Acta Universitatis Carolinae. Mathematica et Physica

M. K. Fage; G. A. Maximey

On approximate solution of systems of linear ordinary differential equations

Acta Universitatis Carolinae. Mathematica et Physica, Vol. 15 (1974), No. 1-2, 17

Persistent URL: http://dml.cz/dmlcz/142318

On Approximate Solution of Systems of Linear Ordinary Differential Equations

M. K. FAGE, G. A. MAXIMEY
Computer Center, Novosibirsk

The Cauchy problem

$$\vec{y}'(x) = P(x)\,\vec{y}(x) + \vec{f}(x), \qquad \vec{y}(x_0) = \vec{y}_0\,, \tag{1}$$

($P(x)$ — a square matrix) has the exact solution

$$\vec{y}(x) = M(x, x_0)\,\vec{y}_0 + \int_{x_0}^{x} M(x, s)\,\vec{f}(s)\,\mathrm{d}s\,, \tag{2}$$

with

$$M(x, s) = E + \int_s^x P(u)\,\mathrm{d}u + \int_s^x P(u)\,\mathrm{d}u \int_s^u P(v)\,\mathrm{d}v + \ldots\,. \tag{3}$$

The authors establish the following method of approximate solution of (1): 1^0 — fix x_0 and $x = x_0 + h(h > 0)$; 2^0 — take a partial sum $M_R(x, s)$ of the series (3) with integrals of multiplicity $\leq R$; 3^0 — substitute $M_R(x, s)$ into (2) and transform all the integrals into the integrals over unit cubes of corresponding dimensions; 4^0 — to the obtained integrals apply Sobol quadrature formulae over Π_τ meshes (J. M. Sobol, Multidimensional quadrature formulae and Haar functions, Moscow, 1969).

The equalization of the error obtained by rejecting the remainder of the series (3), to the error of the quadrature formulae permits of estimating the number of points of quadrature meshes.

The methods of this kind allow as well: (1) — to compare the choice of the step h with the selection of other parameters of the procedure (of an integer R and of quadrature formulae parameters); (2) — to formulate the problem of optimal combination of all parameters.