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An Approximative Method of Solving the Flat Thermal Neutron Flux Problem for an Infinite Cylindrical Homogenized Reactor Fueled with Natural Uranium

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Introduction

In this article, an approximative method is given for solving the flat thermal neutron flux problem for cylindrical onedimensional reactor geometries (in the two-groups diffusion approximation [1], [2], [3]. A special feature of this method is, that it simulates the usual technical procedure for realizing the spatially dependent distribution of fuel concentration [1]. In virtue of the dependence of the resonance escape probability on the concentration of the natural uranium as fuel, this problem will be nonlinear, and therefore the method considered will be an iterative one. A variable outer reactor boundary is taken in account, and its influence together with the influence of the material on the convergence of the method is illustrated. Graphically, the optimal value of the reactor outer boundary is determined. A similar problem was solved in [4] by help of other methods.

Solution in the reflector

The usual two groups diffusion equations in the reflector [1]

$$\Delta q_R - \frac{1}{\tau_R} q_R = 0 \tag{1a}$$

$$\Delta \Phi_R - \frac{1}{L_R^2} \Phi_R = -\frac{1}{D_R} q_R, \qquad L_R^2 = \frac{D_R}{\Sigma_M^a} \tag{2a}$$

for the thermal neutron flux Φ_R and for the slowing-down density q_R have in the case of the infinite homogenized cylinder geometry $\Phi_R = \Phi_R(r)$, $q_R = q_R(r)$, $0 \leq r < \infty$ the following solutions

$$q_R(r) = A_R \left[I_0\left(\frac{r}{\sqrt{\tau_R}}\right) \frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} - K_0\left(\frac{r}{\sqrt{\tau_R}}\right) \right] \tag{1}$$

$$\begin{aligned} \Phi_R(r) = B_R &\left[I_0\left(\frac{r}{L_R}\right) \frac{K_0\left(\frac{a}{L_R}\right)}{I_0\left(\frac{a}{L_R}\right)} - K_0\left(\frac{r}{L_R}\right) \right] + \\ + A_R &\frac{1}{D_R\left(\frac{1}{L_R^2} - \frac{1}{\tau_R}\right)} \left[I_0\left(\frac{r}{\sqrt{\tau_R}}\right) \frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} - K_0\left(\frac{r}{\sqrt{\tau_R}}\right) \right] \end{aligned} \tag{2}$$

which obviously fulfil at the outer end of the reflector (i.e., for $r = a$) the usual boundary conditions

$$q_R(a) = 0, \qquad \Phi_R(a) = 0. \tag{3}$$

The integration constants A_R, B_R are determined by the following conditions valid on the interface $r = b$ between the reactor core and the reflector

$$\Phi_A(b) \equiv \Phi = \Phi_R(b), \quad D_A\Phi'_A(b) = 0 = D_R\Phi'_R(b) \tag{4}$$

from which there follow for $A_R = A_R(\Phi, b, a)$, $B_R = B_R(\Phi, b, a)$ the relations

$$B_R = \Phi \frac{a_{22}}{a_{11}a_{22} - a_{12}a_{21}} = \Phi\widetilde{B} \qquad (\widetilde{B} = \widetilde{B}(a, b)) \tag{5}$$

$$A_R = \Phi \frac{(-a_{21})}{a_{11}a_{22} - a_{12}a_{21}} = \Phi\widetilde{A} \qquad (\widetilde{A} = \widetilde{A}(a, b)) \tag{6}$$

where we have denoted

$$\begin{aligned} a_{11} &= I_0\left(\frac{b}{L_R}\right) \frac{K_0\left(\frac{a}{L_R}\right)}{I_0\left(\frac{a}{L_R}\right)} - K_0\left(\frac{b}{L_R}\right); \\ a_{12} &= \frac{1}{D_R\left(\frac{1}{L_R^2} - \frac{1}{\tau_R}\right)} \left[I_0\left(\frac{b}{\sqrt{\tau_R}}\right) \frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} - K_0\left(\frac{b}{\sqrt{\tau_R}}\right) \right] \end{aligned} \tag{7}$$

$$a_{21} = \frac{1}{L_R} \left[I_1\left(\frac{b}{L_R}\right) \frac{K_0\left(\frac{a}{L_R}\right)}{I_0\left(\frac{a}{L_R}\right)} + K_1\left(\frac{b}{L_R}\right) \right]; \tag{8}$$

$$a_{22} = \frac{1}{D_R\left(\frac{1}{L_R^2} - \frac{1}{\tau_R}\right)} \frac{1}{\sqrt{\tau_R}} \left[I_1\left(\frac{b}{\sqrt{\tau_R}}\right) \frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} + K_1\left(\frac{b}{\sqrt{\tau_R}}\right) \right]$$

For the case of an infinite reflector, it follows from these relations by letting $a \to \infty$

$$a_{11} = -K_0\left(\frac{b}{L_R}\right) \qquad a_{12} = \frac{1}{D_R\left(\frac{1}{L_R^2} - \frac{1}{\tau_R}\right)} \left[-K_0\left(\frac{b}{\sqrt{\tau_R}}\right)\right] \tag{9}$$

$$a_{21} = \frac{1}{L_R} K_1\left(\frac{b}{L_R}\right) \qquad a_{22} = \frac{1}{\sqrt{\tau_R}} \frac{1}{D_R\left(\frac{1}{L_R^2} - \frac{1}{\tau_R}\right)} K_1\left(\frac{b}{\sqrt{\tau_R}}\right) \tag{10}$$

$$\Phi_R(r) = B_R\left[-K_0\left(\frac{r}{L_R}\right)\right] + A_R \frac{1}{D_R\left(\frac{1}{L_R^2} - \frac{1}{\tau_R}\right)} \left[-K_0\left(\frac{r}{\sqrt{\tau_R}}\right)\right] \tag{11}$$

$$q_R(r) = A_R\left[-K_0\left(\frac{r}{\sqrt{\tau_R}}\right)\right]. \tag{12}$$

Formulation of the flux flattening problem in the reactor core

For the reactor core formed by an infinite homogenized cylinder, it follows from the two-groups diffusion equations

$$-D_A \Delta \Phi_A + \Sigma_M^a (1 + M)\, \Phi_A = q_A \tag{13a}$$

$$-\tau_A \Delta q_A + q_A = k \Sigma_M^a M \Phi_A, \qquad k = k(M) \tag{13b}$$

for the thermal neutron flux Φ_A and for the slowing-down density q_A in the core and further from the condition of the radial flattening of the thermal neutron flux in the core

$$\Phi_A(r) = \Phi = \Phi_0 f(z); \qquad f(z) \in C(-\infty, +\infty) \tag{13}$$

that for the relative fuel concentration (neglecting the moderator expelling by the fuel)

$$M(r) = \frac{\Sigma_u^a(r)}{\Sigma_M^a} = \frac{\sigma_u^a}{\Sigma_M^a} N_u(r) \tag{14}$$

the following relations are valid

$$q_A(r) = \Phi \Sigma_M^a [1 + M(r)] \tag{15}$$

$$\frac{d^2}{dr^2} M(r) + \frac{1}{r}\frac{d}{dr} M(r) + f(M) \,.\, M(r) = \frac{1}{\tau_A} \tag{16}$$

where we have denoted

$$f(M) = \frac{k(M) - 1}{\tau_A} > 0, \qquad k(M) = \eta p(M) \tag{16a}$$

and the function $p(M)$ is supposed to be known (e.g., from experiments). From the symmetry reasons, the fuel spatial distribution $M(r)$ must fulfil the boundary condition

$$M'(0) = \frac{d}{dr} M(r) \bigg|_{r=0} = 0 \tag{17}$$

Further, on the interface $r = b$ between the reactor core and the reflector, the usual continuity conditions of the fast neutron flux and current must be valid

$$\frac{1}{(\xi\Sigma_s)_A} q_A(b) = \frac{1}{(\xi\Sigma_s)_R} q_R(b) \tag{18}$$

$$\tau_A \frac{d}{dr} q_A(r) \bigg|_{r=b} = \tau_R \frac{d}{dr} q_R(r) \bigg|_{r=b} \tag{19}$$

From the condition (18), one obtains for the fuel spatial distribution $M(r)$ the following boundary condition

$$M(b) = \vartheta_1(a, b) \tag{20}$$

where we have denoted

$$\vartheta_1(a, b) = \frac{(\xi\Sigma_s)_A}{(\xi\Sigma_s)_R} \left[\frac{D_R}{L_R^2 \Sigma_M^a} \frac{\frac{L_R}{\tau_R} - 1}{\frac{L_R}{\sqrt{\tau_R}} \frac{w_1(a, b)}{w_2(a, b)} - 1} \right] - 1 \tag{20a}$$

$$w_1(a, b) = \frac{I_0\left(\frac{b}{L_R}\right) \frac{K_0\left(\frac{a}{L_R}\right)}{I_0\left(\frac{a}{L_R}\right)} - K_0\left(\frac{b}{L_R}\right)}{I_1\left(\frac{b}{L_R}\right) \frac{K_0\left(\frac{a}{L_R}\right)}{I_0\left(\frac{a}{L_R}\right)} + K_1\left(\frac{b}{L_R}\right)},$$

(20b)

$$w_2(a, b) = \frac{I_0\left(\frac{b}{\sqrt{\tau_R}}\right) \frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} - K_0\left(\frac{b}{\sqrt{\tau_R}}\right)}{I_1\left(\frac{b}{\sqrt{\tau_R}}\right) \frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} + K_1\left(\frac{b}{\sqrt{\tau_R}}\right)}$$

From the condition (19), one obtains the following condition which determines the critical dimension b of the reactor core

$$\frac{d}{dr} M(r) \Bigg|_{r=b} = \vartheta_2(a, b) \tag{21}$$

where we have denoted again

$$\vartheta_2(a, b) = \frac{\tau_R}{\tau_A} \frac{1}{w_2(a, b) \sqrt{\tau_R}} \frac{(\xi\Sigma_s)_R}{(\xi\Sigma_s)_A} [\vartheta_1(a, b) + 1] \tag{21a}$$

The nonlinear differential equation (16) together with the boundary conditions (17), (20) and with the critical condition (21) represents a mathematical model for the thermal neutron flux flattening problem in an infinite cylindrical homogenized critical natural uranium fueled reactor with reflector. It is a nonlinear two-point boundary value problem with eigen value-parameter b, determining the fuel spatial distribution $M(r)$ in the reactor core, which is necessary for the radially flattened thermal neutron flux, and which we shall solve approximately in what follows.

Linearization of the problem and its iterative solution

In most physically interesting cases, it is possible to take, following [5], in first approximation a constant resonance escape probability $p(M) = p$:

$$p = p(M) = p_0 = konst., \quad f(M) = \frac{\eta p_0 - 1}{\tau_A} = f_0 = konst., \quad (1 \leq \eta p_0 = k_0 \leq \eta) \tag{22}$$

so that our problem is in this case a linear one and can be easily solved [5].

Therefore, let us suppose that we know the n-th approximation ($n = 1, 2, 3, \ldots$) $M_n(r)$, b_n (where b_n denotes the n-th approximate radius of the reactor core). Let us divide the interval $(0, b_1)$ in 2^n subintervals

$$0 \leq r_{i-1} < r_i \leq b_1 \qquad (i = 1, 2, \ldots, 2^n) \tag{23}$$

giving rise to 2^n cylindrical zones, and let us compute in each such i-th zone the mean value $\overline{M}_n^{(i)}$ by help of the relation

$$\overline{M}_n^{(i)} = \frac{1}{\frac{1}{2}(r_i^2 - r_{i-1}^2)} \int_{r_{i-1}}^{r_i} r dr M_n(r) \tag{24}$$

Instead of the nonlinear equation (16), we shall consider in each i-th cylindrical zone the following linearized equation for the next approximation $M_{n+1}^{(i)} = M_{n+1}^{(i)}(r)$

$$\frac{d^2}{dr^2} M_{n+1}^{(i)}(r) + \frac{1}{r} \frac{d}{dr} M_{n+1}^{(i)}(r) + f(\overline{M}_n^{(i)}) \, M_{n+1}^{(i)}(r) = \frac{1}{\tau_A} \qquad (i = 1, 2, \ldots, 2^n) \tag{25}$$

which has the general solution

$$M_{n+1}^{(i)} = \frac{1}{\tau_A f(\overline{M}_n^{(i)})} + A_{n+1}^{(i)} J_0\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) + B_{n+1}^{(i)} N_0\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \tag{26}$$

Substituting (26) in (24) and integrating, we obtain

$$\overline{M}_n^{(i)} = \frac{1}{\tau_A f(\overline{M}_{n-1}^{(i)})} + \frac{1}{\frac{1}{2}(r_i^2 - r_{i-1}^2)} \Bigg\{ A_n^{(i)} \frac{1}{\sqrt{f(\overline{M}_{n-1}^{(i)})}} \Big[r_i \mathcal{J}_1 \Big(r_i \sqrt{f(\overline{M}_{n-1}^{(i)})} \Big) -$$

$$- r_{i-1} \mathcal{J}_1 \Big(r_{i-1} \sqrt{f(\overline{M}_{n-1}^{(i)})} \Big) \Big] + B_n^{(i)} \frac{1}{\sqrt{f(\overline{M}_{n-1}^{(i)})}} \Big[r_i N_1 \Big(r_i \sqrt{f(\overline{M}_{n-1}^{(i)})} \Big) - \tag{27}$$

$$- r_{i-1} N_1 \Big(r_{i-1} \sqrt{f(\overline{M}_{n-1}^{(i)})} \Big) \Big] \Bigg\} .$$

If another initial approximation is used as the Goertzel's solution (22), the integration in (27) must be carried out numerically.

On the interface $r = r_i$ of two cylindrical zones, the usual physical conditions for the slowing-down density $q_A(r)$ and its derivative $q'_A(r) = (d/dr) q_A(r)$ must be valid [1 — 3] (ensuring the continuity of the flux and current of the fast neutrons) which are, in virtue of the relation (15) equivalent to the following two conditions

$$M_{n+1}^{(i)}(r_i) = \frac{(\xi\Sigma_s)_A^{(i)}}{(\xi\Sigma_s)_A^{(i+1}} \frac{(\Sigma_M^a)^{(i+1)}}{(\Sigma_M^a)^{(i)}} [1 + M_{n+1}^{(i+1)}(r_i)] - 1 \tag{28}$$

$$\frac{d}{dr} M_{n+1}^{(i)}(r) \Bigg|_{r = r_i} = \frac{\tau_{A^{(i+1)}}}{\tau_{A^{(i)}}} \frac{(\Sigma_M^a)^{(i+1)}}{(\Sigma_M^a)^{(i)}} \frac{d}{dr} M_{n+1}^{(i+1)}(r) \Bigg|_{r = r_i} \tag{29}$$

taking for the case of the core with homogeneous moderator (and if the moderator expelling by the fuel is neglected) the following simple form

$$M_{n+1}^{(i)}(r_i) = M_{n+1}^{(i+1)}(r_i), \quad \frac{d}{dr} M_{n+1}^{(i)}(r) \Bigg|_{r = r_i} = \frac{d}{dr} M_{n+1}^{(i+1)}(r) \Bigg|_{r = r_i} \tag{30}$$

The conditions (28), (29) ((30) resp.) represent linear recursive relations for determination of the integration constants $A_{n+1}^{(i+1)}$, $B_{n+1}^{(i+1)}$. From the general solution (26) of the equation (25), it is obvious, that in each iteration must the solution in the first zone have the simple form (from symmetry and finiteness reasons)

$$M_{n+1}^{(1)} = \frac{1}{\tau_A f(\overline{M}_n^{(1)})} + A_{n+1}^{(1)} \mathcal{J}_0 \Big(r \sqrt{f(\overline{M}_n^{(1)})} \Big) , \quad \begin{matrix} (0 \leq r \leq r_1) \\ (n = 0, 1, 2, \ldots) \end{matrix} \tag{31}$$

so that we have

$$B_{n+1}^{(1)} = 0 \qquad (n = 0, 1, 2, \ldots) \tag{31a}$$

and further

$$M_{n+1}^{(0)} = M_{n+1}^{(1)}(0) = \frac{1}{\tau_A f(\overline{M}_n^{(1)})} + A_{n+1}^{(1)} \Leftrightarrow A_{n+1}^{(1)} = M_{n+1}^{(0)} - \frac{1}{\tau_A f(\overline{M}_n^{(1)})} \tag{32}$$

so that the integration constants $A_{n+1}^{(1)}$, $B_{n+1}^{(1)}$ are in each iteration $(n = 0, 1, 2, \ldots)$ linear functions of the unknown initial value $M_{n+1}^{(0)}$, which we can take as parameter. From the linearity of the recurrent relations (14), (15), it follows clearly, that also in all following cylindrical zones the integration constants $A_{n+1}^{(i+1)}$, $B_{n+1}^{(i+1)}$ $(1 \leq i \leq 2^n - 1)$

are linear functions of the parameter $M_{n+1}^{(0)}$, and therefore must have the form

$$A_{n+1}^{(i+1)} = M_{n+1}^{(0)} \alpha_{n+1}^{(i+1)} + \beta_{n+1}^{(i+1)}, \quad B_{n+1}^{(i+1)} = M_{n+1}^{(0)} \gamma_{n+1}^{(i+1)} + \vartheta_{n+1}^{(i+1)} \tag{33}$$

Introducing the vector of integration constants

$$\omega_{n+1}^{(i+1)} = (\alpha_{n+1}^{(i+1)}, \beta_{n+1}^{(i+1)}, \gamma_{n+1}^{(i+1)}, \vartheta_{n+1}^{(i+1)}) \in E_4 \tag{34}$$

we can write the linear recursive relations (28), (29) for the integration constants in the form

$$\omega_{n+1}^{(i+1)} = H_{n+1}^{(i+1)} \omega_{n+1}^{(i)} + v_{n+1}^{(i)}, \qquad \begin{array}{l} (1 \leq i \leq 2^n - 1) \\ (n = 1, 2, \ldots) \end{array} \tag{35}$$

where the entries $h_{j,k}^{(n+1,i+1)}$ $(j, k = 1, 2, 3, 4)$ of the 4×4 matrices $H_{n+1}^{(i+1)}$ and te coordinates $v_{n+1,j}^{(i)}$ $(j = 1, 2, 3, 4)$ of the vectors $v_{n+1}^{(i)} \in E_4$ are given by the explicite formulae, which can be deduced (for the case of a homogeneous moderator in the reactor core) from the corresponding formulae for the slab reactor core geometry written explicitly (in the same notation) in [3], by help of the following substitutions

$$\cos\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \leftarrow J_0\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \tag{36}$$

$$\sin\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \leftarrow N_0\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \tag{37}$$

$$-\sqrt{f(\overline{M}_n^{(i)})} \sin\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \leftarrow -\sqrt{f(\overline{M}_n^{(i)})}\, J_1\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \tag{38}$$

$$\sqrt{f(\overline{M}_n^{(i)})} \cos\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \leftarrow -\sqrt{f(\overline{M}_n^{(i)})}\, N_1\left(r \sqrt{f(\overline{M}_n^{(i)})}\right) \tag{39}$$

$$\begin{aligned} \Delta = \sqrt{f_{n+1}^{(i+1)}}\, W(\cos(r_i \sqrt{f_{n+1}^{(i+1)}}), \sin(r_i \sqrt{f_{n+1}^{(i+1)}}) \leftarrow \\ \leftarrow \sqrt{f_{n+1}^{(i+1)}}\, W(J_0(r_i \sqrt{f_{n+1}^{(i+1)}}), N_0(r_i \sqrt{f_{n+1}^{(i+1)}}) = \frac{2}{\pi r_i}\,; \; (f_{n+1}^{(i+1)} = f(\overline{M}_n^{(i+1)})\,, \end{aligned} \tag{40}$$

where W denotes the Wronski's determinant of the corresponding fundamental systems of solutions.

For the determination of the unknown parameter $M_{n+1}^{(0)}$ (i.e., the initial value of the fuel concentration) and of the criticality parameter b_{n+1} of the active zone, we have at our disposition two following continuity conditions for the slowing-down density of neutrons $q_A(r)$ and its derivative $(d/dr)q_A(r)$ on the interface $r = b_{n+1}$ between the reactor core and the reflector:

$$\frac{1}{(\xi\Sigma_s)_A}\, q_A(b_{n+1}) = \frac{1}{(\xi\Sigma_s)_R}\, q_R(b_{n+1}) \tag{41}$$

$$\tau_A \frac{d}{dr} q_A(r)\bigg|_{r = b_{n+1}} = \tau_R \frac{d}{dr} q_R(r)\bigg|_{r = b_{n+1}}. \tag{42}$$

The left side of the equation (41) is, in virtue of the relations (15), (26), (33), a linear function of the unknown parameter $M_{n+1}^{(0)}$, so that we can compute this parameter from (41) explicitely:

$$M_{n+1}^{(0)} = \frac{\dfrac{(\xi\Sigma_s)_A}{(\xi\Sigma_s)_R}\dfrac{\widetilde{A}(a, b_{n+1})}{\Sigma_M^a}\left[I_0\left(\dfrac{b_{n+1}}{\sqrt{\tau_R}}\right)\dfrac{K_0\left(\dfrac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\dfrac{a}{\sqrt{\tau_R}}\right)} - K_0\left(\dfrac{b_{n+1}}{\sqrt{\tau_R}}\right)\right] - \left[1 + \dfrac{1}{\tau_A f(\overline{M}_n^{(i)})} + \beta_{n+1}^{(i)}\mathcal{J}_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \vartheta_{n+1}^{(i)}N_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right)\right]}{\alpha_{n+1}^{(i)}\mathcal{J}_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \gamma_{n+1}^{(i)}N_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right)} \tag{43}$$

and substitute it in the condition (42) which, if modified by help of the relations (26), (33), gives us the following criticality condition whose least positive root is the approximate critical core dimension b_{n+1}:

$$\begin{aligned} 0 = F_i(b_{n+1}) \equiv & \left\{-\frac{\tau_R}{\tau_A}\frac{\widetilde{A}(a, b_{n+1})}{\Sigma_M^a\sqrt{\tau_R}}\left[I_1\left(\frac{b_{n+1}}{\sqrt{\tau_R}}\right)\frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} + K_1\left(\frac{b_{n+1}}{\sqrt{\tau_R}}\right)\right]\right\} - \\ & - \sqrt{f(\overline{M}_n^{(i)})}\left[\beta_{n+1}^{(i)}\mathcal{J}_1\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \vartheta_{n+1}^{(i)}N_1\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right)\right] - \\ & - \sqrt{f(\overline{M}_n^{(i)})}\frac{\alpha_{n+1}^{(i)}\mathcal{J}_1\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \gamma_{n+1}^{(i)}N_1\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right)}{\alpha_{n+1}^{(i)}\mathcal{J}_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \gamma_{n+1}^{(i)}N_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right)}\left\{\frac{(\xi\Sigma_s)_A}{(\xi\Sigma_s)_R}\frac{\widetilde{A}(a, b_{n+1})}{\Sigma_M^a}\cdot\right. \\ & \cdot\left[I_0\left(\frac{b_{n+1}}{\sqrt{\tau_R}}\right)\frac{K_0\left(\frac{a}{\sqrt{\tau_R}}\right)}{I_0\left(\frac{a}{\sqrt{\tau_R}}\right)} - K_0\left(\frac{b_{n+1}}{\sqrt{\tau_R}}\right)\right] - \\ & \left. - \left[1 + \frac{1}{\tau_A f(\overline{M}_n^{(i)})} + \beta_{n+1}^{(i)}\mathcal{J}_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \vartheta_{n+1}^{(i)}N_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right)\right]\right\}. \end{aligned} \tag{44}$$

The index $i = N$ of the last cylindrical zone must be determined from the criticality condition: $b_{n+1}^{(N)}$ must be the least eigenvalue-parameter of our mathematical model (16), (17), (20), (21) of the radial thermal neutron flux flattening. Therefore, for N we must take in the sequence of indices $i = 1, 2, \ldots, 2^n, \ldots$ this index, for which first times the inequality

$$r_{i-1} \leq b_{n+1}^{(i)} = b_{n+1}^{(N)} \leq r_i \tag{45}$$

and also the inequalities

$$F_{i-1}(b_{n+1}^{(i-1)}) \cdot F_i(b_{n+1}^{(i)}) < 0, \quad M_{n+1}^{(0)}(b_{n+1}^{(i-1)}) \cdot M_{n+1}^{(0)}(b_{n+1}^{(i)}) < 0 \tag{46}$$

will hold.

For $i = N$, for which one of the inequalities (46) is fulfilled first times, we have to determine the critical parameter value $b_{n+1}^{(N)}$ by an appropriate interpolation and to control, by substituting $b_{n+1}^{(N)}$ in (44), the validity of the critical condition (44). Also in the initial iteration, one must choose carefully the starting point $r = r_1 > 0$ and the mesh size Δr for the interpolation of the criticality parameter from the conditions (46) (see the next paragraph).

Approximative determination of the critical parameter

Numerical evaluation of the first positive root b of the criticality condition on the computer was done in a very simple way, using the properties of the initial parameter $M_{n+1}^{(0)}$ given by (43). Because the poles of this function are very sharp, it is possible to compute the first pole of $M_{n+1}^{(0)}$ in (43) (which is also the first pole of the right side $F_i(b_{n+1})$ of the criticality condition) from the condition

$$\alpha_{n+1}^{(i)} J_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) + \gamma_{n+1}^{(i)} N_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) = 0\,. \tag{47}$$

Because in the case of an infinite cylindrical reactor with constant fuel distribution we have $\gamma_{n+1}^{(i)} = 0$ in (47), we can take for the Bessel functions as a reasonable approximation the linear part of their power series expansion near the first root j_1 of J_0:

$$J_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) \doteq - J_1(j_1)\left[b_{n+1}\sqrt{f(\overline{M}_n^{(i)})} - j_1\right]; \quad J_0(j_1) = 0 \tag{48}$$

$$N_0\left(b_{n+1}\sqrt{f(\overline{M}_n^{(i)})}\right) \doteq N_0(j_1) - N_1(j_1)\left[b_{n+1}\sqrt{f(\overline{M}_n^{(i)})} - j_1\right] \tag{48a}$$

Substituting both the approximations (48), (48a) into (47), we obtain the following explicite formula for the approximative value of the least positive pole of the right side $F_i(b_{n+1})$ of the criticality condition (44)

$$b_{n+1}^{(i)} \doteq \frac{j_1}{\sqrt{f(\overline{M}_n^{(i)})}} + \frac{\gamma_{n+1}^{(i)} N_0(j_1)}{\left[\alpha_{n+1}^{(i)} J_1(j_1) + \gamma_{n+1}^{(i)} N_1(j_1)\right]\sqrt{f(\overline{M}_n^{(i)})}}$$

which can be considered as a reasonable starting value for the searched approximation of the critical parameter $b_{n+1}^{(N)}$ if the index N of the zone is taken so, that the inequality (45) holds.

Numerical results and their physical interpretation

The above mentioned method of solving the problem of radial flattening of the thermal neutron flux in an infinite cylindrical reactor fueled by natural uranium by means of a suitable spatial distribution of the fuel in the critical reactor core was programmed in GIER-ALGOL by A. Mjasnikov and with help of this program, a series of numerical results was obtained for different reflector materials and different outer boundaries, and also for different forms of the dependence $p = p(M)$ of the resonance escape probability p on the fuel concentration M (corresponding to

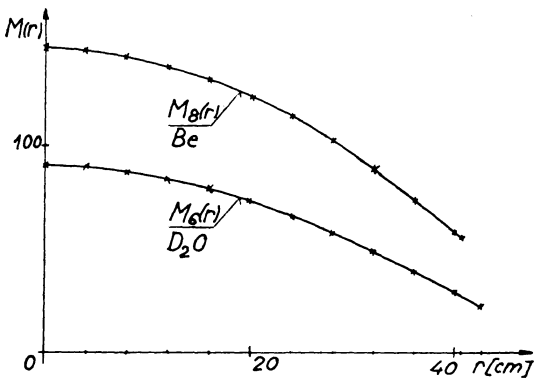

Fig. 1.

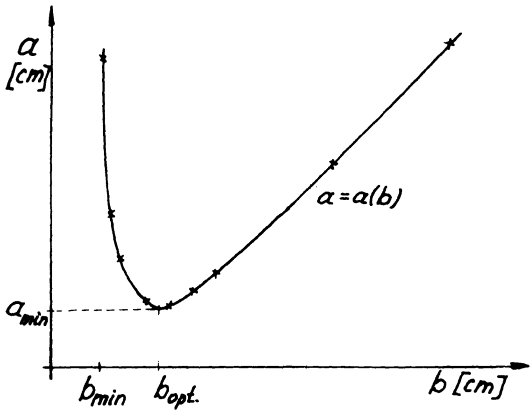

Fig. 2.

various fuel lattice parameters resp. to homogeneous fuel distribution). For illustration, the shape of the highest approximation curve $M_n = M_n(r)$ for two reflector materials is plotted (see Fig. 1) and in Table 1, the convergence of the initial fuel concentration $M_{n+1}^{(0)}$ for increasing iteration index $n + 1$ is shown. For decreasing a, convergence of the method becomes worse and for not having divergences, a suitable initial fuel distribution must be taken.

With help of these calculations, the dependence of the outer reactor boundary parameter a on the critical core parameter b could be plotted (see Fig. 2), from which the existence of a minimal outer boundary of the critical infinite cylindrical reactor with radially flattened thermal neutron flux can be seen — a physically very interesting fact.

Table 1.

	n	b_{Be} [cm]	b_{D_2O} [cm]	$M_{o\ Be}$	$M_{o\ D_2O}$
Be $p_0 = 0{,}85$ $\gamma = 6.10^{-4}$ $a = 900$ [cm] D_2O $p_0 = 0{,}83$ $\gamma = 6.10^{-4}$ $a = 300$ [cm]	1	62,47912	81,04477	198,3325	130,9354
	2	44,70905	46,10718	152,2450	93,11028
	3	41,24521	42,82898	148,0097	88,64672
	4	40,82195	42,56899	146,9427	88,43397
	5	40,74672	42,55202	146,7763	88,39282
	6	40,73202	42,54941	146,7374	88,38902
	7	40,72801		146,7289	
	8	40,72654		146,7265	

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