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Statistical models of earthquake occurrences

Terms of use:

Statistical Models of Earthquake Occurrences

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Statistickými zákony bylo vyšetřováno časové rozdělení zemětřesného výskytu v Evropě. Negativně binomické rozdělení popisuje lépe výskyt mělkých otřesů než jednoduchý Poissonův proces, protože dovoluje v časové jednotce výskyt více než jednoho zemětřesení.

The time distribution of earthquake occurrence in the European area was investigated by the statistical laws. The negative binomial distribution describes the occurrence of shallow shocks better the simple Poisson process because it allows for the occurrence of more than one earthquake in a time unit.

Статистическими законами было исследовано временное распределение появления землетрясений в Европе. Отрицательное биномальное распределение описывает лучше появление мелких толчков чем простое распределение Пуассона, потому что позволяет появление больше чем одного землетрясения во временном участке.

Earthquake occurrences can be modelled as a stochastic point process. Earthquakes correspond to points randomly distributed along a time axis. The probability structure of a point process is described by the distribution of the number of events within a particular interval. The process is stationary if its probability structure is invariant under translations of the time axis. Because of its simplicity and of its success in describing many phenomena the Poisson process is one used as the first approximation of earthquake data. The Poisson process is defined by the basic conditions:

(a) independence — the number of events in any time interval is independent of the number of events in any other nonoverlapping interval,

(b) stationarity — the probability of one event occurring in a short time interval dt is $\lambda \, dt$, where λ is a constant in time,

(c) orderliness — the probability of more than one event occurring in the same time interval is asymptotically negligible.

The probability distribution of a Poisson process is

$$P(\lambda)_i = \exp\left(-\lambda\right) \lambda^i/i! \,, \quad \lambda > 0, \, i = 0, 1, 2, \ldots, \tag{1}$$

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where $P(\lambda)_i$ is the probability that in an arbitrary interval of unit length i events will occur with the intensity λ. The mean and variance of the number of events per unit time interval are both equal to the parameter λ.

For testing the actual distribution with the Poisson distribution the χ^2 test is used under the assumption that the theoretical frequencies $n\,P(\lambda)_i \geqq 2$, where n is the length of the observation period. If we denote by s_i the number of years with the i-th annual number of earthquakes, then the test statistics

$$\chi_0^2 = \sum_{i=0}^{t} [s_i - n\,P(\lambda)_i]^2 / n\,P(\lambda)_i \tag{2}$$

has a χ^2 distribution with $t - 2$ degrees of freedom and the parameter of the Poisson distribution

$$\lambda = \sum_{i=0}^{t} i s_i / \sum_{i=0}^{t} s_i . \tag{3}$$

The parameters of the Poisson distribution λ, the test statistics χ_0^2 and the critical values χ_α^2 are tabulated.

The most important characteristics of earthquake time series, which the Poisson process does not take into account, is the tendency of earthquakes to occur in groups. The assumption inherent in the Poisson series is that the probability of an event remains constant, equal to the parameter λ, which in practice is rarely true.

Any variation in the expectation of an event, in particular for one event to increase the probability of another one will increase the variance of the distribution relative to the mean and a negative binomial distribution will invariably describe the data better.

Writing the negative binomial distribution in the form $p^k(1 - q)^{-k}$, the probability of i events is given by

$$NB(k, p)_i = \binom{i + k - 1}{k - 1} p^k q^i, \quad i = 0, 1, 2, \ldots \tag{4}$$

when k, p are parameters, $p + q = 1$. The mean of the negative binomial distribution is
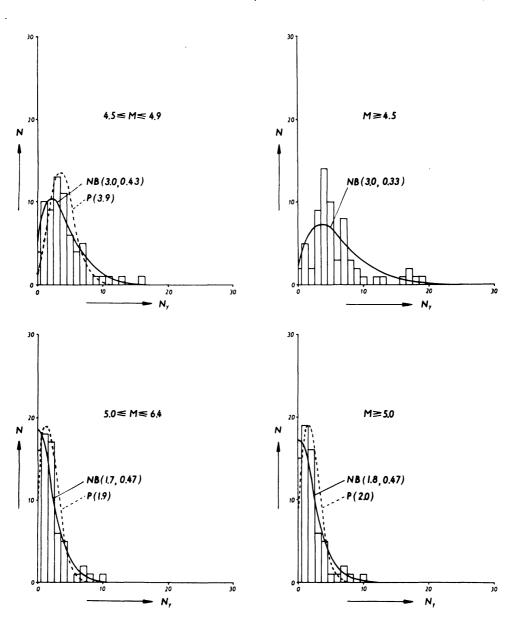
$$M = kq/p , \tag{5}$$

and the variance

$$V = kq/p^2 . \tag{6}$$

Thus, as p is necessarily less than unity, the variance is always greater than the mean, while for the Poisson distribution the variance is equal to the mean λ. Having derived the negative binomial distribution as a generalized Poisson series, it is evident that Poisson distribution is obtained as a limiting form of $p^k(1 - q)^{-k}$.

There are several methods of estimating the parameters p and k. One of them is the method of moments. If the first two moments of the negative binomial distribu-

tion are estimated from the sample moments, then the ratio of the mean to the variance provides an estimate of p, i.e. if the mean of the samples is m and the variance is s^2, then

$$p = m/s^2 \, , \tag{7}$$

Fig. 1. Histogram for frequency of occurrence of earthquakes in the Alps and the Apennines.

when $m = kq/p$ and noting that $q = 1 - p$ an estimate of k is given by

$$k = mp/(1 - p).\qquad(8)$$

The values of the parameters p, k are tabulated.

To test the observed distribution with the negative binomial distribution the χ^2 test is used, too.

In Figure 1 the histograms for the frequency of earthquake occurrence, in one-year intervals, are given for one European region (number of years with the given annual number of earthquakes N versus number of shocks in one-year intervals N_y). In those cases where the hypotheses are not rejected the curves for the Poisson distribution with the parameter λ are shown by a dashed line and the curves for the negative binomial distribution with the parameters p, k by a solid line.

It is evident that the negative binomial distribution fits the observed frequencies of occurrence better than the Poisson distribution. If the lower magnitude limits is raised the mean time interval between events becomes longer and longer. This step decreases the likelihood of a possible interaction between successive earthquakes, and one can therefore expect the time series to tend toward a simple Poisson sequence. The rate of occurrence of aftershocks decays roughly as the reciprocal of the time elapsed after the main shock. Their maximum magnitude is usually one or two units below the magnitude of the main shock, hence a large proportion of aftershocks will be filtered out every time the magnitude threshold of the data is raised. It can be generally concluded that the process with the negative binomial entries as a model describing the occurrence of shallow earthquakes in Europe is more appropriate than a simple Poisson process. It has a more general validity for different European earthquake provinces because it can accommodate also the occurrence of aftershocks.