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A Note on the Cometary Nucleus

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Recent perihelion and a unique aphelion photometric data of Comet Encke are used for a preliminary discussion concerning the nature and size of the cometary nucleus. In the empirical photometric formula the exponent $n \sim 4$ indicates very moderate evaporation heat. Two different processes of the gas liberation, one acting only near the perihelion passage, are proposed. The size of the nucleus is estimated from the assumed gas production rate and compared with results obtained by another method based on the known dimensions of Phobos. The obtained results indicate that a considerable part of the nucleus surface is nonvolatile. The effect of surface inhomogeneities and variability of albedos on the secular changes of the gas production is discussed.

I. Introduction

A comparison of photometric and spectroscopic data of a comet observed near the aphelion point with similar observational results obtained during the perihelion passage may provide some valuable information about the cometary nuclei. A considerable decrease of the gas production at a large heliocentric distance for a comparatively long time interval means that the contribution of the cometary atmosphere to the intrinsic luminosity of the objects is comparable to or even negligible in respect to the brightness of the entire nucleus. Therefore photometric data obtained at the aphelion passage should be considered most representative for the estimation of the nucleus size. The very prospective methods including photometric measurements in the visual range combined with those in the infrared, as proposed for satellite studies [1], should be used in the future. However, recently obtained data may be exploited, too.

Observations of periodic comets near their aphelia are exceptional. Only one of these comets, Schwassmann-Wachmann 1, may be observed every year near its opposition with the Sun, the eccentricity of the orbit of this comet being $e = 0.132$ only, perihelion distance $q = 5.538$ AU and aphelion distance $Q = 7.21$ AU. The periodic comet Oterma was also observable during the period 1943—1962 each year near its opposition with the Sun. During this period Comet Oterma moved also in a nearly circular orbit with the eccentricity $e = 0.144$ ($q = 3.388$ AU, $Q = 4.54$ AU). The perturbations due to the approach to Jupiter at 10^{-1} AU in 1963 changed its orbit and the former aphelion became the perihelion ($q = 5.88$ AU, $Q = 8.31$ AU). Bouška [2] showed that the apparent brightness of Comet Oterma is now far beyond the possibilities of the largest telescopes and therefore the comet must be considered a lost one. It has not been observed after 1962, indeed.

However, it is obvious that these objects are in some sense exceptional among comets because of considerable dust and gas production, large heliocentric distances and small orbital eccentricity. Fortunately a third comet recently observed at aphelion, P/Encke, has physical as well as orbital properties which are most suitable for the above mentioned studies.

Comet Encke moves in a relatively eccentric orbit [3] ($e = 0.847$, $q = 0.339$ AU) with the aphelion distance $Q = 4.09$ AU. It is the most observed periodic comet which has been recovered during 48 returns after its discovery by P.F.A. Méchain in 1786. The last perihelion passage of Comet Encke occurred on 1971 January 9.9 ET. The period of this comet is $P = 3.302$ years, and therefore it was at aphelion on 1972 September 3.9. The recovery observation on 1972 August 15 by Roemer and McCorkle [4] occurred 19 days before the aphelion passage.

2. Recent Observations of Comet Encke

At the last perihelion return Comet Encke was recovered on 1970 September 26.4 UT by Roemer [5] with the Steward Observatory reflector on Kitt Peak. The comet was of stellar appearance and its photographic magnitude was 18^m. After this recovery observation the comet was photographed by van den Bergh [6] on 1970 October 4, with the 122-cm Palomar Schmidt telescope. The comet, of magnitude about 17^m, had a stellar nucleus and a very faint coma. Waterfield and South [6] photographed the comet on 1970 October 21.8 UT at the Woolston Observatory. The 80-min. exposure showed the comet as a circular central condensation 20″ in diameter, with an extremely faint, diffuse outer coma about 1.5′ in diameter. The "total" magnitude was $m_1 = 14.5^m$.

Further observations were made by Chernykh at the Crimean Astrophysical Observatory [7]. On 1970 November 1.7 UT the comet was diffuse, with a circular coma of diameter 3′ and a starlike eccentric nucleus. The "nuclear" magnitude was $m_2 = 15^m$. Another observation was made on 1970 December 21.7 UT; the comet was diffuse with a very faint condensation and the "total" magnitude was 7–8^m [3].

Bortle [7], Stormville, New York, observed the comet on 1970 November 17.0 UT as a very large, nebulous mass of magnitude 9.1^m. Further the following observations were reported [8]: On 1970 November 30.7 UT Locher, Grüt-Wetzikon, estimated the magnitude $m_1 = 9.5^m$ and on December 6.7 UT $m_1 = 9.5^m$.

respectively. According to T. Seki, Kochi Observatory, on 1970 December 8.5 UT the "total" magnitude was $m_1 = 8.5^m$(*).

An attempt to observe the comet at aphelion was successful in 1972. Roemer [4] reported that this comet was photographed by herself and McCorkle on 1972 August 15 with the Steward Observatory 229-cm reflector on Kitt Peak. On the two

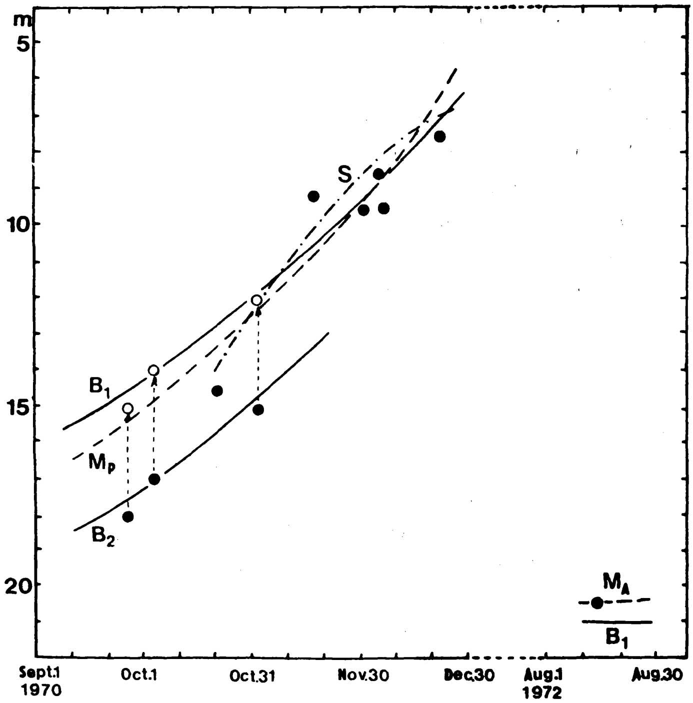

Fig. 1.

90-min. exposures (in particularly good seeing), taken on August 15.3 and 15.4 UT, the images of the comet were round and stellar, of magnitude of about 20.5^m. Comet Encke was also photographed by McCrosky and Shao [4] on 1972 September 5.2 UT with the 155-cm reflector at the Harvard College Observatory, Agassiz Station, and further by Roemer and Gonzales [4] on 1972 September 13.2 and September 13.3 UT. These last exposures were taken primarily for positional purposes and no data about magnitudes or physical appearance were given.

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*) Marsden (Comets in 1970, Quart. J. R. Astr. Soc. 12, 260, 1971) quoted other following observations: "By December 7, Bortle was estimating a total magnitude of 6.9^m. On December 15 and 21, Beyer noted visually some faint rays extending 4′ somewhat north of west, the total magnitude being 6.3^m on the latter date. Also on December 21, Simmons estimated a magnitude of 5.9^m, while Beyer made it 6.1^m the next evening." Those data are not plotted in Fig. 1. However, they lead practically to the same conclusion as follows from the used observations. (Note added in proof.)

Nevertheless, these observations at least confirm that the successful detection of the comet during the aphelion passage was not due to some unusual brightness outburst.

It is also interesting to compare the predicted magnitudes. Marsden [9] computed the brightness of P/Encke using the classical formula and assuming the absolute magnitude $m_0 = 11.5$ and the photometric exponent $n = 6$. The magnitudes according to this formula are shown in Fig. 1 by the curve denoted M_P.

Another brightness ephemeris was computed by Sekanina [10]. His predicted magnitudes are "total" (i.e. m_1), in the international photovisual system, and refer to observations by the method applied by Beyer [11]. These predictions were based on an analysis of Beyer's preperihelion observations at the apparitions of 1937, 1947, 1951 and 1961, and a supposed secular brightness decrease, on the average of 0.1^m per revolution, was taken into account as well. The author notes that the post-perihelion prediction is very uncertain and it is based mostly on the 1964 apparition and reflects the observed perihelion asymmetry in the comet's light curve. The mentioned computed magnitudes are also shown in Fig. 1, by the curve denoted S. It seems to be a somewhat unadequately elaborate method for determining the predicted magnitude.

In the "aphelion" ephemeris Marsden [12] computed the comet's magnitude assuming the values of absolute magnitude $m_0 = 9$ and of photometric exponent $n = 6$. A different absolute magnitude in regard to the former value [9] evidently was used to get a good agreement with the observed brightness of the comet by Roemer. These magnitudes of Marsden are denoted by M_A in Fig. 1.

3. Analysis of the Observed Brightness

All the observed magnitudes of Comet Encke were estimated only. Bouška [13] showed that such estimates of a comet's brightness may be influenced by different effects not only of instrumental character. Relatively large and entirely unknown errors may be caused by observational conditions and instrumental as well as individual errors of different observers. Such observational data are therefore considerably inhomogeneous and their accuracy somewhat problematic. Moreover, it is known that photographs of faint comets at larger heliocentric distances yield mostly the magnitudes of the nucleus, i.e. the "nuclear" magnitudes m_2 only. These "nuclear" magnitudes of Comet Encke may differ from the "total" magnitudes m_1 (i.e. magnitude of nucleus + coma) by up to 5 magnitudes, as shown by Vanýsek [14].

The expressions "nuclear" and "total" are somewhat misleading in the case of faint comets. Although the "nuclear" magnitude provides some information concerning the brightness of the central condensation of a moderate bright comet, this definition of magnitude is very vague for a faint object (and has no sense at all) due to the instrumental effects. Therefore, the "nuclear" magnitudes are almost

Table 1.

Date (UT)	*m*	m_1	m_2	$m_1' = m_1 - 5\log\Delta$	$m_2' = m_2 - 5\log\Delta$	*log r*	*Observer*
1970 Sept. 26.4	18	(15)	18	(15.1)	18.1	+0.270	Roemer
1970 Oct. 4	17	(14)	17	(14.4)	17.4	+0.249	v. d. Bergh
1970 Oct. 21.8		14.5		15.6		+0.186	Waterfield, South
1970 Nov. 1.7		(12)	15	(13.5)	16.5	+0.147	Chernykh
1970 Nov. 17.0	9.1	(9.1)		(10.9)		+0.073	Bortle
1970 Nov. 30.7		9.5		11.3		—0.016	Locher
1970 Dec. 5.4		8.5		10.3		—0.051	Seki
1970 Dec. 6.7		9.5		11.3		—0.064	Locher
1970 Dec. 21.7		7—8		9.2		—0.228	Chernykh
1972 Aug. 15.3	20.5	(20.5)		(18.0)		+0.612	Roemer, McCorkle

identical with the "total" ones for comets say of $m > 15^m$ when only the central condensation determines the entire luminosity of the object.

Table 1 contains the discussed observational material. In most cases the observers give the "total" magnitudes m_1, only one magnitude is denoted as "nuclear" m_2. In three cases there are given "magnitudes" only. It is very probable that the magnitude of the recovery observation of Roemer and the magnitude by van den Bergh are "nuclear" magnitudes m_2. On the other hand, the magnitude by Bortle is probably "total" m_1. The aphelion observation by Roemer is very probably also the "total" magnitude.

From Fig. 1 it is evident that the difference between the "total" and "nuclear" magnitudes of Comet Encke is $m_2 - m_1 \sim 3^m$. If the magnitudes m_2 are reduced to the magnitudes m_1 under this assumption, the "total" absolute magnitude of Comet Encke is obtained $m_{0,1} = 11.5$ which is in good agreement with Marsden's absolute magnitude [9]. The "nuclear" absolute magnitude is $m_{0,2} = 14.5$ and the photometric exponent in both cases is $n = 4.60$, somewhat smaller than that by Marsden. The "total" magnitude of Comet Encke may be computed using the improved empirical formula

$$m_{1,calc.} = 11.5 + 5 \log \Delta + 11{,}5 \log r.$$

This formula yields the computed magnitude 21.0^m for 1972 August 15 which differs only by about 0.5^m from Roemer's observed magnitude. The magnitudes computed according to this formula are shown in Fig. 1 by the curve denoted B_1, the magnitudes computed with $m_{0,2} = 14.4^m$ by the curve denoted B_2, respectively. From Fig. 1 it is evident that the calculated magnitudes are in good agreement with the estimated magnitudes. Moreover, the determined "total" absolute brightness is in very good agreement with the results by Vsekhsvyatskij [15] and Kresák [16] for the last but one return (1967 XIII). Kresák found that the mean secular decrease amounts to not more than $1.0–1.1^m$ per century, i.e. about 0.03^m per revolution, much less than derived previously by some other authors. This result seems to be real and in agreement with a similar study done by Meisel [17] for the periodic comet Tempel 2, but it is in contradistinction of the opinion of Whipple [18] who supposes, on the basis of the observational material collected by Vsekhsvyatskij [19], a complete desintegration of Comet Encke about 2000. The present absolute magnitude of Comet Encke is by about 3.5^m brighter than that predicted by Whipple. In view of the mentioned results Whipple's estimation of comets' ages seems to be underestimated.

The behaviour of the brightness change with heliocentric distance of Comet Encke may be interpreted as a combined effect of two sources of gaseous components of the comet's atmosphere. Let us denote these two components *a* and *b*. Under the simplified assumption that the intensity $I(r)$ of the entire cometary atmosphere is formally determined by

$$I(r) = I_a r^{-n_a} + I_b r^{-n_b}$$

where $n_a = n_a(r)$; $n_b = n_b(r)$. If $I_a(r = 1) = I_b(r = 1)$, then $I(r)$ for a near-perihelion r is determined virtually by the component for which the exponent is larger. Supposing for instance $n_a > n_b$ then it is obvious that the component referring to I_a determines the luminosity of the comet near the perihelion passage in the range of r which is generally indirectly proportional to $I_a(r = 1)$ and n_b. If $I_a(r = 1)$ is proportional to the source power of material requiring larger energy than the source producing the component *b*, then the secular variation of the power source of the *a* component causes the secular variation of the "absolute magnitude" determined from the near-perihelion observations. Consequently, the secular drops of intrinsic cometary brightness determined from the perihelion observations are due to the secular decay of the source of the component *a* which liberates free radicals by a different mechanism than the source of the *b* component does. This should be valid for an icy conglomerate nucleus, as follows from the following qualitatively described process:

The size distribution of grains liberated from the nucleus depends on the cohesion of the surface layers. A high cohesion of the surface layer of the old comet P/Encke does not permit a sufficient production of icy grains until the gas production, which is regulated by the sublimation rate of snows, is below some critical value, i.e. at large perihelion distances.

Icy particles liberation is followed by an increasing of the area of ice exposed to the solar radiation and by an increasing of the gas production due to sublimating icy grains. Such a process may be extremely effective if icy grains are carriers of free radicals in the form of clathrate hydrates.

If the brightness change due to the production from liberated icy grains is approximately determined by the values I_a and n_a, while the production from the nucleus is determined by I_b and n_b, then obviously $I_a \sim I_b$ only at $r = \min$. The comets exhibits some kind of "surge" or "flare" lasting only a few days close to the perihelion passage and with a strong perihelion asymmetry of the light curve.

The apparent fast secular decrease of the "absolute" magnitudes of comets estimated from near-perihelion observations should be considered as the secular decay of one kind of the mechanisms which lead to radicals liberation from the nucleus. The above shortly described process seems to be compatible with the behaviour of Comet Encke.

The exponent $n \sim 4$ in the empirical formula found from the observations which seems to be valid for a gaseous atmosphere (because of the continuum absence in the spectra of P/Encke) indicates that the heat of sublimation of volatiles should be less than 6000 cal mol^{-1}. Moreover, if we interpret the empirical n as n_b, the mechanism producing the gaseous component b seems to be a two-step process. Therefore the photodissociation of parent molecules liberated directly from the nucleus remains still one source of the observed radicals. The second mechanism — for which the empirical $n > 4$ is dominant only in a small range of r near the perihelion — may be interpreted as a three-step process where the sublimation of icy grains plays an important role.

4. Size of the Nucleus

The dimensions of the cometary nucleus determined from the emission bands luminosity are based on the supposition that the gas production rate per unit area is independent of the size of the nucleus [20], [21]. The production rates for various values of the vaporization heat and heliocentric distances r were determined by Huebner [22]. For $r = 1$ AU the production rates for sublimation heat lower than 8000 cal mol^{-1} vary between 10^{17} to 10^{18} molecules cm^{-2} sec^{-1}. It must be noted, however, that in view of the dominant H and OH abundance proved in cometary atmospheres, the production rate is determined by ice sublimation. On the other hand, however, the nucleus size is estimated from the luminosity of the C_2 band of the $A^3\Pi_g - X^3\Pi_u$ transitions, eventually from the CN $\Delta v = 0$ band. Therefore, the method implies the assumption of a constant ratio of observed radicals to H_2O.

If the relative abundance of C_2 to H_2O in comets is approximately the same as can be expected from the thermochemical reaction in the primordial solar nebula, then the mean value of $N_{C_2}/N_{H_2O} \sim 10^{-6}$ seems to be quite an acceptable ratio and is in good agreement with preliminary results obtained from the comparison of Ly-alpha and visual observations of bright comets. Therefore the production rate $n_{C_2} \sim 10^{12}$ molecules cm^{-2} sec^{-1} may be accepted.

If L is the luminosity of a particular C_2 band (expressed in photons sec^{-1}), then the effective surface area S_{eff} of the comet's nucleus is

$$S_{eff} = L(fn\beta)^{-1}$$

where f is the oscillator strength of the measured band, $n = n_{C_2}$ and $\beta = v\tau$,

v being the mean expansion velocity of the gaseous cometary atmosphere and τ the mean lifetime of a C_2 molecule.

If the luminosity L is expressed in magnitudes $m_{o,c}$ for unit distance 1 AU and $f = 3 \times 10^{-3}$, then for the effective radius $R_{eff} = (S_{eff}/4\pi)^{1/2}$ one gets

$$\log R_{eff} = 16.8 - 0.2\, m_{o,c} - Q \qquad \text{(in cm)}$$

where $Q = \frac{1}{2} \log (n\beta)$, if $n = n_{C_2}$ and β are the values for unit distance, too. Supposing that $m_{o,c}$ differs only slightly from $m_{o,1}$ and $\beta = 10^{10\pm0.5}$ cm, then for $Q = 11$ $R_{eff} = 10^{6-0.2m_{o,c}}$ cm. Assuming for $m_{o,c}$ of Comet Encke the value obtained in the previous paragraph, then $R_{eff} = 5 \times 10^3$ cm. Even if the factor Q is overestimated by 1, and the rate of n_{C_2} is about 10^{10} molecules cm^{-2} sec^{-1}, R_{eff} remains < 1 km. Values $R_{eff} < 1$ km were obtained for several periodic comets by Konopleva and Shul'man [21] by means of a similar method, but $n_{C_2} \leq$ $\leq 10^{11}$ molecules cm^{-2} sec^{-1}, which must be assumed in such a case, seems to be too low.

If some part of the nucleus surface S_o corresponding to the real radius R_o is inactive in the gas production, then $S_{eff}/S_o < 1$. Shul'man [20] therefore introduced a so-called spotted nucleus which is created by progressive covering of the icy surface by a nonvolatile crust. The reality of such a kind of nucleus can be proved only by determining R_o (or S_o) by some other method.

The method for the diameter estimation may be based on the assumption that the cometary nucleus has some physical similarity to satellites. For instance, the estimated aphelion magnitude can be used for the estimation of the size of the real nucleus of Comet Encke assuming that the general features of Mars' satellite Phobos and those of cometary nuclei are very similar. The mean diameter of Phobos directly measured by Mariner 9 is about 21 km. Comparing the apparent magnitudes of Phobos and Comet Encke and reducing them to unit distance, the radius of the comet nucleus $R \sim 4$ km, and if the albedo of the nucleus is considerably higher than the albedo of the Martian satellite, then the radius of the comet nucleus $R < 4$ km, but above 1 km, seems to be the most reasonable result.

This result is in good agreement with the radius of the nucleus which was found by Roemer [23] from observations of Comet Encke during its returns in the years 1957 to 1960. Roemer computed the radii using the formula of Houziaux [24]. The radius of the nucleus corresponding to an assumed albedo of 0.02 was found to be 3.5 km, or 0.6 km if the albedo 0.7 was used.

Under the assumption of the same values of the albedo, and on the basis of the "nuclear" absolute magnitude discussed in this paper, the following values of the radius of the nucleus were obtained: $R(A = 0.02) = 6.0$ km and $R(A = 0.7) =$ $= 1.0$ km. These values are in good agreement not only with the results of Roemer, but also with the above mentioned results obtained from the comparison of brightness of Phobos and Comet Encke.

All "direct" estimation lead to a real $R > 1$ km. Therefore, the ratio of the real radius of the P/Encke nucleus to the effective radius will be $R_o/R_{eff} = 2$ to 10 if the low limit for the factor Q is used, and increases up to several orders with increasing C_2 relative abundance. Therefore it seems to be very likely that, in the case of such an old comet as P/Encke obviously is, a considerable part of the surface is nonvolatile or covered by a nonvolatile crust.

5. The Effect of Surface Inhomogeneity

A cometary nucleus can be supposed to be a body with numerous individual surface areas acting under the solar radiation entirely as individual sources of the gaseous (and dust) component of the cometary atmosphere. The heat transfer between these areas is negligibly small due to the low conductivity of the nucleus. Any discontinuity between two areas means that energy balance requirements for individual areas are essentially independent of each other. Generally on a "spotted" nucleus two different kinds of areas can be expected which differ in physical structure and, consequently, in albedos and vaporization rates.

Even if the energy absorbed by the surface of a nucleus depends on the absorbitivity in wavelengths near 0.5 μm, the vaporization rates are, as follows from the energy balance equations, moderated considerably by the emissivity of the nucleus near the maximum of Planck's curve for the nucleus surface temperature $\lambda_{max} = 10$ to 100 μm. Delsemme and Miller [25] computed the production rates for various ices assuming a "grey" rotating nucleus with an albedo $A = 0.9$ independent of the wavelength, from which follows that the absorption coefficient $\varkappa_v$ in the visual region is equal to the emissivity ε_i in the infrared region. Their production rates $Z \equiv Z(r)$ for water snow are practically identical as follows from Huebner's graph [26] for evaporation heat 10 kcal mol^{-1}. However, if $\varkappa_v < \varepsilon_i$ and the visual albedo is for instance 0.7 (which seems to be the upper limit for a real case) and the infrared emissivity remains 0.9, then the production rate decreases very fast with increasing heliocentric distance. The constructed curve resembles the curve obtained from Huebner's data for vaporization heat 13 kcal mol^{-1}. On the other hand, the increase of infrared albedo means that the evaporation rates follow the inverse square law up to $r \sim 3$ to 4 AU. These results are not surprising and qualitatively can easily be understood directly from the energy balance equations. (Some details have recently been discussed by Delsemme [27], [28].) For instance if the visual albedo $A_v \sim 0.1$ and the infrared one $A_i \sim 0.9$, $\log Z(r)$ plotted as a function of $\log r$ is almost identical for vaporization rates of highly volatile material with sublimation heat 6 kcal mol^{-1} up to 5 AU.

Therefore the change of the absorbitivity in visual and of the emissivity in infrared may simulate a change of the kind of ices from water snow to highly volatile material.

In a real case, however, a considerable change of absorbitivity can be expected in the visual spectral range rather than in the infrared one. Contemporary available infrared measurements of bright comets indicate that colour temperatures derived from the continuum near 10 μm differ from the black-body spectral distribution so that $T_{col} > T_{b.b}$ (see [29], [30]). However, this must be interpreted as an effect due to optical properties of an optically thin cloud of small particles with the scattering efficiency in infrared which may strongly deviate from the reflectivity of the solid surface of a large area.

(Nevertheless, the infrared emissivity on earthly material shows very remarkable features which should be taken into consideration in a more detailed study.)

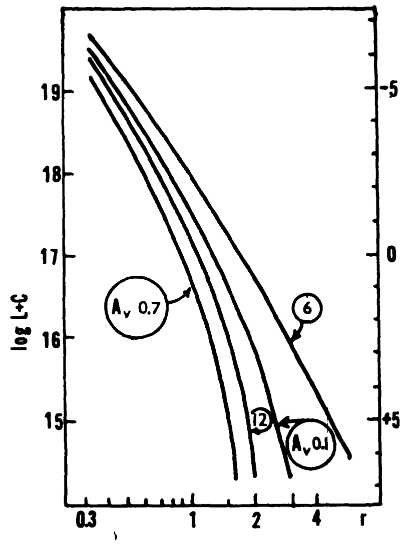

Fig. 2.

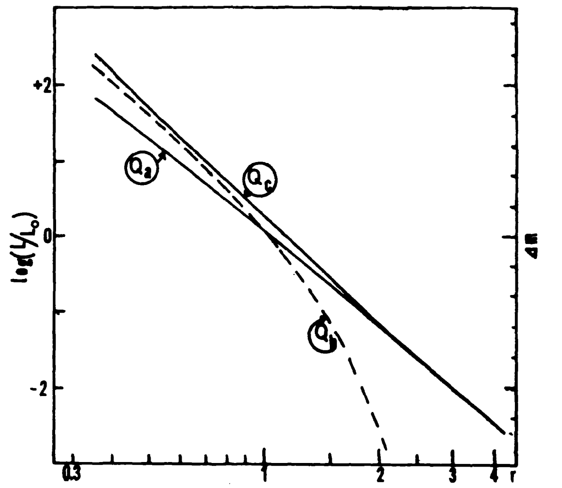

Fig. 3.

When the surface of a real cometary nucleus is covered by two groups of areas of different A_v and A_i (or with material with different evaporation heats) which occupy effective areas $S_{eff,a}$ and $S_{eff,b}$, respectively, then by a secular change of $S_{eff,a}/S_{eff,b}$ a considerable change can be expected in the brightness behaviour of a comet near the perihleion passage. Such an example is illustrated in Figs. 2 and 3.

If only a pure gaseous coma is considered for the visual range we get approximate monochromatic magnitudes dependent on the $C_2(\Delta v = 0)$ band luminosity. Let us suppose that the production rate of parent particles for C_2 $Z_{C_2}(r) = kZ(r)$, where $Z(r)$ is the vaporization rate for ice. Then the luminosity $L(r)$ is proportional to the effective area S_{eff} and to the oscillator strength, as well as to the solar photon flux in the spectral range of dissociation continuum and in the resonance wavelength of a given band which governs the lifetime of the molecular and band emission, and also to some extent, the expansion velocity of particles in a cometary coma. If we take into consideration two areas $S_{eff,a}$, $S_{eff,b}$ with different $Z(r)$'s, then the total relative production rate at unit heliocentric distance $r = 1$ is proportional to $Q_a + Q_b$ where $Q_a = Z_{r=1,a} \times S_{eff,a}$ and $Q_b = Z_{r=1,b} \times S_{eff,b}$. (Note that the meaning of Q in § 4 is different.) The luminosity $L(r)$ may be formally expressed as

$$\log L = \log (Q_a + Q_b) - C - (n_1 + n_2) \log r$$

where C is constant and $n_1 = n_1(r)$ expresses the law of production rate, i.e. $Q(r) = Q_{r=1}\, r^{-n}$ which can directly be estimated from the production rate curve. For highly volatile ices $n_1 = 2$ for a wide range of r. The second exponent $n_2 = n_2(r)$ involves the photon flux dillution $\sim r^{-2}$ which causes the emission decrease and the increase of the scale length of the coma and of the dimensions of the effectively measured coma. The value of n_2 is virtually unknown if visual or photographic methods are used for estimating the apparent brightness. Only an assumed value $n_2 = 2$ was accepted for constructing the $\log L$ curves in Figs. 2 and 3. In Fig. 2 the curve denoted by 6 is for ices with sublimation heat of 6 kcal, curve 12 for 12 kcal, respectively. The curve denoted by $A_v = 0.1$ is for water snow with the visual albedo as well as the infrared one: $A_v = A_i = 0.1$, while the curve $A_v = 0.7$ is for $A_v = 0.7$ and $A_i = 0.1$, respectively. The two last mentioned curves may approximately be replaced (by a small shift in $\log L$) by the curve for sublimation heat $\sim$ 10 and 13 kcal. Similarly, curve 6 may approximately be replaced by the curve for water snow if $A_v < A_i$. The numerical value of $\log L$ in Fig. 2 is chosen so that it approximately coincides with the numerical value of the production rate $Z_{r=1}$ (in molecules cm^{-2} sec^{-1}); therefore, if L should be expressed in photons sec^{-1}, then C involves the effective surface area, the oscillator strength and the scale length of the coma.

Fig. 3 shows the luminosity curve for $Q_a = Q_b = 1$ and $Q_c = Q_a + Q_b$, where Q_a is for the case of $A_v > A_i$, while Q_b is for $A_v = A_i < 0.1$.

The curve Q_b represents a case when the cometary nucleus is covered by two different kinds of material which, of course, may be either water snow with different visual albedos, or ices with different vaporization heats.

It is quite evident from the graph that when for instance the albedo exhibits a secular decrease then the original value of $n_1 > 2$ becomes 2 and is constant over a large range of r. If the areas $S_{eff,a}$ with higher albedo are not replaced by adequately increased low-albedo areas $S_{eff,b}$ then the decrease with time t in $Q_c(t)$ asymptotically approaches Q_b if $\dot{Q}_a t \gg \dot{Q}_b t$.

Therefore a periodic comet with such a secular change in surface structure exhibits a secular decrease of the maximum value of $L(r)$ but this decrease is almost exponentially slowed down and the comet behaves as a body containing volatile material which follows the r^{-2} law of the evaporation region. This seems to be the case of Comet Encke.

In this paper we have no intention to speculate about the entire structure of cometary nuclei, because it seems to be very difficult to distinguish between two physically meaningful reasons for the secular change of comets' behaviour — the change of the visual (and eventually infrared) albedo or the change of composition of ices. The change of albedo seems to be very likely one of the dominant effects. However, the depletion of "ice lakes" from the extremely irregular and porous core of cometary nuclei containing ices of quite different relative abundances of volatile material, must be considered another important event which determines the behaviour of comets in their final stages.

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