Acta Universitatis Carolinae. Mathematica et Physica

S. Kříž

A note on seemingly different models of the disk accretion in binary stars

Acta Universitatis Carolinae. Mathematica et Physica, Vol. 21 (1980), No. 2, 3--6

Persistent URL: http://dml.cz/dmlcz/142449

1980 ACTA UNIVERSITATIS CAROLINAE — MATHEMATICA ET PHYSICA Vol. 21. No. 2

A Note on Seemingly Different Models of the Disk Accretion in Binary Stars

S. KŘÍŽ

Department of Astronomy and Astrophysics, Faculty of Mathematics and Physics, Charles University, Prague*)

Received 14 January 1980

The accretion model given in [3] and [4] differs considerably from the models given in [1] and [2] because an incorrect solution of equations has been used in [3] and [4]. The proper treatment leads to the perfect agreement.

Модель диска, которая была рассмотренна в [3] и [4] значительно отличается от моделей из [1] и [2] из-за плохого решения уравнений в работах [3] и [4]. Правильный подход дает хорошое согласие.

Akreční model uvedený v [3] a [4] se značně liší od modelů uvedených v [1] a [2], protože v [3] a [4] bylo provedeno chybné řešení rovnic. Správný postup vede k dokonalé shodě.

Long ago before the theory of accretion disks was developed by Shakura and Sunyaev [1] and by Lynden-Bell nad Pringle [2] Gorbatsky had published a pioneer work [3] concerning a rotating disk-shaped envelope around one component of a binary star. The principal significance of the viscosity and turbulence has been corectly recognized in [3] but the resulting formulae are quite different from the corresponding formulae given in [1] and [2]. Recently, Gorbatsky [4] (chapter 6, §4 of his book) has again repeated his model and concluded that the results obtained in [2] cannot be applied to the steady disks in close binaries. Because of the great importance of accretion processes in close binaries we shall try to explain why Gorbatsky's results are so different from the other ones.

The principal equations of [1] and [2] were obtained from considerations concerning the transport of the angular momentum in the disk while Gorbatsky's [3], [4] formulae governing the steady-state accretion were derived directly from the basic equation of motion. Let us follow Gorbatsky's process. The equation of motion can be written

$$\varrho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla p - \varrho \nabla \Phi + \mathbf{F}^{\mathrm{v}}, \tag{1}$$

*) 150 00 Praha 5, Švédská 8, Czechoslovakia.

where v is the velocity, ϱ the density, p the pressure, t time and Φ the gravitational potential. $\boldsymbol{F}^v$ is the viscosity force given in Gorbatsky's book [4] [see his equations (1.66), (1.67), (1.69)] in the following form:

$$F_i^v = \sum_j \frac{\partial}{\partial x_j}\left[\eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}(\nabla v)\right]. \tag{2}$$

η is the dynamic viscosity. We shall assume that the disk is stationary, the velocities in the disk are independent of the height z above the central plane, $v_z = 0$, the gravity is given by a central body only and there is a cylindrical symmetry. Then in cylindrical coordinates (R, φ) we find

$$v_R \frac{\mathrm{d}v_R}{\mathrm{d}R} - \frac{v^2}{R} = -\frac{1}{\varrho}\frac{\mathrm{d}p}{\mathrm{d}R} - \frac{\mathrm{d}\Phi}{\mathrm{d}R} + \frac{1}{\varrho}\left\{\frac{\mathrm{d}}{\mathrm{d}R}\left[\frac{2}{3}\eta\left(2\frac{\mathrm{d}v_R}{\mathrm{d}R} - \frac{v_R}{R}\right)\right] + \right.$$
$$\left. + \frac{\eta}{R}\left(\frac{\mathrm{d}v_R}{\mathrm{d}R} - \frac{2v_R}{R}\right)\right\}, \tag{3}$$

$$v_R \frac{\mathrm{d}v_\varphi}{\mathrm{d}R} + \frac{v_R v_\varphi}{R} = \frac{1}{\varrho}\left\{\frac{\mathrm{d}}{\mathrm{d}R}\left[\eta\left(\frac{\mathrm{d}v_\varphi}{\mathrm{d}R} - \frac{v_\varphi}{R}\right)\right] + \frac{2\eta}{R}\left(\frac{\mathrm{d}v_\varphi}{\mathrm{d}R} - \frac{v_\varphi}{R}\right)\right\}. \tag{4}$$

Equation (3) differs somewhat from the corresponding one given in [3], [4]. This likely means that an another expression of the stress tensor was used than the one given in chapter 1 of Gorbatsky's book [4]. However, this difference is insignificant. Providing $v_R \ll v_\varphi$ and the pressure to be negligible equation (3) only tells us that v_φ corresponds to the Keplerian motion in accord with the conclusion of [3] and

$$v_\varphi = \sqrt{\left(\frac{GM}{R}\right)}, \tag{5}$$

where M is the mass of the central star and G the constant of gravity.

However, the next Gorbatsky's step is logically wrong. He has inserted thus derived Keplerian v_φ back into the equation corresponding to (3) and determined v_R. One equation does not allow us to compute two unknowns in such a way. Consequently, Gorbatsky's formula for v_R is erroneous.

Let us introduce the surface density in the usual way as

$$\sigma = \int_{-\infty}^{+\infty} \varrho \,\mathrm{d}z\,, \tag{6}$$

kinematic viscosity $\nu = \eta/\varrho$, and its mean value

$$\bar{\nu} = \int_{-\infty}^{+\infty} \nu\varrho \,\mathrm{d}z/\sigma\,. \tag{7}$$

Peforming the integration of (4) over z we find

(8) $$\sigma v_R\left(\frac{\mathrm{d}v_\varphi}{\mathrm{d}R}+\frac{v_\varphi}{R}\right)=\frac{\mathrm{d}}{\mathrm{d}R}\left[\sigma\bar{\nu}\left(\frac{\mathrm{d}v_\varphi}{\mathrm{d}R}-\frac{v_\varphi}{R}\right)\right]+2\sigma\bar{\nu}\left(\frac{\mathrm{d}v_\varphi}{\mathrm{d}R}-\frac{v_\varphi}{R}\right).$$

The outward flux of material through a cylinder of radius R is given by the equation of continuity as

(9) $$F = 2\Pi R\sigma v_R\,.$$

Equations (8) and (9) agree well with the corresponding ones by Gorbatsky. Notice that Gorbatsky's $\tilde{\eta} = \sigma\bar{\nu}$.

Gorbatsky has assumed $\tilde{\eta}$ to be constant. But equation (8) can be solved without this unfounded assumption in the following way. Substituting the angular velocity $\Omega = v_\varphi/R$ and the specific angular momentum $h = v_\varphi R$ for the complicated functions of v_φ in (8) and using (9) we obtain

(10) $$(-F)\frac{\mathrm{d}h}{\mathrm{d}R}=\frac{\mathrm{d}g}{\mathrm{d}R}\,,$$

where

(11) $$g = -2\Pi\sigma\bar{\nu}R^3\frac{\mathrm{d}\Omega}{\mathrm{d}R}\,.$$

Since F is constant we have

(12) $$(-F)\,h = g - g_0\,.$$

Here g_0 is a constant of integration. Our equations (10), (11), (12) perfectly corresponds to the equations (4), (1), (5) by Lynden-Bell and Pringle [2]. Therefore the further quantities follow from [2]. For the sake of clarity we shall yet write some resulting formulae:

(13) $$\sigma = \frac{(-F)\,[1-\sqrt{(R_*/R)}]}{3\Pi\bar{\nu}}$$

and

(14) $$v_R = -\frac{3\bar{\nu}}{2R[1-\sqrt{(R_*/R)}]}\,.$$

R_* is the radius of the mass-gaining star. (13) and (14) can be derived from (12) and (9) using (5) and the relation $g_0 = F\sqrt{(GMR_*)}$ derived in [2].

On the contrary to our approach Gorbatsky [3], [4] has solved (8) as a second order differential equation using the above mentioned assumption $\tilde{\eta} = \text{const}$. He has derived

(15) $$v_\varphi = \frac{C_1}{2+C/\tilde{\eta}}R^n + C_2/R\,,$$

where $n = 1 + C/\tilde{\eta}$ and C is related to our F by the obvious relation $F = 2\Pi C$. To obtain an agreement with (5) Gorbatsky has concluded that $C = -3\tilde{\eta}/2$. This condition combined with (9) could provide us with a good approximation $\sigma = (-F)/(3\Pi\bar{v})$ which is valid providing $R \gg R_*$. Unfortunately, Gorbatsky has combined an equation corresponding to (9) with his uncorrectly derived expression for v_R and concluded that $\sigma \sim 1/\sqrt{R}$.

Consequently, the resulting formulae of [3] and [4] are incorrect. But it should be mentioned that the equations derived here as well as in [1] and [2] are also incorrect if we study the very outer region of the accretion disk because our basic assumptions are not fullfiled. The cylindrical symmetry is disturbed by the gaseous stream from the secondary component as well as by the gravity of the secondary component which is not negligible. In this region, the excessive angular momentum – which is transported from the inner parts of the disk due to the viscous stress – is probably removed away by the tidal effects mentioned by Paczynski [5] and/or by a direct interaction of the disk with the secondary component. Of course, some loss of matter from the binary to its vicinity is not also excluded.

References

[1] Shakura N. I., Sunyaev R. A., Astron. Astrophys., *24* 1973, 337.

[2] Lynden-Bell L., Pringle J. E., Monthly Not. Roy. Astron. Soc., *168* 1974, 603.

[3] Gorbatsky V. G., Trudy Astron. Obs. Leningrad Univ., *22* 1965, 16.

[4] Gorbatsky V. G., Kosmicheskaya Gazodinamika, Moscow, 1977.

[5] Paczynski B., Astrophys. J. *216* 1977, 822.