldots x_n) \subseteq x'_1 \ldots x'_{n-1}$. Conversely, suppose $z \in x'_1 \ldots x'_n$ and $z' \in i(z)$; then $z \in i(z')$ and with respect to the first part of the proof, we have $z' \in x_1 \ldots x_n$, thus $z \in i(x_1 \ldots x_n)$.

Theorem 7. If $H \in F.C.$, then $\forall (x, y, z, t) \in H^4$, from $x \,.\, y \cap z \,.\, t \neq \emptyset$, it follows that for any $y' \in i(y)$ and any $z' \in i(z)$, we have $x \,.\, z' \cap t \,.\, y' \neq \emptyset$.

Suppose $u \in x \,.\, y \cap z \,.\, t$, $u' \in i(u)$, $z' \in i(z)$, $t' \in i(t)$. It is clear that $u \,.\, u' \subseteq$ $\subseteq x \,.\, y \,.\, z' \,.\, t' = x \,.\, z' \,.\, y \,.\, t'$. But if $e \in E(H)$, $e \in u \,.\, u'$ then $e \in x \,.\, z' \,.\, y \,.\, t'$ and there exist an element $v \in x \,.\, z'$ and an element $v' \in y \,.\, t'$ such that $e \in v \,.\, v'$. Thus, since $i(y \,.\, t') = t \,.\, y'$, one obtains $x \,.\, z' \cap t \,.\, y' \neq \emptyset$.

Definition 2. Let H be a feebly canonical hypergroup and let K be a subhypergroup of H . K is called a feebly canonical subhypergroup of H if $K \cap E(H) \neq \emptyset$.

Theorem 8. If $H \in F.C.$ and K is a subhypergroup of H, the following six conditions are equivalent:

1) K is a feebly canonical subhypergroup of H.
2) There exists an element $x \in K$ such that $K \cap i(x) \neq \emptyset$.
3) $E(H) \subseteq K$.
4) $\forall x \in K,\ i(x) \subseteq K$.
5) K is invertible.
6) K is closed as a subhypergroup of H.

It is clear that 1) and 2) are equivalent.

1) implies 3). Suppose $e_1 \in E(H) \cap K$, $e_2 \in E(H)$. Then from $e_1 \in e_1 . e_2 \cap e_1 . e_1$ it follows $e_2 \in e_1 . e_1 \subseteq K$, hence 3).

3) implies 4). If $x \in K$, $e \in E(H)$, there exists $x' \in K$ such that $e \in x . x'$. Then from Th. 3, one obtains $i(x) = e . x' \subseteq K$.

4) implies 5). Suppose $a \in K$ and $x \in a . y$. Since H is reversible, there exists $a' \in i(a)$ such that $y \in a' . x$ and thus $y \in K . x$.

5) implies 6). It is always true.

6) implies 1). Trivial.

Theorem 9. If $H \in F.C.$, then $E(H)$ is an invertible hypergroup.

Suppose $\{e_1, e_2\} \subseteq E(H)$, $q \in H$. Then for $z \in e_1 . e_2$, we have $z/e_1 \cap q/q \neq \emptyset$, since $q \in q . e_2$. Consequently, $z . q \cap e_1 . q \neq \emptyset$, and thus $q \in z . q$ and $z \in E(H)$. Therefore $E(H)$ is a subsemihypergroup. Now, let $x \in H$ be such that $e_2 \in e_1 . x$. Then, since $e_1 \in e_1 . e_1$, we have $e_2/x \cap e_1/e_1 \neq \emptyset$. further $e_1 . e_2 \cap e_1 . x \neq \emptyset$, and hence $x \in e_1 . e_2$ and $x \in E(H)$. One has shown that $E(H)$ is a closed subhypergroup, and therefore $E(H)$ is invertible by Th. 8.

Theorem 10. If $H \in F.C.$, then $\forall (x, y) \in H^2$, $\forall x' \in i(x)$, $\forall y' \in i(y)$, $\forall e \in E(H)$, we have:

1) $i(x . y) = x' . y' = i(x) . i(y)$,
2) $I_p(x . y) \subseteq I_p(x) . I_p(y)$,
3) $I_p^n(x) \subseteq (I_p(x))^n$,
4) $\forall k \geqq 1,\ i^{2k}(x) = e . x,\ i^{2k+1}(x) = i(x) = e . x'$.

1) follows from Th. 6.

2) If $a \in I_p(x . y)$ then there exists $z \in x . y$ such that $a \in z . z' \subseteq x . y . x' . y' = x . x' . y . y' = I_p(x) . I_p(y)$.

3) follows from 2) and Th. 5.

4) From 1) one obtains $i(e . x') = i(e) . i(x') = e . x$ and $i(e . x) = i(e) . i(x) = e . x'$. The rest is clear by induction.

Definition 3. A hypergroup H is called 1-hypergroup if $|\omega_H| = 1$.

We shall denote by $1 - \Omega$ the class of the 1-hypergroups.

Theorem 11. If H is an 1-hypergroup and we set $\omega_H = \{e\}$, then we have:

(1) the β^*-classes are the products $e \,.\, a$ where $a \in H$,

(2) H is regular and reversible,

(3) $\forall x \in H$, $\forall x' \in i(x)$, $i(x) = \mathscr{C}(x')$.

(1) It is obvious.

(2) $\forall x \in H$ we have by (1), $x \,.\, e = x \,.\, e = \mathscr{C}(x)$ and $x \in \mathscr{C}(x)$; it follows $e \in E(H)$. For any $x \in H$, there exists an element $x' \in H$ such that $e \in x' \,.\, x$. But $x \,.\, x' \subseteq \omega_H = \{e\}$, then $x \,.\, x' = e$, hence also $x' \,.\, x = e$ and thus H is regular. Further, if $y \in x \,.\, z$, there exists $v \in H$ such that $x \in y \,.\, v$, therefore $y \in y \,.\, v \,.\, z$, and hence there exists $a \in v \,.\, z$ such that $y \in y \,.\, a$. Consequently, $v \,.\, z \cap \omega_H \neq \emptyset$, then $v \,.\, z = e$ and thus $v = z'$ is an inverse of z. Finally, $x \in y \,.\, z'$, i.e. H is reversible.

(3) First, we prove that $\mathscr{C}(x') \subseteq i(x)$. If $y \in \mathscr{C}(x')$, then, by (1), $y \in x' \,.\, e$, hence $x \,.\, y \subseteq x \,.\, x' \,.\, e = e \,.\, e = e$ and we have $y \in i(x)$. Now, the inclusion $i(x) \subseteq \mathscr{C}(x')$. If $x \,.\, z = e$, we have $x' \,.\, x \,.\, z = x' \,.\, e = C(x')$, therefore $e \,.\, z = \mathscr{C}(x')$ and $z \in \mathscr{C}(x')$.

Theorem 12. Let H be an 1-hypergroup. If $|H| \leqq 4$, then H is either a group or, up to isomorphic copies, one of the following hypergroups (1), (2), (3) (in this case, H is complete).

(1)

	e	b	c
e	e	b, c	b, c
b	b, c	e	e
c	b, c	e	e

(2)

	e	a	b	c
e	e	H'	H'	H'
a	H'	e	e	e
b	H'	e	e	e
c	H'	e	e	e

where $H' = \{a, b, c\}$ and

(3)

	e	a	b	c
e	e	a, b	a, b	c
a	a, b	c	c	e
b	a, b	c	c	e
c	c	e	e	a, b

We shall show that in the case $|H| = 4$, if one supposes $e \,.\, a = \{a, b\}$, one obtains necessarily the hypergroup (3). In the other cases, one proceeds similarly. From the hypothesis, it follows $e \,.\, b = \{a, b\} = b \,.\, e$, $e \,.\, c = c \,.\, e = c$. Since $\omega_H = e$, we have $a \,.\, c \cap \{a, c\} = \emptyset$ and then $a \,.\, c = e$, otherwise $a \,.\, c = b$ would imply, by Th. 11(2), $c \in a' \,.\, b$, where $a' \in i(a)$, and hence $\varphi_H(c) = 1$. Moreover, similarly, one obtains $a \,.\, a = a \,.\, b = b \,.\, a = b \,.\, c = c \,.\, b$. Finally, from Th. 11(2), it follows $c \,.\, c = \{a, b\}$. To complete the proof, one has only to verify the associativity.

Theorem 13. For any cardinal $n \geqq 5$, there exists a 1-hypergroup which is not complete and has cardinality n.

Let's set $H_n = \{e\} \cup A \cup B$, where $|A| \geqq 2 \leqq |B|$, $A \cap B = \emptyset$, $e \notin A \cup B$, and let's fix an element b_1 in B. Define the following hyperoperation in H_n:

$\forall a \in A,\ a \cdot a = b_1,\ \forall (a_1, a_2) \in A^2$ such that $a_1 \neq a_2,\ a_1 \cdot a_2 = B$.

$\forall (a, b) \in A \times B,\ a \cdot b = b \cdot a = e,\ \forall (b_1, b_2) \in B^2,\ b_1 \cdot b_2 = A$.

H_n is a hypergroup which is clearly not complete but satisfies the condition $|\omega_H| = 1$. The hypergroups obtained in this way are not the unique non-complete 1-hypergroups as one can see from the following examples; S for n = 5 and T for n = 7 where $A = \{a, b\}$, $C = \{c, d\}$, $P = \{p, q\}$ and $T = A \cup C \cup P \cup \{e\}$.

S	e	a	b	c	d
e	e	a, b	a, b	c, d	c, d
a	a, b	c, d	c	e	e
b	a, b	c	c, d	e	e
c	c, d	e	e	a, b	a, b
d	c, d	e	e	a, b	a, b

T	e	a	b	c	d	p	q
e	e	A	A	C	C	P	P
a	A	c	d	P	P	e	e
b	A	d	c	P	P	e	e
c	C	P	P	e	e	A	A
d	C	P	P	e	e	A	A
p	P	e	e	A	A	c	d
q	P	e	e	A	A	d	c

Theorem 14. Let H be a hypergroup which satisfies the following condition:
(t) $\forall (x, y) \in H^2,\ x \cdot y \cap \omega_H \neq \emptyset \Rightarrow x \cdot y = \omega_H$.

Let A, B be subhypergroups of H. Then the following conditions are equivalent:

(1) $A \cap B = \omega_H$.

(2) $\forall (a_1, a_2) \in A^2,\ \forall (b_1, b_2) \in B^2,\ a_1 \cdot b_1 \cap a_2 \cdot b_2 \neq \emptyset \Rightarrow \mathscr{C}(a_1) = \mathscr{C}(a_2)$, $\mathscr{C}(b_1) = \mathscr{C}(b_2)$.

(1) $\Rightarrow$ (2)

The condition (t) is equivalent to (t'): $\forall n \geqq 2,\ \forall (x_1, \ldots, x_n) \in H^n,\ \prod_{i=1}^{n} x_i \cap \omega_H \neq \emptyset \Rightarrow \prod_{i=1}^{n} x_i = \omega_H$. Suppose now $a_1 \cdot b_1 \cap a_2 \cdot b_2 \neq \emptyset$ and let a'_2, b'_1 be such that $a'_2 \cdot a_2 = \omega_H = b_1 \cdot b'_1$. Then, since $a'_2 \cdot a_1 \cdot b_1 \cdot b'_1 \cap a'_2 \cdot a_2 \cdot b_2 \cdot b'_1 \neq \emptyset$, one obtains $a'_2 \cdot a_1 \cdot \omega_H \cap \omega_H \cdot b_2 \cdot b'_1 \subseteq A \cap B = \omega_H$. Thus there exists $\{z, w\} \subseteq \omega_H$ such that $a'_2 \cdot a_1 \cdot z = w \cdot b_2 \cdot b'_1 = \omega_H$ and it follows $a'_2 \cdot a_1 = \omega_H = b_2 \cdot b'_1$. If one multiplies this equality by a_2 on the left, one obtains $\omega_H \cdot a_1 = a_2 \cdot \omega_H = \omega_H \cdot a_2$. Similarly, $\omega_H \cdot b_1 = \omega_H \cdot b_2$.

(2) $\Rightarrow$ (1)

Suppose $e \in A \cap \omega_H$ and let a be an element of A. There exists $a' \in A$ such that $e \in a \cdot a'$ and then $a \cdot a' \cap \omega_H \neq \emptyset$. Consequently, $a \cdot a' = \omega_H$ and $\omega_H \subseteq A$. Similarly, $\omega_H \subseteq B$ and $\omega_H \subseteq A \cap B$. Conversely, let q be in $A \cap B$. Let e_1 be a partial left identity of q in A and let e_2 be a partial right identity of q in B. We have $q \in e_1 \cdot q \cap q \cdot e_2$, then (2) implies $\mathscr{C}(e_1) = \mathscr{C}(q)$, therefore $q \in \omega_H$ and $A \cap B \subseteq \omega_H$.

Remark. The class of 1-hypergroups and that of complete hypergroups (Co) are contained in the class T of hypergroups satisfying (t).

Theorem 15. If H is a quasicanonical hypergroup then H is a group if and only if $H \in 1 - \Omega$.

Suppose $H \in 1 - \Omega$ and $\{a, b\} \subseteq H$. If $\{c, d\} \subseteq a . b$, we have $a \in c . b' \cap d . b'$, therefore $a . b \subseteq c . b' . b \cap d . b' . b$; but $c . b' . b = c . e = d . e$, and thus $a . b = c = d$.

Theorem 16. If $H \in F.C.$ then, from $H \in 1 - \Omega$, it follows $H \in Co$.

We shall show that $\forall (u, v) \in H^2$, $\forall x \in u . v$, $u . v = \mathscr{C}(x)$. From $x \in u . v$ it follows $u . v \subseteq \mathscr{C}(x) = e . x$. If $y \in x . e$ then $y . e = \mathscr{C}(y) = x . e$, $u . v \subseteq e . y$, $u . v \cap e . y \neq \emptyset$, and hence $y \in u . v$ and $u . v = \mathscr{C}(x)$.

Theorem 17. Let H be in $JS \cap C(i)$. Then $H \in 1 - \Omega$ implies $H \in Co$.

Since $H \in 1 - \Omega$, we have $i(x) = e . x'$ $\forall x \in H$, $\forall x' \in i(x)$ by Th. 3. Then, Th. 4 implies that H is reversible. Consequently, $H \in F.C.$ and $H \in Co$ by Th. 16.

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