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Determination of Fermi Level and Some Parameters of Semiconductors from Measurements of Transport Properties

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The methods of determination of the Fermi level and some of semiconductor parameters for the case of mixed scattering of charge carriers by ionized impurities and acoustical or optical lattice vibrations are proposed. Electrical conductivity, Hall coefficient and thermoelectric power measurements are applied for this purpose. Diagrams and tables necessary for the determination are compiled.

I. Introduction

Some parameters of semiconductors such as the Fermi level, the carrier concentration, their mobility and the effective mass are very often determined from the measurement of transport effects. The accuracy of the computation is reduced by inaccurate determination of scattering factors. The approximate values are used very often. The computations are considerably easier for one kind of carrier scattering than for the mixed one. The mixed scattering by the optical lattice vibrations and ionized impurities is still causing great difficulties. A uniform relaxation time for all transport effects [1] cannot be introduced for the scattering by the lattice optical vibrations in the whole temperature range (except for higher temperaturs than that of Debye). This fact makes the computation of the required scattering factors more complicated and extensive. Amith, Kudman and Steigmeier [2] give the calculation of these factors for higher than the characteristic temperature, i.e. for degenerate and non-degenerate semiconductors.

Some calculations have already been performed [3–8, 19, 21] for the mixed scattering of charge carriers by the longitudinal acoustic modes of the lattice vibrations and ionized impurities.

This paper summarizes the majority of results obtained so far and describes methods for determining the Fermi level and the scattering factors in a relatively simple way either in the case of mixed scattering by ionized impurities and by optical or acoustical lattice vibrations.

We are primarily concerned with the determination of the concentration of charge carriers, their mobility and the effective mass. Measurements of the electrical conductivity, Hall coefficient and thermoelectric power will be employed for their determination. For the sake of simplicity we suppose the n-type semiconductor to be isotropic with scalar

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effective mass. The methods given are valid also for an anisotropic semiconductor. Only some formulae need to be amended by a known factor.

2. The Fundamental Transport Coefficients

The electrical conductivity, Hall coefficient and the differential thermoelectric power (Seebeck coefficient) will be considered as fundamental transport coefficients.

The following relation are valid for electrical conductivity σ by the following equation:

(1)
$$\sigma = e\mu n$$

where e is the electron charge, n – the concentration of electrons defined by the following expression:

(2)
$$n = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta)$$

μ is the conductivity mobility defined by

(3)
$$\mu = \frac{e\langle\tau\rangle}{m}$$

(4)
$$N_c = 2 \left(\frac{2\pi\, m_d\, k_0 T}{h^2}\right)^{3/2}$$

m_d – 'density-of-states' effective mass, m – the effective mass of electron, k_0 the Boltzmann constant, h – the Planck constant, T the absolute temperature, $F_k(\eta)$ the Fermi-Dirac integrals and $\langle\tau\rangle$ – the average value of the relaxation time.

$\langle\tau\rangle$ the average value of the k-th power of the relaxation time defined by the following equation:

(5)
$$\langle\tau^k\rangle = \frac{3F_{1/2}(\eta)}{2} \int\limits_{0}^{\infty} [\tau(x)]^k x^{3/2} \left(-\frac{\partial f_0}{\partial x}\right) dx$$

where η is the reduced Fermi energy $E_F \left(\eta = \dfrac{E_F}{k_0 T}\right)$, x – the reduced electron energy $\left(x = \dfrac{E}{k_0 T}\right)$, f_0 – the Fermi-Dirac distribution function.

The function $F_k(\eta)$ is very often used instead of the Fermi-Dirac integrals. They are defined by the following equation:

(6)
$$F_k(\eta) = \frac{F_k(\eta)}{\Gamma(k+1)}$$

where $\Gamma(k+1)$ is the gamma function.

For the Hall constant R is valid the following equation:

(7)
$$R = \frac{r_H(\eta)}{en}$$

Seebeck coefficient α is equal to:

(8)
$$\alpha = \frac{k_0}{e} \{B(\eta) - \eta\}$$

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The scattering factors $r_H(\eta)$ (Hall coefficient factor) and $B(\eta)$ can be calculated from the following equations:

$$(9) \qquad r_H(\eta) = \frac{\langle \tau^2 \rangle}{\langle \tau \rangle^2}$$

$$(10) \qquad B(\eta) = \frac{\langle \tau x \rangle}{\langle \tau \rangle}$$

The accuracy in calculating the concentration of the charge carrier and their mobility depend on the correct determination of these factors.

3. Scattering of Charge Carriers by Lattice Vibrations and Ionized Impurities

a) Scattering by acoustic lattice vibrations. The relaxation time τ_L for the scattering of charge carriers by the acoustical phonons of the lattice may be computed from the relationship [9–12]:

$$(11) \qquad \tau_L = C_L x^{-1/2}$$

where

$$(12) \qquad C_L = \frac{\pi \, \hbar^4 \varrho \, u_e^2}{\sqrt{2} \, E_1^2 \, (m k_0 \, T)^{3/2}}$$

ϱ is the density of the solid, u_e the velocity of sound, E_1 the deformation potential.

The scattering factors (10), (9) are then equal to

$$(13) \qquad r_{H,L}(\eta) = \frac{3}{4} \, \frac{F_{1/2}(\eta) \, F_{1/2}(\eta)}{[F_0(\eta)]^2}$$

$$(14) \qquad B_L(\eta) = \frac{2 F_1(\eta)}{F_0(\eta)}$$

and the mobility to the following equation:

$$(15) \qquad \mu_L = \frac{2e \, C_L}{3m} \, \frac{F_0(\eta)}{F(\eta)}.$$

b) Scattering by optical lattice vibrations. The question of scattering of carriers by optical lattice vibrations is substantially more complicated as the scattering in this case on the contrary to the previous one is not elastic, and the relaxation time can thus not be introduced in a simple manner. The computation of the transport coefficients is carried out for example by means of the variational method. By this manner Howarth and Sondheimer [13] obtained the following expression for the electrical conductivity:

$$(16) \qquad \sigma = \frac{4 V_a \, M \omega_0 \, (k_0 T)^2}{3\pi \, (e^\star)^2 \, h^2} \, G(\eta, z) \, [\exp z - 1]$$

where

$$(e^\star)^2 = (4\pi)^{-1} M \omega_0 \, V_a (K_0^{-1} - K_s^{-1})$$

M is the reduced mass of the atoms in the unit cell, V_a – the volume of the unit cell, ω_0 the angular frequency of the longitudinal optical vibrations, $z = \Theta/T$ (Θ is the Debye temperature), K_s the dielectric constant and K_0 the high-frequency dielectric constant.

The mobility of charge carriers in a non-degenerate semiconductor is then equal to

$$
(17) \qquad \mu_p = \frac{4V_a \, M\hbar\omega_0 \, (k_0 T)^{1/2}}{3(2\pi m)^{3/2} \, e(e^\star)^2} \; G(z) e^{-\eta} \, [e^\star - 1]
$$

where $Ge^{-\eta}$ is the so-called Howarth-Sondheimer function. Its dependence on Θ/T is shown in Fig. 1 in the first (N = 1) and second (N = 2) approximations for non-degenerate semiconductors. The third and further approximations do not differ substantially
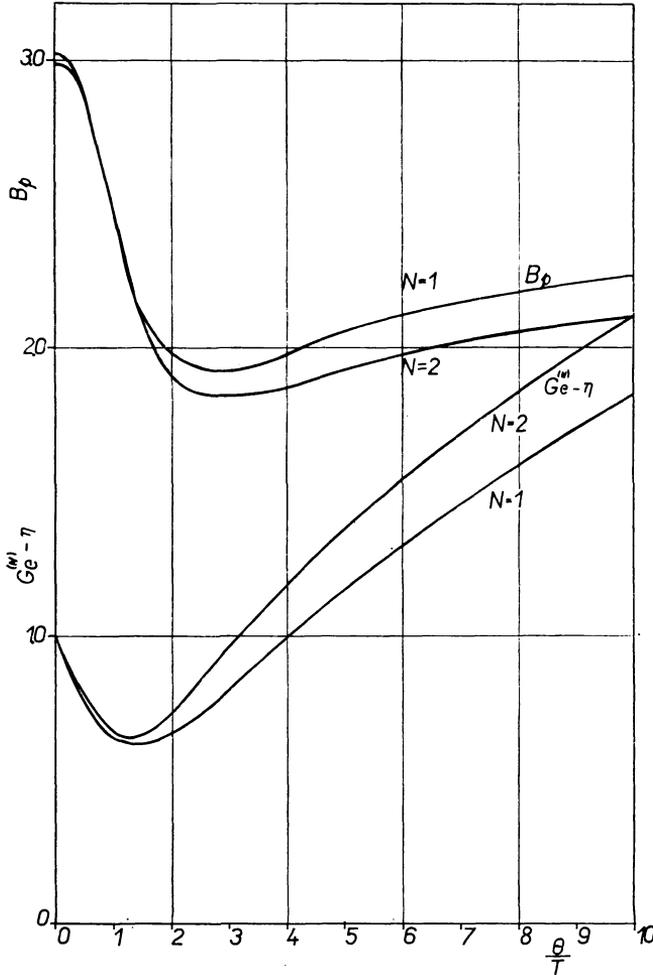
Fig. 1. The Dependence of Howarth-Sondheimer's Function $G^{(N)}e^{-\eta}$ and the Scattering Factor B_p (the kinetic term in the Seebeck coefficient α) on the Reduced Reciprocal Temperature in the First and Second Approximation

from the second one for $T > \Theta$. Lewis and Sondheimer [14] (Fig. 2) determined the scattering factor $r_{Hp}(z)$ for this type of scattering. Delves [15] calculated the scattering factors values for degenerate semiconductors. The dependence of the scattering factor $B_p(z)$ is also plotted for the first and second approximations in Fig. 1 [13].

To simplify computations Ehrenreich investigated whether the relaxation time could be presented by equation

$$(18) \qquad \tau_p = \tau_p^0 \left(\frac{E}{\hbar \omega_0} \right)^r = C_p x^r$$

even when the exponent $r(T)$ is temperature dependent. He also determined that the temperature region for $r(T)$ is practically the same for all transport effects. τ_p^c and C_p are determined from the conditions that $r = 0$ at low temperatures ($T \ll \Theta$) and $r = 1/2$ at high temperatures ($T \gg \Theta$).

$$(19) \qquad \tau = \frac{M V_a (\hbar \omega_0)^{5/2}}{4\pi (2m)^{1/2} e^2 (e^\star)^2 k_0 T} \cdot \frac{e^z - 1}{e}$$
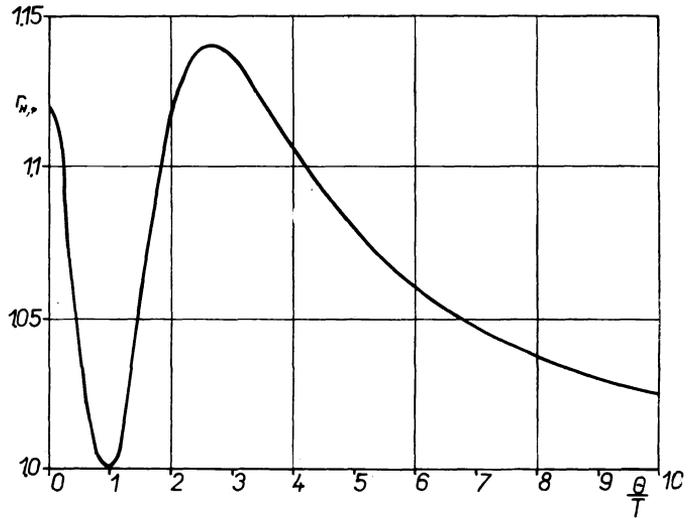
Fig. 2. The Dependence of the Hall Coefficient Factor $r_{H,\,p}$ on the Reduced Reciprocal Temperature in the First Approximation

$$(20) \qquad C_p = \left[\frac{T}{\Theta} \right]^r \frac{M V_a (k_0 \Theta)^{3/2}}{4\pi (2m)^{1/2} e^2 (e^\star)^2} [e^{\Theta/T} - 1].$$

For the mobility of charge carriers in the non-degenerate semiconductors we can determine the values of exponent r from the following equation:

$$(21) \qquad G^{(N)} e^{-\eta} = \frac{1}{2} z^{(1/2 - r)} \Gamma (5/2 + r).$$

where N is the order of approximation.

From the thermoelectric power we can ascertain the exponent r if we put the scattering factor B_p (see Fig. 1) equal to ($5/2 + r$)

$$(22) \qquad B_p = 5/2 + r$$

Similarly we can compute the exponent r from the scattering factor $r_{H,p}$ from the following equation

$$(23) \qquad r_{H,p} = \Gamma(5/2) \Gamma(5/2 + 2r) / \Gamma^2(5/2 + r)$$

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The dependence of the exponent $r(T)$ on temperature for these three transport phenomena is shown in Fig. 3. When $z < 1$ all three methods for calculating r give nearly identical results. The curves calculated from the mobility and thermoelectrical powers in the second approximation cover each other. The values of r differ from each other greatly for temperatures $T < \Theta$ and no solution can be found for r in the region $1.3 < z < 3.0$. There is also a great difference between the first and the second approximation.
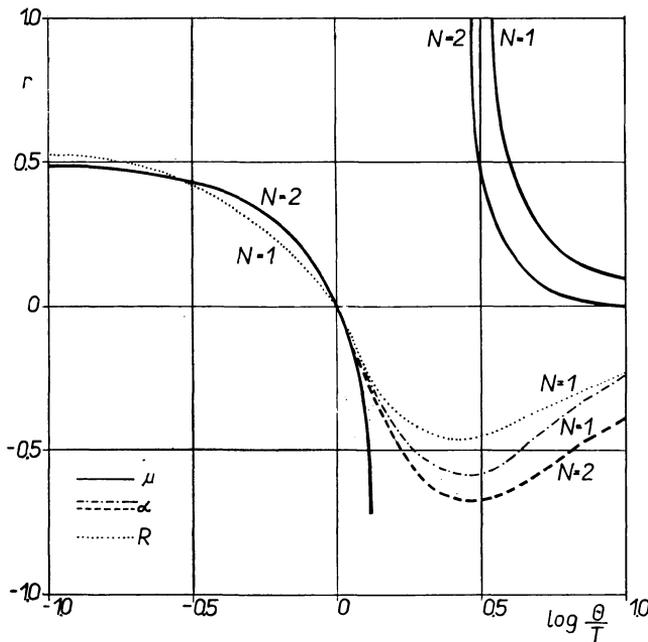
Fig. 3. The Dependence of the Exponent $r(T)$ on the Reduced Reciprocal Temperature for Some Transport Coefficients in the First and Second Approximation

The mobility of current carriers μ_p for the temperature region where the relaxation time may be presented by relationship (18) is equal

$$(24) \qquad \mu_p = \frac{2}{3} \frac{e}{m} \frac{(^3/_2 + r) \, C_p F_{(^1/_2 + r)}(\eta)}{F_{^1/_2}(\eta)}$$

c) Scattering by Ionized Impurities.

The relaxation time for the scattering of charge carriers by ionized impurities τ_I may be expressed by relationship

$$(25) \qquad \tau_I = C_I x^{^3/_2}$$

where

$$(26) \qquad C_I = \frac{(2m)^{^1/_2} \, K_s^2 \, (k_0 T)^{^3/_2}}{\pi Z^2 \, e^4 \, N_I \, g(n^\star, T, x)} = \frac{\tau_I^0}{g(n^\star, T, x)}$$

N_I is the concentration of ionized impurities and Ze the charge of an impurity.

According to the Conwell-Weisskopf theory [16] relationship

$$(27) \qquad g(n^\star, T, x) = \ln\left[1 + (K_s k_0 T/e^2 N_I^{1/3})^2 \cdot x^2\right]$$

is valid for the function $g(n^\star, T, x)$.

Besides this, according to Brooks and Herring [17] and Dingle [18]

$$(28) \qquad g(n^\star, T, x) = \ln(1 + b) - \frac{b}{1 + b}$$

$b = (2 k_e r_0)^2$, k_e is the magnitude of the wave vector of the charge carrier and r_0 the Debye length.

Born's approximation, together with the assumption $b \gg 1$ is applied for deriving formula (28). Dingle [18] denotes b as

$$(29) \qquad b = \frac{2 K_s m (k_0 T)^2}{e^2 \hbar^2 n} \frac{F_{1/2}(\eta)}{F_{-1/2}(\eta)} x$$

The same relation for b in a non-degenerate semiconductor is given by Brooks and Herring with the only difference that electron concentration n is substituted by $n^\star$. For partial ionization of impurities in an uncompensated semiconductor

$$(30) \qquad n^\star = n(2 + n/N_D)$$

and for total ionization of a compensated semiconductor is valid:

$$(31) \qquad n^\star = n + (n + N_A)\left[1 + (n + N_A)/N_D\right]$$

where N_D and N_A are the concentrations of donors and acceptors.

It is necessary to know the average values of (5) in order to determine the scattering
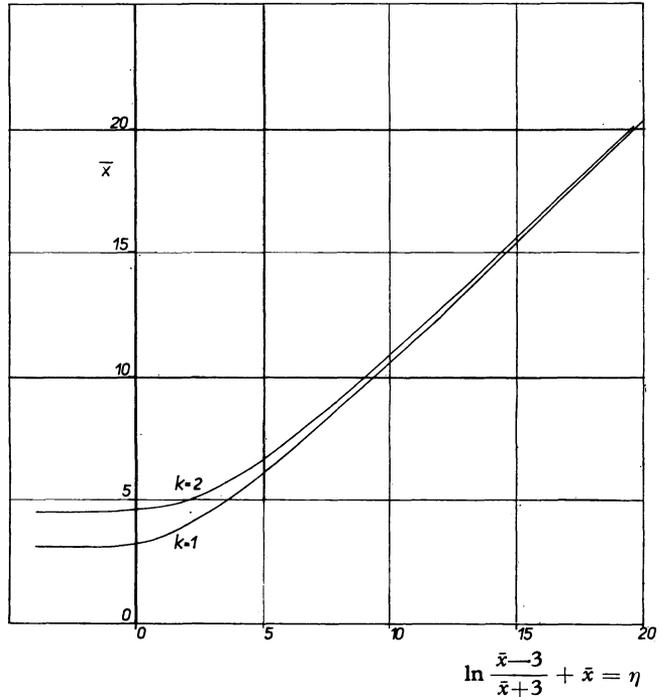
Fig. 4. The dependence of the value $\bar{x}$ on η (for $k = 1$ and 1)

$$\ln \frac{\bar{x} - 3}{\bar{x} + 3} + \bar{x} = \eta$$

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factors and the mobility of charge carriers. An exact computation is difficult because of the logarithmic term containing x, (27), (28). According to Mansfield [4, 19] the computation may be carried out by substituting for x into the function $g(n^{\star}, T, x)$ the mean value $\bar{x}$ for which the integrand of integral (5) attains a maximum. The dependence of this mean value $\bar{x}$ on η is shown in Fig. 4 for $k = 1$ and 2.

The mobility of current carries will then be expressed by

$$(32) \qquad \mu_I = \frac{2e}{m} \, C_I(\bar{x}) \, \frac{F_2(\eta)}{F_{1/2}(\eta)} = \frac{2e}{m} \, \frac{\tau_I^0}{g(n^{\star}, T, \bar{x})} \, \frac{F_2(\eta)}{F_{1/2}(\eta)}$$

Resibois and Dewel [20] have considered the influence of polarization effects on the energy of interaction between an electron and an imperfection under the influence of an external electric field. They obtained the following relationship for the carrier mobility in a non-degenerate semiconductor

$$(33) \qquad \mu_I = \frac{8e}{\sqrt{\pi} \, m} \, C_I \left[1 - 2\pi^{1/2} \, e^3 \, \sqrt{N_I} / 3 \, (K_s k_0 T)^{3/2} \right]$$

When they assumed the concentration of foreign impurities to be small $(e^2 N_I^{1/2} / K_s k_0 T \ll 1)$ and $\hbar^2 r_0^2 / 2mk_0 T \ll 1$). For GaAs with $N_I = 3.3 \cdot 10^{17}$ cm^{-3}, $T = 300°$K and $K_s = 12.5$ we obtain from equation (33) a value of mobility about 20% lower than from eq. (32).

5. The Mixed Scattering of Charge Carriers by Lattice Vibrations and Ionized Impurities

If several types of interaction between electrons and phonons or impurities take part in the scattering process, we can express the total relaxation time as follows:

$$(34) \qquad \tau^{-1} = \sum_i \tau_i^{-1}$$

The resultant relaxation time can be quite simply calculated in such cases where the particular relaxation time τ_i exists. It is somewhat more difficult with regard to scattering by lattice optical vibrations, when the uniform relaxation time can be introduced only for low $(T \ll \Theta)$ and high $(T \gg \Theta)$ temperatures. In the vicinity of the Debye temperature Θ, the Boltzmann equation had to be solved without introducing the relaxation time. We can simplify this task by introducing the relaxation time after Ehrenreich [1]. Then we can also use the equation (34), so that $\tau^{(p)}$ will be equal

$$(35) \qquad \tau^{(p)} = \frac{C_p \, x^{3/2}}{C_p/C_I + x^{(3/2 - r)}}$$

and its average value

$$(36) \qquad \langle \tau^{(p)k} \rangle = \frac{2C_p^k}{3F_{1/2}(\eta)} \int_0^\infty \frac{x^{3(k+1)/2}}{[\gamma^2 + x^{(3/2 - r)}]^k} \left(-\frac{\partial f_0}{\partial x} \right) \, dx$$

where

$$(37) \qquad \gamma^2 = \frac{C_p}{C_I} \doteq \frac{C_p}{\tau_I^0} \, g(n^{\star}, T, \bar{x})$$

We substitute the value $\bar{x}$ instead of x in the expression for C_I (26), for which the integrand of the respective value $\langle \tau^k \rangle$ reaches the maximum. The values $\bar{x}$ in the dependence

on η and γ will be mentioned in further papers. The calculation of some scattering factors at temperatures $T \lesssim \Theta$ for this kind of scattering was carried out by Amith, Kudman and Steigmeier [2].

We can express the particular transport coefficients by means of C_I, C_p and η. As C_I and C_p vary from 0 to ∞ it is more convenient to introduce further parameters q_p defined by the following expression:

$$(38) \qquad q_p = \frac{\mu_p}{\mu_I + \mu_p}$$

which varies from 0 to 1. The value $q_p = 1$ corresponds to the scattering due to ionized impurities, and the value $q_p = 0$ corresponds to scattering by optical mode lattice vibrations. μ_p and μ_I are given by equations (24) and (32).

Between the coefficients C_I and C_p and the parameter q_p the following equation is valid

$$(39) \qquad \frac{C_p}{C_I} = \frac{q_p}{1 - q_p} \frac{3F_2(\eta)}{(^3/_2 + r) F_{(^1/_2 + r)}(\eta)}$$

The scattering factors $r_H^{(p)}(q_p, \eta)$, $B^{(p)}(q_p, \eta)$ as well as the Hall and Seebeck's coefficients (8) depend on the ratio C_p/C_I or on the parameter q_p and on the Fermi energy η. The electrical conductivity is a function of C_p, C_I and η. When ascertaining the value of exponent $r(T)$ it is sufficient for the determination of q_p and η to measure the Hall and Seebeck coefficients. The respective scattering factors $r_H^{(p)}(q_p, \eta)$ and $B^{(p)}(q_p, \eta)$ for various values r are mentioned in tables [2].

We are often interested in the values of the coefficients C_p and C_I. For their determination the electrical conductivity has to be known, too (1), from which we can calculate the mobility μ. Between this mobility and the mobility μ_{II} defined by the equation

$$(40) \qquad \mu_{II}^{-1} = \mu_p^{-1} + \mu_I^{-1}$$

the following relation is valid [2]

$$(41) \qquad \frac{\mu}{\mu_{II}} = \left[\frac{1}{3C_I F_2(\eta)} + \frac{1}{(^3/_2 + r)C_p F_{(^1/_2 + r)}(\eta)} \int_0^\infty x^{3/_2} \tau^{(p)} \left(-\frac{\partial f_0}{\partial x} \right) \right] \mathrm{d}x$$

The ratio μ/μ_{II} is a function of q_p and η and is also tabulated [2]. We can estimate μ_{II} from the known mobility μ and the ratio μ/μ_{II} and by means of relations (40), (24), (32) and (39) we can calculate C_p and C_I.

The resultant relaxation time $\tau^{(L)}$ for the case of scattering by acoustical lattice vibrations and ionized impurities

$$(42) \qquad \tau^{(L)} = \frac{C_I x^{3/_2}}{C_L/C_I + x^2}$$

according to equations (34), (25) and (11). And the average value

$$(43) \qquad \langle (\tau^{(L)})^k \rangle = \frac{2}{3} \frac{C_L}{F_{1/_2}(\eta)} \int_0^\infty \frac{x^{3(k+1)/2}}{\left[x^2 + \frac{C_L}{\tau_I^0} g(n^\star, T, x) \right]} \left(-\frac{\partial f_0}{\partial x} \right) \mathrm{d}x \cdot$$

In the function $g(n^\star, T, x)$ we substitute $\bar{x}$ for x for which the integrand of the respective value $\langle \tau^k \rangle$ attains a maximum. The condition for $\bar{x}$ is

$$(44) \qquad \frac{(3 - k)\bar{x}^2 - 3(k + 1)\beta^2}{2(\bar{x}^3 + \beta^2 \bar{x})} = \tanh \frac{\bar{x} - \eta}{2}$$

where

$$(45) \qquad \beta^2 = \frac{C_I}{\tau_I^0} g(n^\star, T, \bar{x})$$

Mansfield [19] introduced the values x in dependence on β^2 and η for $k = 1$.

Equations (36–41) can be used also for the mixed scattering of charge carriers by ionized impurities and acoustical lattice vibrations. It is sufficient to substitute index L instead of index p and $-1/2$ instead of r.

6. The Determination of Fermi Energy and Scattering Factors from Measurement of the Hall Constant and Thermoelectric Power

From measurements of the Hall coefficient and the thermoelectric power it is possible to determine the necessary parameters for the more accurate calculations of charge carrier concentration and mobility on the assumption that the effective mass is knovn.

The determination of the scattering factors of non-degenerate semiconductors is relatively easy. Let us suppose that the effective mass m and exponent r are known. From the measurement of the Hall coefficient and thermoelectric power the value of the left hand side of the following equation can be calculated (Table 1).

$$(46) \qquad \frac{e}{k_0} \alpha - \ln eN_c R = B^{(p)}(q_p) - \ln r_H^{(p)}(q_p)$$

$B^{(p)}(q_p)$ and $r_H^{(p)}(q_p)$ are the scattering factors (9), (10) for the mixed scattering by optical phonons and ionized impurities. The values of the right-hand side of the equation are plotted in Fig. 5. In this diagram we estimate the parameter q_p for the measured values of the thermoelectric power and Hall coefficient. A certain Hall coefficient factor $r_H^{(p)}$ corresponding to this value q_p can be seen in this diagram, too. Then the carrier concentration can be calculated from equation (7). When measuring also the electrical conductivity we can calculate the mobility μ from the relationship (1). If we want to determine the factors C_p and C_1 we estimate for known q_p the ratio μ/μ_{II} (Fig. 6), the values of which are tabulated in [2] and from this μ_{II}. C_p can be calculated from (38), (39).

The relation between C_p/C_I and q_p is also plotted in Fig. 7.

The evaluation is rather more complicated in the case of degenerate semiconductors as the scatte-

Table 1. *Values of expression $\{B^{(p)}(q_p) - \ln r_H^{(p)}(q_p)\}$ for non-degenerate semiconductor*

q_p \ r	0.0	0.1	0.2	0.3	0.4
0.0	2.500	2.596	2.682	2.762	2.834
0.2	2.801	2.851	2.899	2.945	2.988
0.4	2.953	2.990	3.026	3.059	3.090
0.6	3.079	3.107	3.133	3.156	3.178
0.8	3.204	3.209	3.235	3.248	3.260
0.9	3.271	3.280	3.288	3.295	3.302
1.0	3.341	3.341	3.341	3.341	3.341

ring factors $r_H^{(p)}(q_p, \eta)$, $B^{(p)}(q_p, \eta)$ are functions of q_p and η. The respective values o $\langle \tau^k \rangle$ may be computed for semiconductors having $\eta > 5$ by means of the Bethe-Sommerfeld method. The integral in expression (5) may be expressed by the expansion

$$\int_0^\infty [\tau(x)]^k \, x^{3/2} \left(-\frac{\partial f_0}{\partial x} \right) dx = [\tau(\eta)]^k \, \eta^{3/2} + 2 \sum_{l=1}^\infty \left| \frac{d^{2l}\,(\tau^k x^{3/2})}{dx^{2l}} \right|_{x=\eta} \frac{(2^{(2l-1)} - 1)\,\pi^{2l}\,B_l}{(2l)!}$$

(47)

where B_l are the Bernoulli numbers $\left(B_1 = \dfrac{1}{6} ; \; B_2 = \dfrac{1}{30} \right)$. Relationship (36) is then
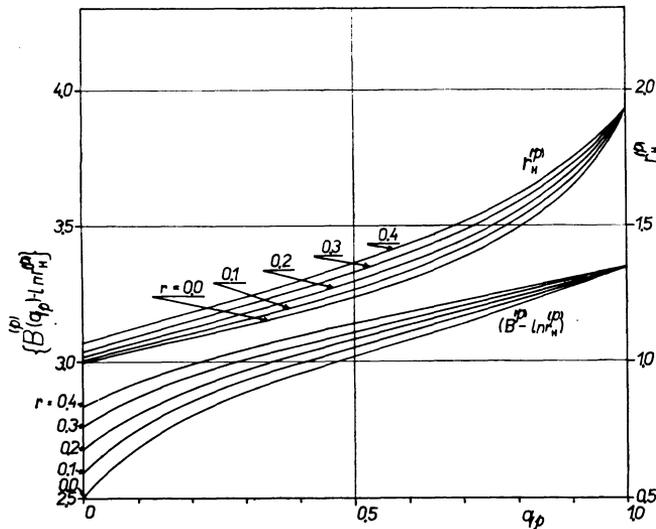
Fig. 5. The Dependence of the Expression $\{B^{(p)}\,(q_p) - \ln\,r_H^{(p)}\,(q_p)\} = (e/k_0)\alpha - \ln eN_cR$ and the HallCoefficient Factor $r_H^{(p)}(q_p)$ on the Parameter q_p for Various Values of r (non-degenerate semiconductor)

Fig. 6. The Ratio μ/μ_{II} as a Function of q_p for Various Values of r (non-degenerate semiconductor) [2]

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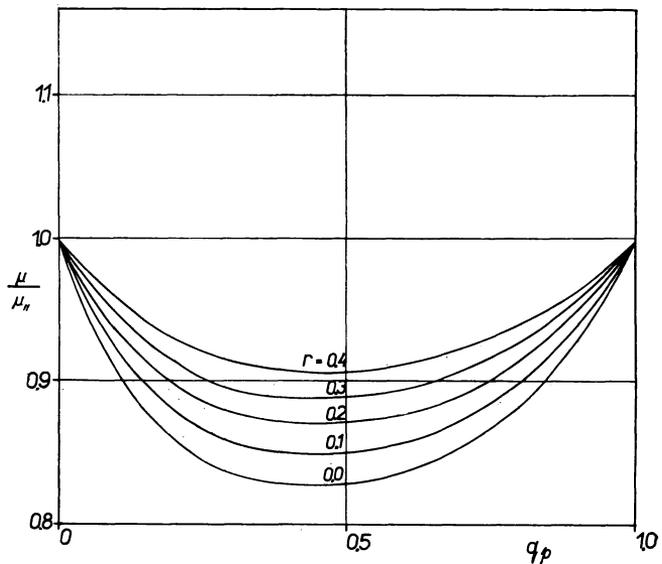
equal

$$
\langle \tau^{(p)k} \rangle \doteq \frac{C_p^k}{36\,F_{1/2}(\eta)} \left\{ \frac{24\eta^{3(k+1)/2} + (3k+3)(3k+1)\pi^2\eta^{\frac{3k-1}{2}}}{[\gamma^2 + \eta^{(3/2-r)}]^k} - \right.
$$

(48)

$$
\left. - \frac{(6k-2r+7)(3-2r)k\pi^2\eta^{\frac{3k-2r+2}{2}}}{[\gamma^2 + \eta^{(3/2-r)}]^{k+1}} + \frac{(3-2r)^2(k+1)k\pi^2\,\eta^{\frac{3k-4r+5}{2}}}{[\gamma^2 + \eta^{(3/2-r)}]^{k+2}} + \dots \right\}
$$

Some calculations of scattering factors for $\eta \leq 20$ are carried out in [2]. In order to determine the scattering factors of interest to us, it will be necessary to determine q_p and η. This can be done in the following manner:

The parameter $(e/k_0)\alpha$ is determined from measurement of the thermoelectric power (Table 2), and from the Hall coefficient is computed the parameter (Table 3) eN_cR

(49)
$$
\frac{e}{k_0}\,\alpha = B^{(p)}(q_p, \eta) - \eta
$$

(50)
$$
eN_cR = \frac{\sqrt{\pi}}{2F^{1/2}(\eta)}\,r_H^{(p)}(q_p, \eta) = \frac{r_H^{(p)}(q_p, \eta)}{F_{1/2}(\eta)}
$$
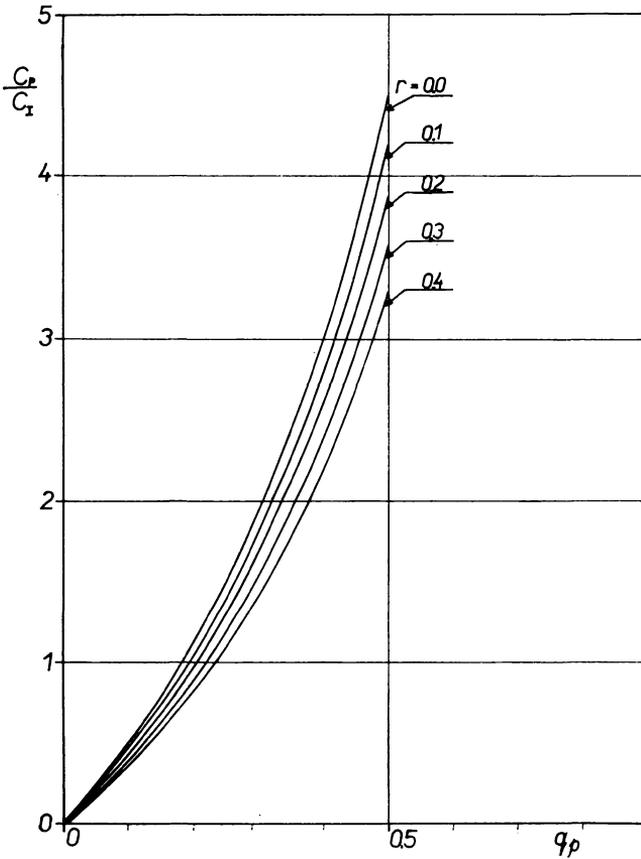
Fig. 7. The Ratio C_p/C_I as a Function of q_p for Various Values of r (non-degenerate semiconductor) [2]

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Table 2. $\{B^{(p)}(q_p\,\eta) - \eta\}$ as a function of q_p and η for various values of r

q_p \ η	—4	—3	—2	—1	0	1.5	3	5	10	20
$r = 0$										
0	6.50	5.51	4.55	3.644	2.833	1.913	1.339	0.912	0.484	0.246
0.2	6.88	5.89	4.93	4.005	3.172	2.197	1.567	1.070	0.581	0.295
0.4	7.12	6.13	5.16	4.236	3.395	2.399	1.742	1.206	0.670	0.353
0.6	7.34	6.35	5.38	4.456	3.608	2.592	1.910	1.342	0.757	0.380
0.8	7.60	6.61	5.65	4.708	3.849	2.807	2.092	1.491	0.845	0.438
0.9	7.77	6.78	5.80	4.867	4.000	2.937	2.198	1.576	0.892	0.463
1.0	8.00	7.00	6.03	5.085	4.202	3.101	2.350	1.674	0.940	0.490
$r = 0.1$										
0	6.60	5.61	4.65	3.740	2.925	1.992	1.405	0.963	0.515	0.261
0.2	6.95	5.95	4.99	4.070	3.233	2.252	1.614	1.108	0.604	0.307
0.4	7.17	6.19	5.22	4.292	3.447	2.444	1.779	1.236	0.687	0.352
0.6	7.39	6.40	5.43	4.504	3.652	2.629	1.939	1.365	0.769	0.397
0.8	7.64	6.64	5.67	4.743	3.882	2.833	2.111	1.505	0.852	0.442
0.9	7.79	6.80	5.83	4.891	4.022	2.954	2.209	1.584	0.895	0.464
1.0	8.00	7.00	6.03	5.085	4.202	3.101	2.350	1.674	0.940	0.490
$r = 0.2$										
0	6.70	5.71	4.74	3.836	3.016	2.071	1.471	1.015	0.546	0.277
0.2	7.01	6.02	5.06	4.135	3.296	2.307	1.661	1.147	0.628	0.320
0.4	7.23	6.24	5.27	4.347	3.500	2.489	1.817	1.267	0.705	0.362
0.6	7.44	6.45	5.48	4.550	3.695	2.665	1.968	1.387	0.781	0.403
0.8	7.67	6.68	5.71	4.776	3.912	2.857	2.129	1.519	0.859	0.445
0.9	7.82	6.83	5.85	4.913	4.042	2.969	2.220	1.592	0.898	0.466
1.0	8.00	7.00	6.03	5.085	4.202	3.101	2.350	1.674	0.940	0.490
$r = 0.3$										
0	6.80	5.81	4.85	3.932	3.107	2.150	1.537	1.066	0.576	0.280
0.2	7.08	6.09	5.13	4.202	3.360	2.364	1.709	1.185	0.651	0.333
0.4	7.29	6.30	5.33	4.403	3.552	2.535	1.854	1.297	0.723	0.372
0.6	7.49	6.50	5.53	4.595	3.738	2.701	1.996	1.410	0.794	0.410
0.8	7.71	6.71	5.74	4.807	3.941	2.880	2.147	1.532	0.865	0.448
0.9	7.84	6.84	5.87	4.932	4.060	2.983	2.230	1.599	0.902	0.468
1.0	8.00	7.00	6.03	5.085	4.202	3.101	2.350	1.674	0.940	0.490
$r = 0.4$										
0	6.90	5.91	4.94	4.027	3.198	2.229	1.602	1.117	0.607	0.309
0.2	7.16	6.16	5.20	4.270	3.425	2.421	1.757	1.225	0.675	0.346
0.4	7.35	6.35	5.39	4.458	3.605	2.581	1.892	1.327	0.741	0.381
0.6	7.54	6.54	5.56	4.640	3.779	2.736	2.024	1.432	0.806	0.416
0.8	7.74	6.74	5.77	4.837	3.968	2.903	2.164	1.545	0.872	0.451
0.9	7.86	6.87	5.89	4.950	4.076	2.996	2.240	1.606	0.905	0.469
1.0	8.00	7.00	6.03	5.085	4.202	3.101	2.350	1.674	0.940	0.490

Table 3. $r_H^{(p)}(q_p \; \eta)/F_{1/2}(\eta)$ as a function of q_p and η for various values of r

q_p \ η	0	0.2	0.4	0.6	0.8	0.9	1.0
				$r = 0$			
—4.0	54.975	59.54	64.87	71.63	82.02	90.60	105.92
—3.8	45.056	48.79	53.12	58.66	67.17	74.16	86.65
—3.6	36.955	60.02	43.57	48.12	55.06	60.79	71.13
—3.4	30.312	32.83	35.74	39.44	45.13	49.83	58.31
—3.2	24.883	26.95	29.34	32.37	36.97	40.86	47.81
—3.0	20.436	22.13	24.07	26.57	30.37	33.52	39.22
—2.8	16.807	18.19	19.77	21.85	24.96	27.55	32.18
—2.6	13.812	14.944	16.24	17.94	20.48	22.58	26.42
—2.4	11.380	12.30	13.37	14.75	16.84	18.59	21.70
—2.2	9.372	10.122	11.003	12.137	13.860	15.28	17.83
—2.0	7.734	8.353	9.057	10.010	11.41	12.57	14.67
—1.8	6.394	6.899	7.478	8.255	9.406	10.36	12.08
—1.6	5.294	5.702	6.189	6.824	7.761	8.550	9.960
—1.4	4.394	4.728	5.119	5.646	6.424	7.070	8.220
—1.2	3.656	3.930	4.256	4.680	5.323	5.850	6.798
—1.0	3.050	3.273	3.545	3.893	4.423	4.860	5.632
—0.8	2.554	2.733	2.960	3.246	3.688	4.046	4.676
—0.5	1.970	2.104	2.273	2.488	2.817	3.083	3.555
—0.3	1.666	1.774	1.916	2.091	2.366	2.589	2.969
0.0	1.307	1.388	1.492	1.627	1.834	1.999	2.286
0.5	0.895	0.943	1.007	1.095	1.225	1.333	1.507
1.0	0.635	0.664	0.705	0.762	0.846	0.914	1.023
1.5	0.466	0.485	0.512	0.550	0.605	0.649	0.717
2.0	0.354	0.366	0.384	0.409	0.447	0.476	0.520
3.0	0.223	0.228	0.237	0.249	0.267	0.281	0.300
5.0	0.113	0.114	0.117	0.120	0.126	0.130	0.135
10.0	0.0415	0.0417	0.0420	0.0425	0.0432	0.0437	0.0442
20.0	0.01482	0.01483	0.01487	0.01490	0.0 1499	0.01503	0.01523

Table 3 (*cont.*)

q_p \ η	0	0.2	0.4	0.6	0.8	0.9
			$r = 0.1$			
—4.0	55.21	60.47	66.24	73.28	83.78	92.08
—3.8	45.26	49.66	54.29	60.07	68.66	75.49
—3.6	37.71	40.65	44.49	49.19	56.21	61.79
—3.4	30.46	33.34	36.50	40.31	46.07	50.62
—3.2	25.00	27.37	29.96	33.07	37.82	41.53
—3.0	20.53	22.48	24.58	27.10	31.04	34.07
—2.8	16.87	18.49	20.20	22.27	25.48	27.98
—2.6	13.88	15.19	16.59	18.27	20.93	22.96
—2.4	11.42	12.52	13.66	15.02	17.21	18.88
—2.2	9.414	10.31	11.23	12.36	14.14	15.51
—2.0	7.769	8.499	9.250	10.180	11.64	12.76
—1.8	6.422	7.008	7.641	8.408	9.591	10.51
—1.6	5.317	5.797	6.321	6.946	7.915	8.677
—1.4	4.413	4.811	5.229	5.752	6.547	7.167
—1.2	3.672	3.992	4.336	4.771	5.426	5.937
—1.0	3.063	3.323	3.610	3.973	4.506	4.930
—0.8	2.564	2.779	3.014	3.307	3.752	4.099
—0.5	1.978	2.137	2.315	2.533	2.866	3.124
—0.3	1.672	1.799	1.949	2.132	2.406	2.621
0.0	1.312	1.407	1.519	1.659	1.865	2.026
0.5	0.898	0.958	1.025	1.115	1.249	1.345
1.0	0.637	0.673	0.717	0.776	0.860	0.924
1.5	0.467	0.490	0.519	0.558	0.613	0.655
2.0	0.355	0.370	0.389	0.415	0.452	0.479
3.0	0.223	0.230	0.239	0.252	0.270	0.283
5.0	0.113	0.115	0.117	0.121	0.126	0.130
10.0	0.0415	0.0417	0.0421	0.0426	0.0433	0.0437
20.0	0.01482	0.01483	0.01487	0.01492	0.1499	0.01503
			$r = 0.2$			
—4.0	55.94	61.85	67.73	75.04	85.49	93.45
—3.8	45.87	50.69	55.46	61.50	70.06	76.59
—3.6	37.76	41.57	45.45	50.37	57.35	62.73
—3.4	30.86	34.10	37.28	41.30	46.98	51.41
—3.2	25.33	27.99	30.60	33.86	38.57	42.17
—3.0	20.80	22.99	25.12	27.79	31.63	34.60
—2.8	17.09	18.91	20.64	22.86	25.97	28.40
—2.6	14.06	15.52	16.92	18.75	21.33	23.30
—2.4	11.47	12.77	13.94	15.42	17.53	19.17
—2.2	9.537	10.17	11.47	12.70	14.41	15.74

Table 3 (*cont.*)

q_p \ η	0	0.2	0.4	0.6	0.8	0.9
			$r = 0.2$			
—2.0	7.769	8.662	9.443	10.46	11.87	12.95
—1.8	6.504	7.148	7.801	8.632	9.783	10.66
—1.6	5.385	5.908	6.448	7.120	8.073	8.801
—1.4	4.468	4.899	5.339	5.888	6.670	7.272
—1.2	3.717	4.076	4.431	4.890	5.530	6.021
—1.0	3.100	3.387	3.686	4.070	4.605	5.009
—0.8	2.595	2.830	3.078	3.384	3.823	4.163
—0.5	2.000	2.177	2.360	2.591	2.920	3.172
—0.3	1.691	1.833	1.988	2.179	2.449	2.657
0.0	1.325	1.431	1.549	1.690	1.895	2.051
0.5	0.906	0.972	1.052	1.135	1.266	1.363
1.0	0.642	0.747	0.708	0.787	0.873	0.933
1.5	0.471	0.497	0.527	0.566	0.621	0.660
2.0	0.357	0.374	0.394	0.420	0.458	0.483
3.0	0.224	0.232	0.242	0.254	0.272	0.284
5.0	0.114	0.115	0.118	0.122	0.127	0.131
10.0	0.0416	0.0418	0.0422	0.0427	0.0434	0.0438
20.0	0.01483	0.01484	0.01488	0.01493	0.01500	0.01504
			$r = 0.3$			
—4.0	57.11	63.74	69.40	76.71	87.09	95.94
—3.8	46.83	49.83	56.88	62.85	71.41	78.62
—3.6	38.39	42.83	46.65	51.55	58.57	64.48
—3.4	31.50	35.09	38.20	42.29	48.05	52.84
—3.2	25.85	28.79	31.35	34.71	39.39	42.68
—3.0	21.23	23.60	25.75	28.49	32.29	35.05
—2.8	17.44	19.40	21.12	23.38	26.53	28.76
—2.6	14.34	15.94	17.35	19.20	21.77	23.62
—2.4	11.81	13.10	14.27	15.81	17.91	19.39
—2.2	9.73	10.778	11.74	13.00	14.71	15.93
—2.0	8.026	8.879	9.675	10.712	12.111	13.109
—1.8	6.632	7.320	7.991	8.822	9.986	10.804
—1.6	5.490	6.051	6.604	7.293	8.232	8.921
—1.4	4.554	5.009	5.470	6.042	6.810	7.360
—1.2	3.788	4.159	4.533	5.009	5.648	6.098
—1.0	3.159	3.462	3.775	4.156	4.683	5.064
—0.8	2.643	2.892	3.149	3.463	3.897	4.208
—0.5	2.037	2.221	2.414	2.650	3.975	3.202
—0.3	1.721	1.874	2.033	2.224	2.496	2.687
0.0	1.348	1.460	1.581	1.725	1.928	2.073

Table 3 (*cont.*)

q_p \ η	0	0.2	0.4	0.6	0.8	0.9
			$r = 0.3$			
0.5	0.9201	0.9908	1.065	1.157	1.282	1.369
1.0	0.6505	0.6950	0.7343	0.8013	0.8822	0.9362
1.5	0.4767	0.5044	0.5358	0.5748	0.6285	0.6658
2.0	0.3611	0.3789	0.4002	0.4267	0.4622	0.4866
3.0	0.2261	0.2342	0.2443	0.2570	0.2741	0.2855
5.0	0.1140	0.1160	0.1188	0.1225	0.1275	0.1309
10.0	0.04163	0.04189	0.04227	0.04277	0.04341	0.04379
20.0	0.01484	0.01485	0.01489	0.01494	0.01500	0.01504
			$r = 0.4$			
—4.0	58.70	64.95	71.16	79.95	88.85	95.99
—3.8	48.12	53.26	58.35	65.56	79.85	78.67
—3.6	39.46	43.64	47.85	52.77	59.68	64.48
—3.4	32.37	35.77	39.23	43.23	48.96	52.90
—3.2	26.56	29.36	32.17	35.46	40.12	43.37
—3.0	21.81	24.11	26.36	29.12	32.90	35.56
—2.8	17.92	19.82	21.66	23.93	27.04	29.19
—2.6	14.73	16.27	17.78	19.65	22.18	23.96
—2.4	12.12	13.38	14.61	16.15	18.24	19.67
—2.2	9.99	11.01	12.04	13.29	14.99	16.18
—2.0	8.239	9.087	9.923	10.94	12.34	13.32
—1.8	6.809	7.493	8.183	9.027	10.17	10.96
—1.6	5.634	6.194	6.761	7.453	8.391	9.042
—1.4	4.673	5.132	5.602	6.169	6.933	7.469
—1.2	3.885	4.259	4.643	5.115	5.740	6.178
—1.0	3.239	3.551	3.862	4.249	4.767	5.125
—0.8	2.708	2.963	3.224	3.537	3.965	4.259
—0.5	2.086	2.276	2.473	2.707	3.123	3.246
—0.3	1.761	1.916	2.078	2.274	2.541	2.716
0.0	1.378	1.495	1.617	1.764	1.960	2.091
0.5	0.939	1.011	1.087	1.178	1.302	1.383
1.0	0.663	0.708	0.755	0.815	0.895	0.926
1.5	0.484	0.513	0.545	0.584	0.636	0.671
2.0	0.366	0.385	0.406	0.434	0.467	0.491
3.0	0.229	0.237	0.247	0.260	0.276	0.287
5.0	0.115	0.117	0.120	0.123	0.125	0.131
10.0	0.0417	0.0420	0.0424	0.0429	0.0435	0.0439
20.0	0.01485	0.01486	0.01490	0.01495	0.01501	0.01504

The values of q_p and η that correspond to these variables at a given value of $r(r = 0.4)$ are then determined from Fig. 8 and 9. ($eN_cR = 0.6$; ($e/k_0\alpha = 2.7$). We plot the dependences of q_p on η (Fig. 10e) for these parameters and determine the demanded values of q_p and η from the intersection point of the curves. (—). These curves are plotted for some of the variables in Fig. 10e.

Evaluation is rather complicated by the fact that the charts must be constructed for definite values of $r(T)$ that vary from 0 to $1/2$ for $T \gtrless \Theta$. The respective values for $r = 0; 0.1; 0.2; 0.3; 0.4$ are shown in Tables 2 and 3, and in the charts 10a, b, c, d, e.

We shall determine the scattering factors for the mixed scattering of charge carrier by acoustical phonons and ionized impurities, in a similar manner to that applied in the
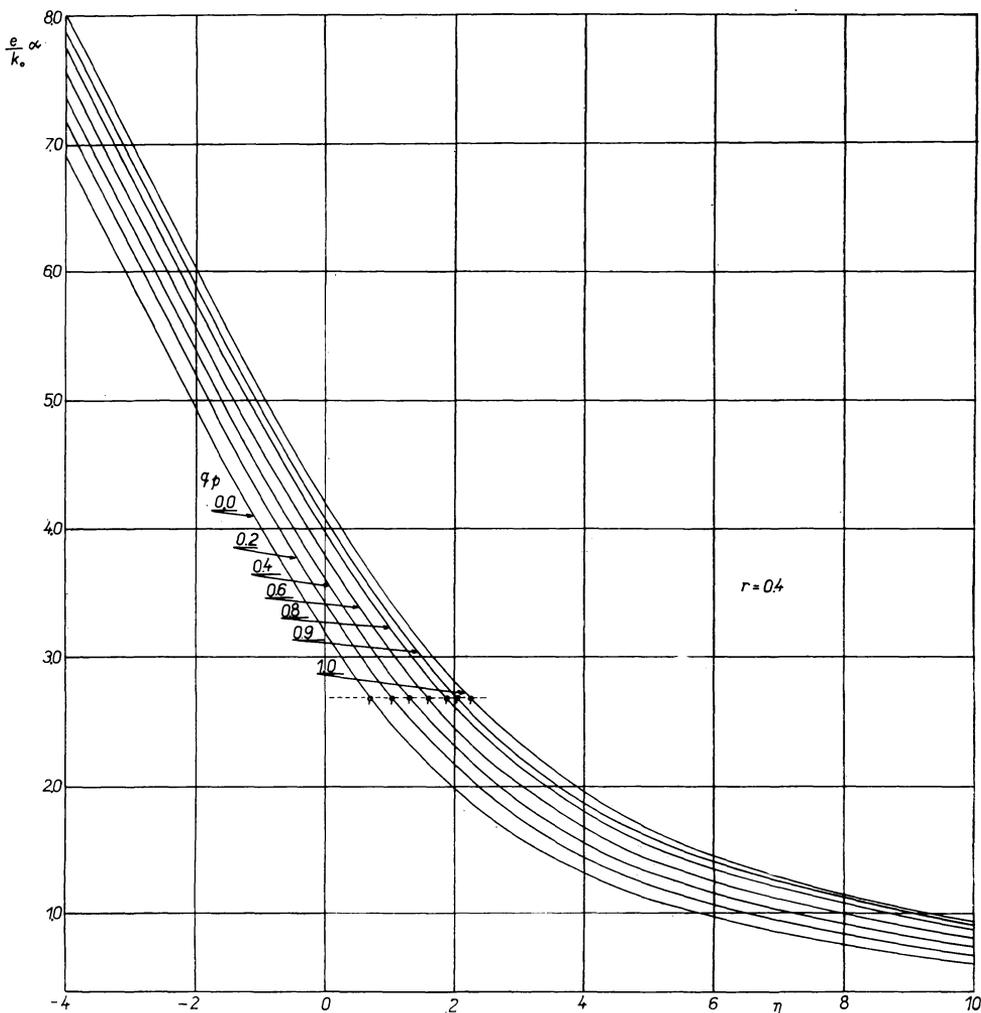
Fig. 8. The Dependence of the Parameter $(e/k_0)\alpha$ on the Reduced Fermi energy η for Various Values of $q_p(r = 0.4)$

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previous case. For a non-degenerate semiconductor the left-hand side of equation (46) is determined from measurement results and $q_L = \dfrac{\mu_L}{\mu_L + \mu_I}$ is determined from Fig. 11.

Table 4. *Values of expression* $\{B^{(L)}(q_L) - \ln r_H^{(L(}(q_L)\}$ *for non-degenerate semiconductor*

β^2	q_L	$B(q_L) -$ $- \ln r_H^{(L)}$	β^2	q_L	$B(q_L) -$ $- \ln r_H^{(L)}$
0	0	1.856	6.25	0.5102	2.831
0.04	0.00662	1.987	9.00	0.6000	2.908
0.16	0.02597	2.130	12.25	0.6712	2.965
0.36	0.05660	2.250	16.00	0.7273	3.020
0.64	0.09638	2.351	20.25	0.7714	3.060
1.00	0.1428	2.438	25.00	0.8064	3.096
1.44	0.1935	2.512	36.00	0.8571	3.148
1.96	0.2462	2.578	49.00	0.8909	3.189
2.56	0.2990	2.636	100.00	0.9434	3.256
3.24	0.3506	2.689	400.00	0.9852	3.318
4.00	0.4000	2.734	∞	1.000	3.341
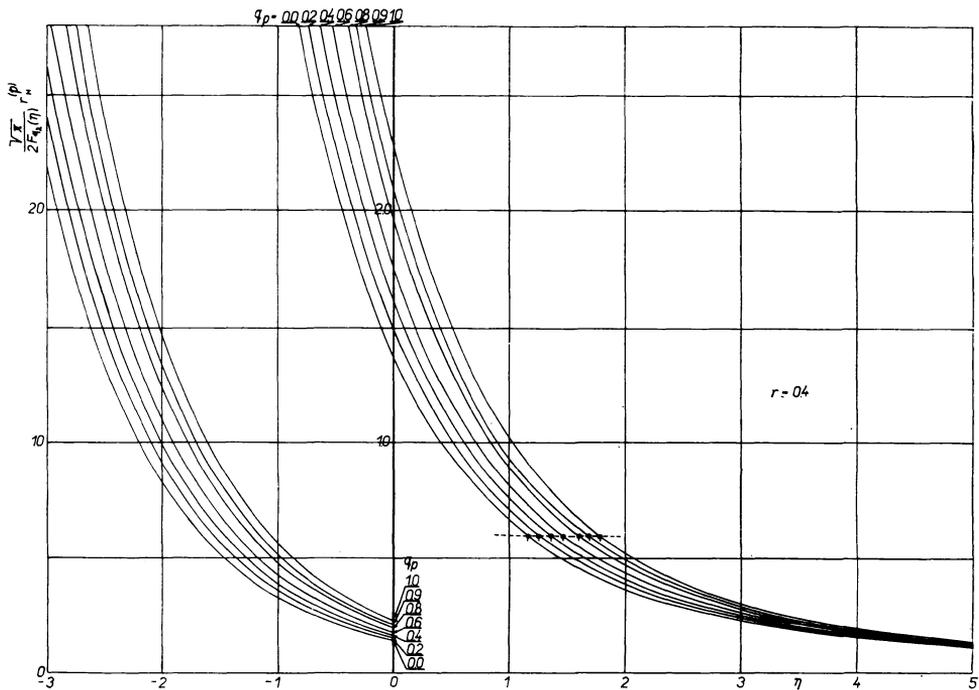
Fig. 9. The Dependence of the Parameter $r_H^{(p)}(q_p, \eta)/F^{1/_2}(\eta) = eN_c R$ on the Reduced Fermi Energy for Various Values of $q_p(r = 0.4)$

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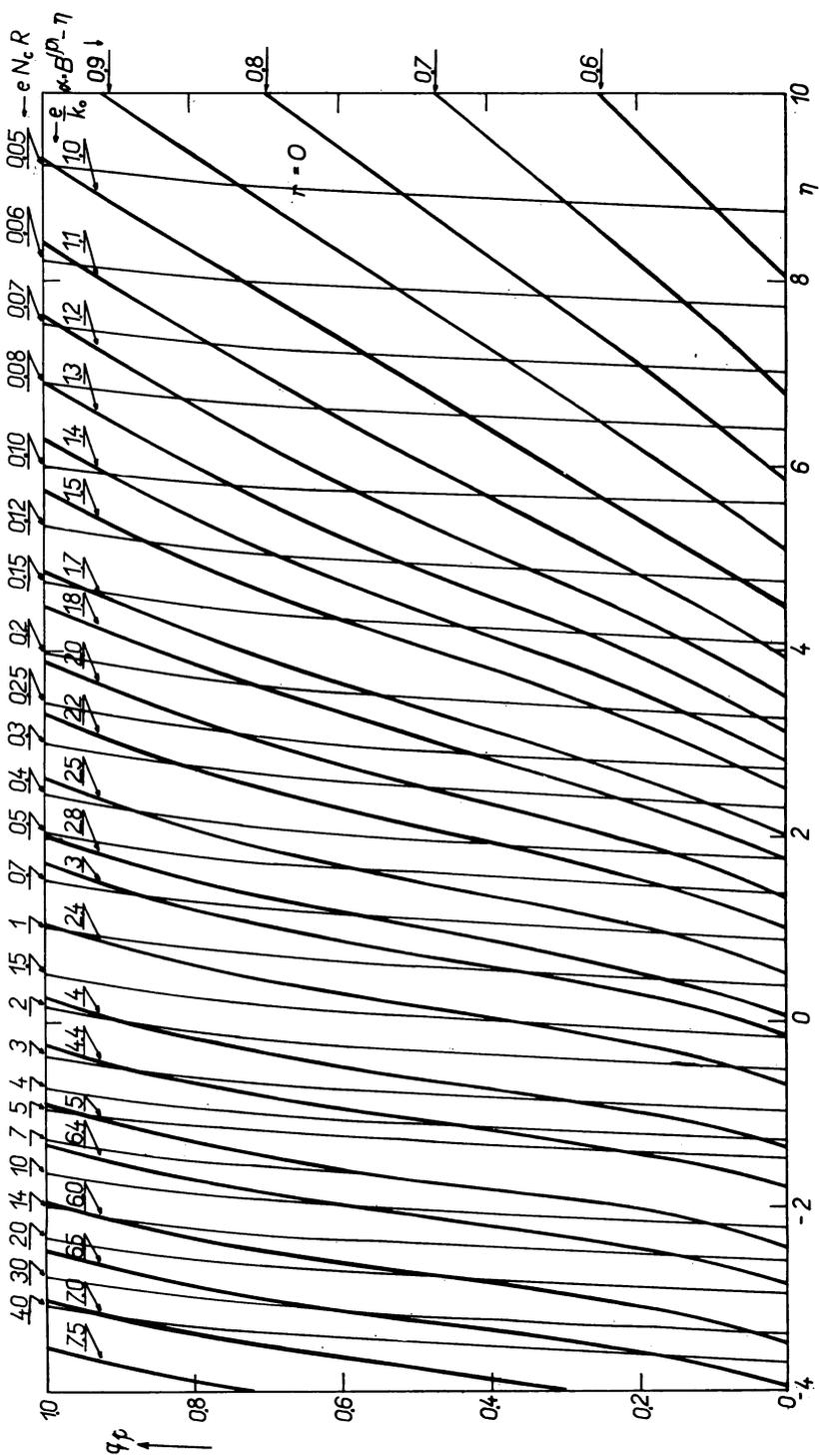
Fig. 10a. The Dependence of q_p on the Reduced Fermi Energy η. Parameters are $(e/k_0)\alpha$ and $r_H^{(p)} (q_p, \eta)/F_{1/2}(\eta)$ $(r = 0)$
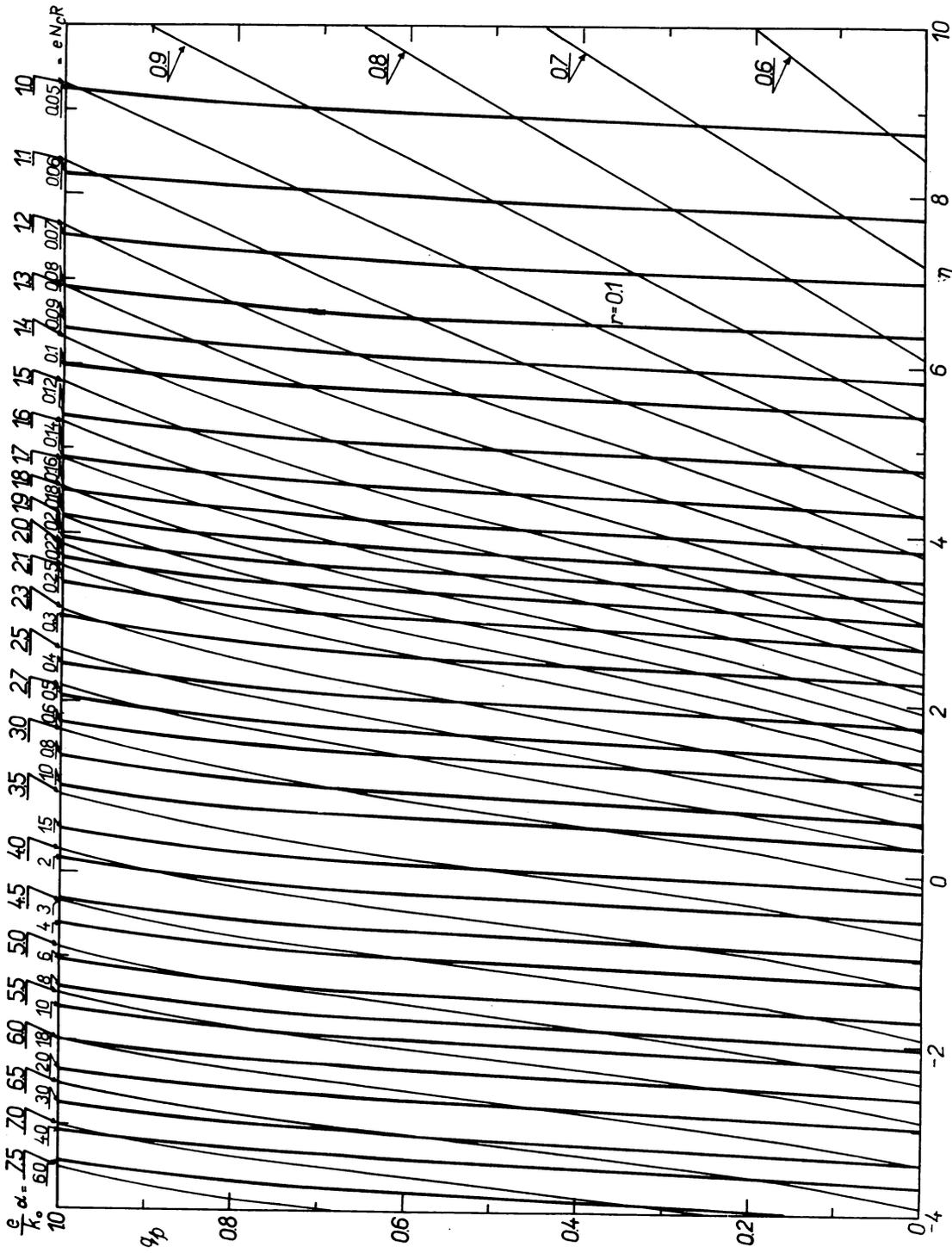
Fig. 10b ditto ($r = 0.1$)
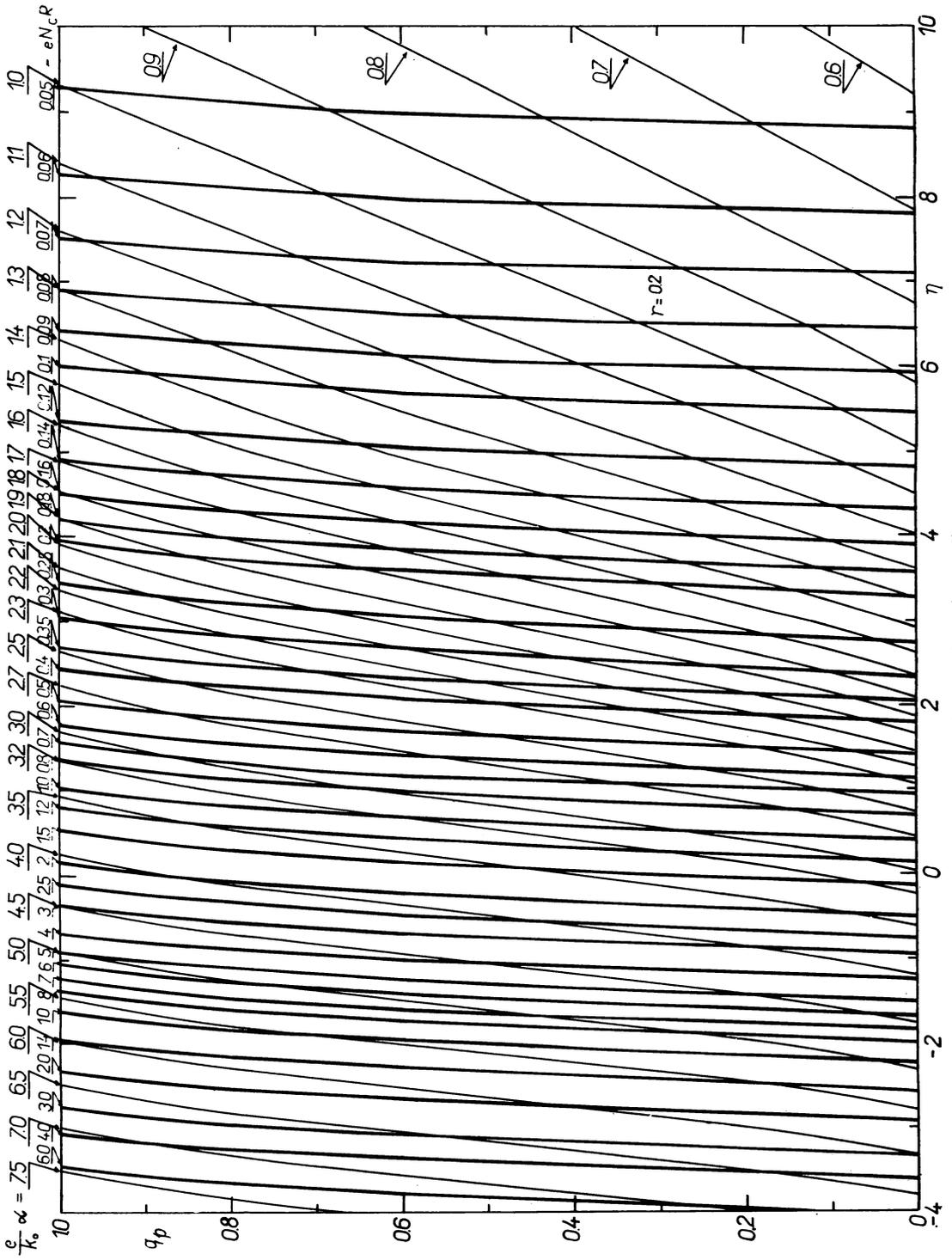
Fig. 10c ditto ($r = 0.2$)

Fig. 10d ditto (r = 0.3)

$\frac{e}{k_o}\alpha =$

q_p

$= e N_c R$

$r = 0.4$

η
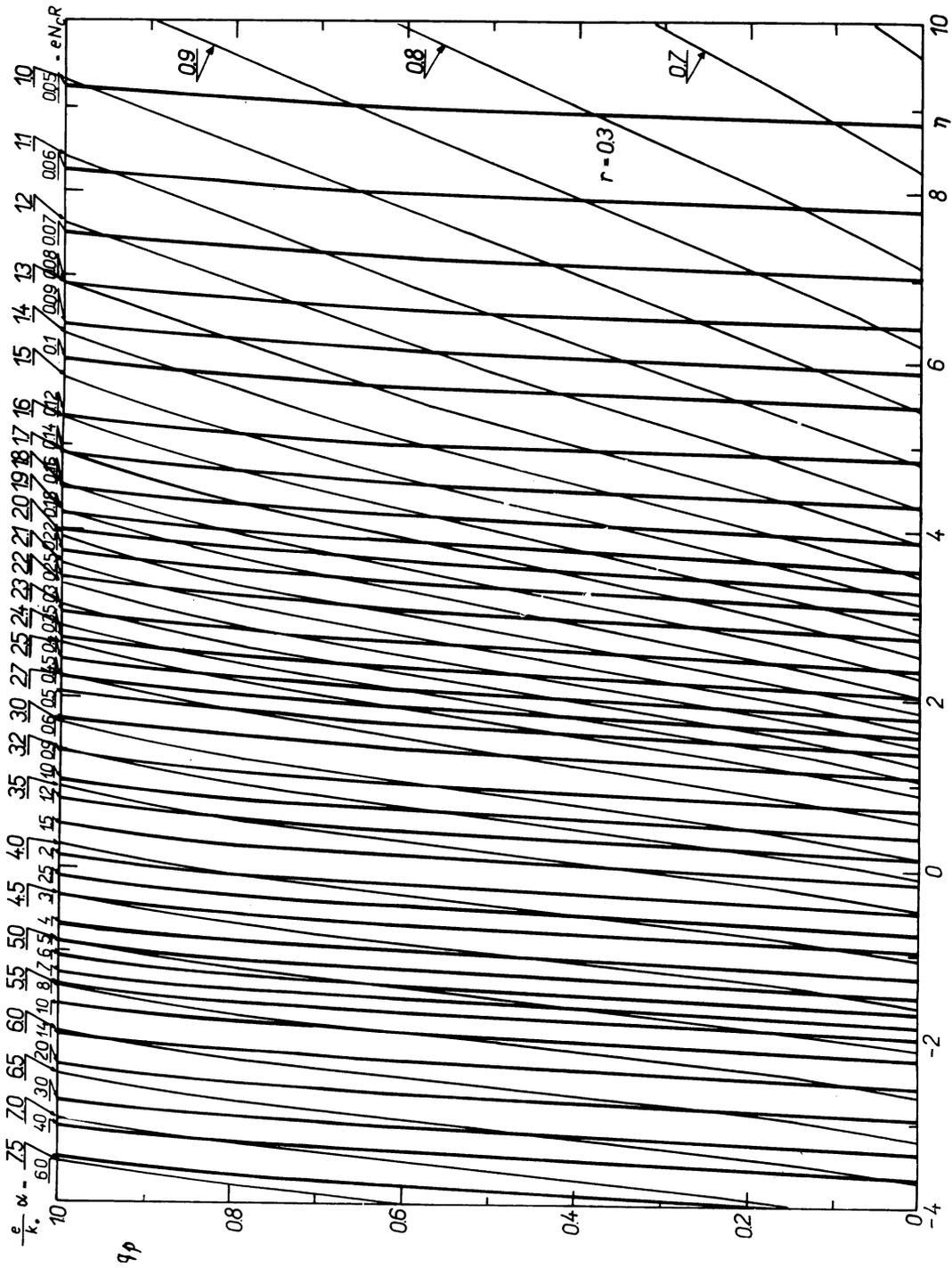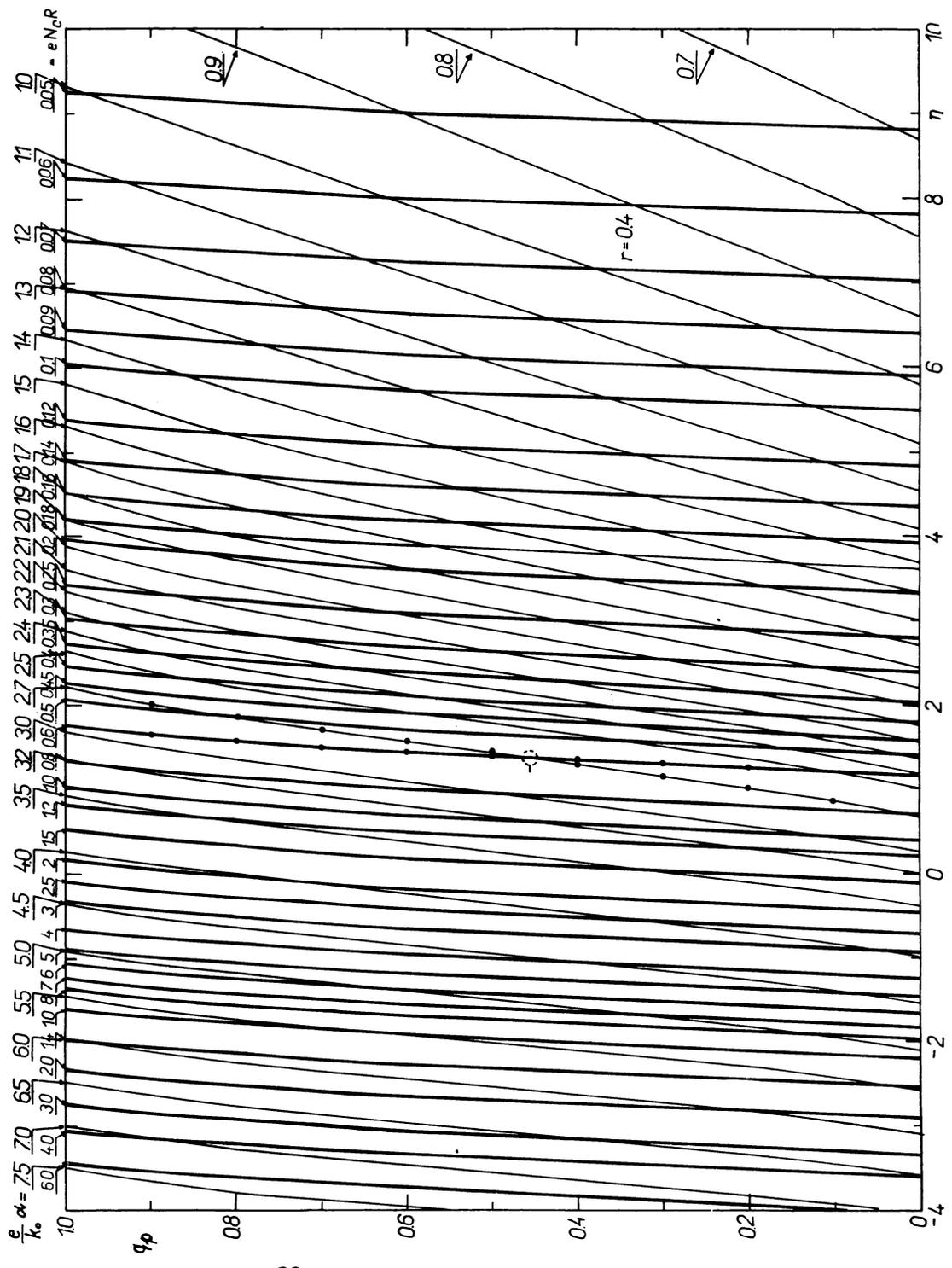
Fig. 10e ditto ($r = 0.4$)

The corresponding values are mentioned in Table 4. Tables elaborated by Dingle, Arndt and Roy were used for their calculations [5].

All the scattering factors have not been computed for non-degenerate semi-conductors. Fistul, Omeliamovski, Tatarov (21) mention the average values $\langle \tau^{(L)} \rangle$ and $\langle (\tau^{(L)})^2 \rangle$. The scattering factors $r_H^{(L)}(\beta, \eta)$ for the mixed scattering by acoustical lattice vibrations and ionized impurities are plotted for various values of β in Fig. 12. The values of the factor $B^{(L)}(q_L, \eta)$, have to our knowledge, not yet been calculated.
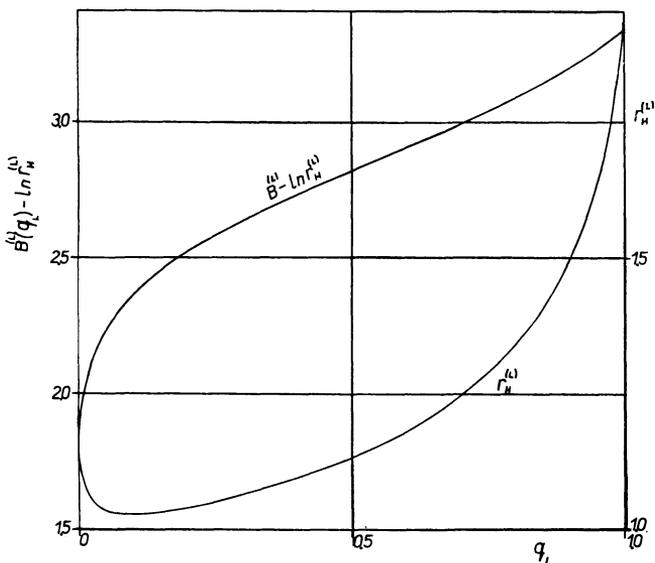
Fig. 11. The Dependence of the Expression $\{B^{(L)}(q_L) - \ln r_H^{(L)}(q_L)\} = (e/k_0)\alpha - \ln eN_cR$ and the Scattering Factor $r_H^{(L)}(q_L)$ on the Parameter q_L for non-degenerate Semiconductor
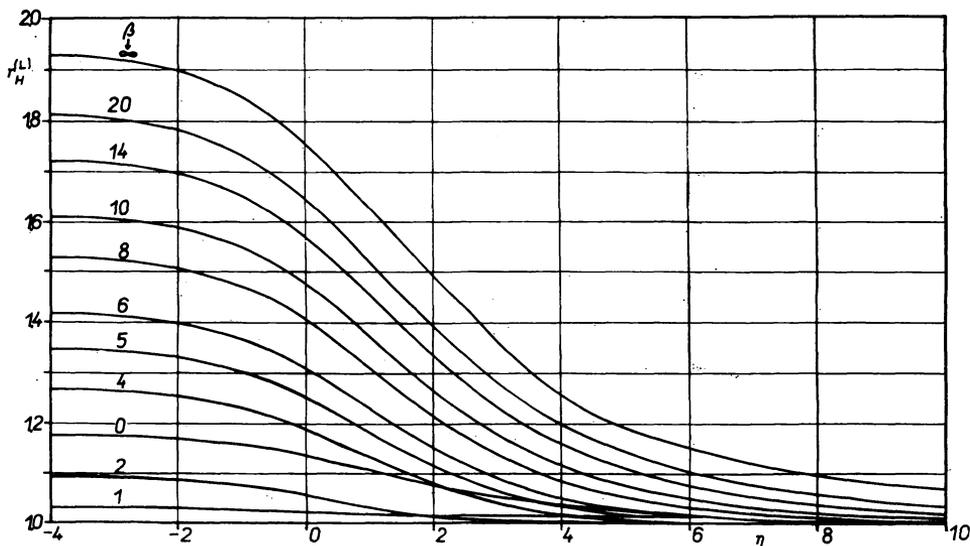

Fig. 12. The Dependence of the Hall Coefficient Factor on the Reduced Fermi Energy for Various Values of β

7. The Determination of Fermi Energy and Scattering Factors from Measurement of Electrical Conductivity and Hall Coefficient

In this chapter there is described the method of the determination of results, especially of the carrier concentration and mobility due to the impurity scattering from the measurements of electrical conductivity and Hall coefficient only, on the assumption that the value and the temperature dependence of lattice mobility, as well as the effective mass m are known. This assumption is often fulfilled very well. Further, we shall assume that the predominating fundamental type of scattering is either scattering by acoustical or only by optical mode lattice vibrations.

The value of the Fermi energy and the scattering factor $r_H^{(p)}$ can be determined by the following method. We shall estimate the Hall mobility $\mu_H^{(p)}$ from the Hall coefficient and electrical conductivity; it is:

$$(51) \qquad \mu_H^{(p)} = R\sigma = r_H^{(p)} \mu^{(p)}$$

and thus we calculate the ratio $\mu_H^{(p)}/\mu_p^*$ which we may write in the form

$$(52) \qquad \frac{\mu_H^{(p)}}{\mu_p^*} = \frac{\langle(\tau^{(p)})^2\rangle}{\langle\tau^{(p)}\rangle\,\langle\tau_p\rangle^*} = \frac{3\sqrt{\pi}\,M_{{}^{1}\!/_{2}}^r(\gamma,\eta)}{4\Gamma\,(^5/_2 + r)\,M_3^r(\gamma,\eta)}$$
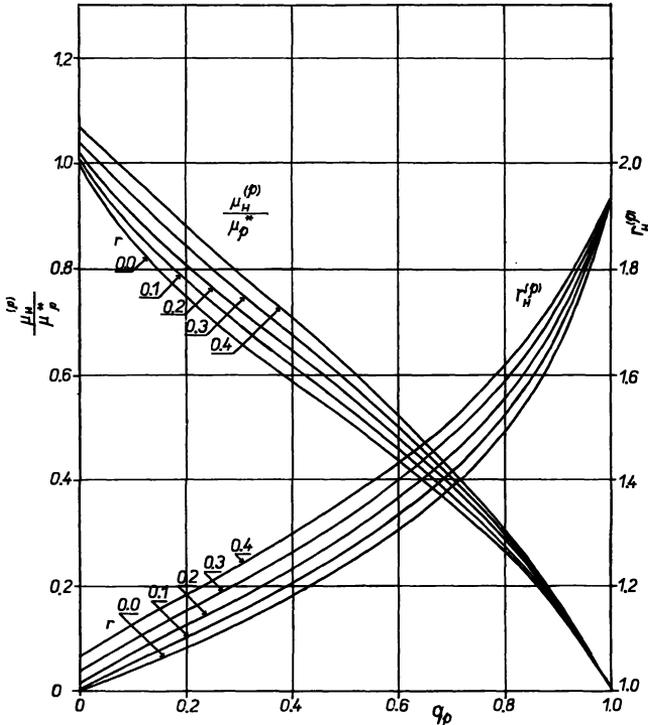

Fig. 13. The Dependence of the Mobility Ratio $\mu_H^{(p)}/\mu_p^*$ and the Hall Coefficient Factor $r_H^{(p)}(q_p)$ on the Parameter q_p for Various Values of r (non-degenerate semiconductor)

Table 5. *The ratio $\mu_H^{(p)}/\mu_p^*$ as a function of q_p and η for various values of r*

$r = 0$

q_p \ η	$-\infty$	-1	0	1.5	3	5	10	20
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.2	0.7456	0.7488	0.7527	0.7625	0.7733	0.8002	0.7941	0.7983
0.4	0.5874	0.5874	0.5874	0.5885	0.5908	0.6176	0.5973	0.5551
0.6	0.4370	0.4340	0.4304	0.4227	0.4154	0.4337	0.4032	0.4449
0.8	0.2640	0.2592	0.2536	0.2409	0.2282	0.2349	0.2061	0.2017
0.9	0.1531	0.1489	0.1441	0.1330	0.1224	0.1242	0.1046	0.1013
1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

$r = 0.1$

q_p \ η	$-\infty$	-1	0	1.5	3	5	10	20
0.0	1.0047	1.0116	1.0202	1.0427	1.0711	1.1074	1.1749	1.2557
0.2	0.7759	0.7822	0.7901	0.8119	0.8398	0.8928	0.9354	1.0032
0.4	0.6155	0.6176	0.6205	0.6298	0.6441	0.6910	0.7045	0.7533
0.6	0.4580	0.4564	0.4548	0.4524	0.4529	0.4853	0.4755	0.5039
0.8	0.2744	0.2705	0.2661	0.2563	0.2477	0.2622	0.2427	0.2535
0.9	0.1558	0.1538	0.1497	0.1405	0.1321	0.1381	0.1230	0.1272

$r = 0.2$

q_p \ η	$-\infty$	-1	0	1.5	3	5	10	20
0.0	1.0181	1.0309	1.0469	1.0902	1.1399	1.2236	1.3753	1.5697
0.2	0.8081	0.8175	0.8294	0.8634	0.9099	0.9930	1.0974	1.2551
0.4	0.6448	0.6490	0.6548	0.6728	0.7003	0.7703	0.8273	0.9428
0.6	0.4794	0.4793	0.4796	0.4830	0.4922	0.5411	0.5583	0.6306
0.8	0.2847	0.2817	0.2784	0.2719	0.2679	0.2915	0.2846	0.3171
0.9	0.1614	0.1585	0.1552	0.1480	0.1422	0.1530	0.1441	0.1591

$r = 0.3$

q_p \ η	$-\infty$	-1	0	1.5	3	5	10	20
0.0	1.0394	1.0573	1.0799	1.1428	1.2293	1.3497	1.6041	1.9523
0.2	0.8424	0.8550	0.8710	0.9186	0.9840	1.1012	1.2825	1.5633
0.4	0.6753	0.6817	0.6904	0.7174	0.7593	0.8560	0.9675	1.1743
0.6	0.5011	0.5026	0.5049	0.5144	0.5332	0.6010	0.6527	0.7854
0.8	0.2946	0.2927	0.2906	0.2876	0.2887	0.3228	0.3323	0.3947
0.9	0.1652	0.1629	0.1604	0.1554	0.1524	0.1689	0.1681	0.1980

$r = 0.4$

q_p \ η	$-\infty$	-1	0	1.5	3	5	10	20
0.0	1.0683	1.0906	1.1191	1.2005	1.3171	1.4863	1.8645	2.4206
0.2	0.8799	0.8951	0.9149	0.9742	1.0623	1.2178	1.4932	1.9395
0.4	0.7072	0.7158	0.7274	0.7638	0.8213	0.9481	1.1270	1.4571
0.6	0.5233	0.5263	0.5306	0.5466	0.5760	0.6652	0.7600	0.9746
0.8	0.3043	0.3034	0.3026	0.3033	0.3102	0.3561	0.3865	0.4895
0.9	0.1687	0.1672	0.1654	0.1626	0.1629	0.1858	0.1953	0.2455

where

$$(53) \qquad M_3^r (\gamma, \eta) = \int_0^\infty \frac{x^3 \, e^{(x-\eta)} \, \mathrm{d}x}{[\gamma^2 + x^{(3/2 - r)}]^2 \, [1 + \exp (x - \eta)]^2}$$

$$(54) \qquad M_{1/2}^r (\gamma, \eta) = \int_0^\infty \frac{x^{3/2} \, e^{(x-\eta)} \, \mathrm{d}x}{[\gamma^2 + x^{(3/2 - r)}]^2 \, [1 + \exp (x - \eta)]^2}$$

Instead of μ_p^* and $\langle \tau_p \rangle^*$ we take the values (5), (24) for the non-degenerate semiconductors ($\eta < -4$). In order to be able to use the tables compiled by Amith, Kudman and Steigmeier [2] we rewrite equation (52) as follows:

$$(55) \qquad \frac{\mu_H^{(p)}}{\mu_p^*} = r_H^{(p)} \, \frac{\mu}{\mu_{II}} \, (1 - q_p) \, \frac{F_{(1/2 + r)} (\eta)}{F_{1/2} (\eta)}$$

The values on the right-hand side of expression (55) are given in Table 5 in dependence on the Fermi energy and the parameter q_p. As the tables of the Fermi-Dirac integrals
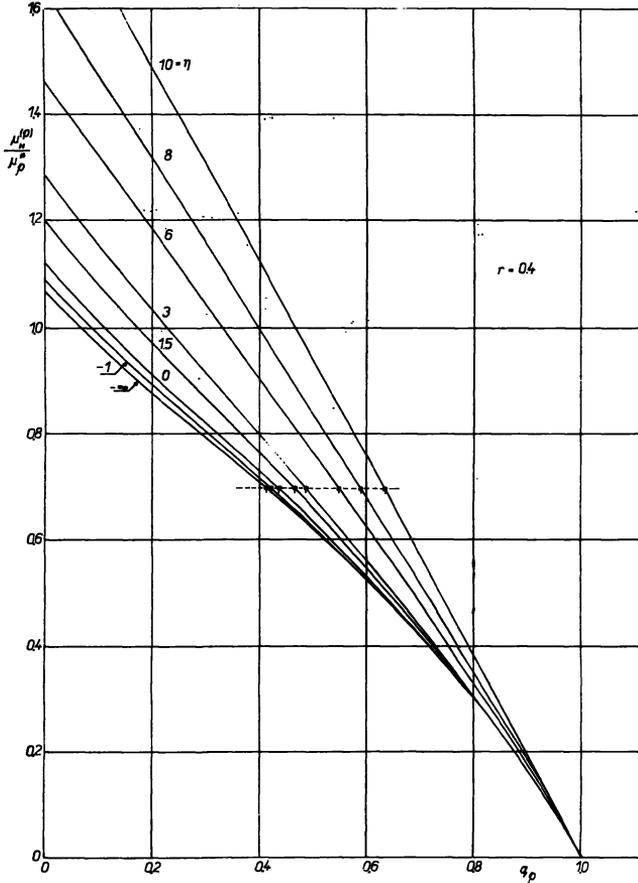
Fig. 14. The Dependence of the Mobility Ratio $\mu_H^{(p)}/\mu_p^*$ on the Parameter q_p for Various Values of Reduced Fermi Energy η ($r = 0.4$)
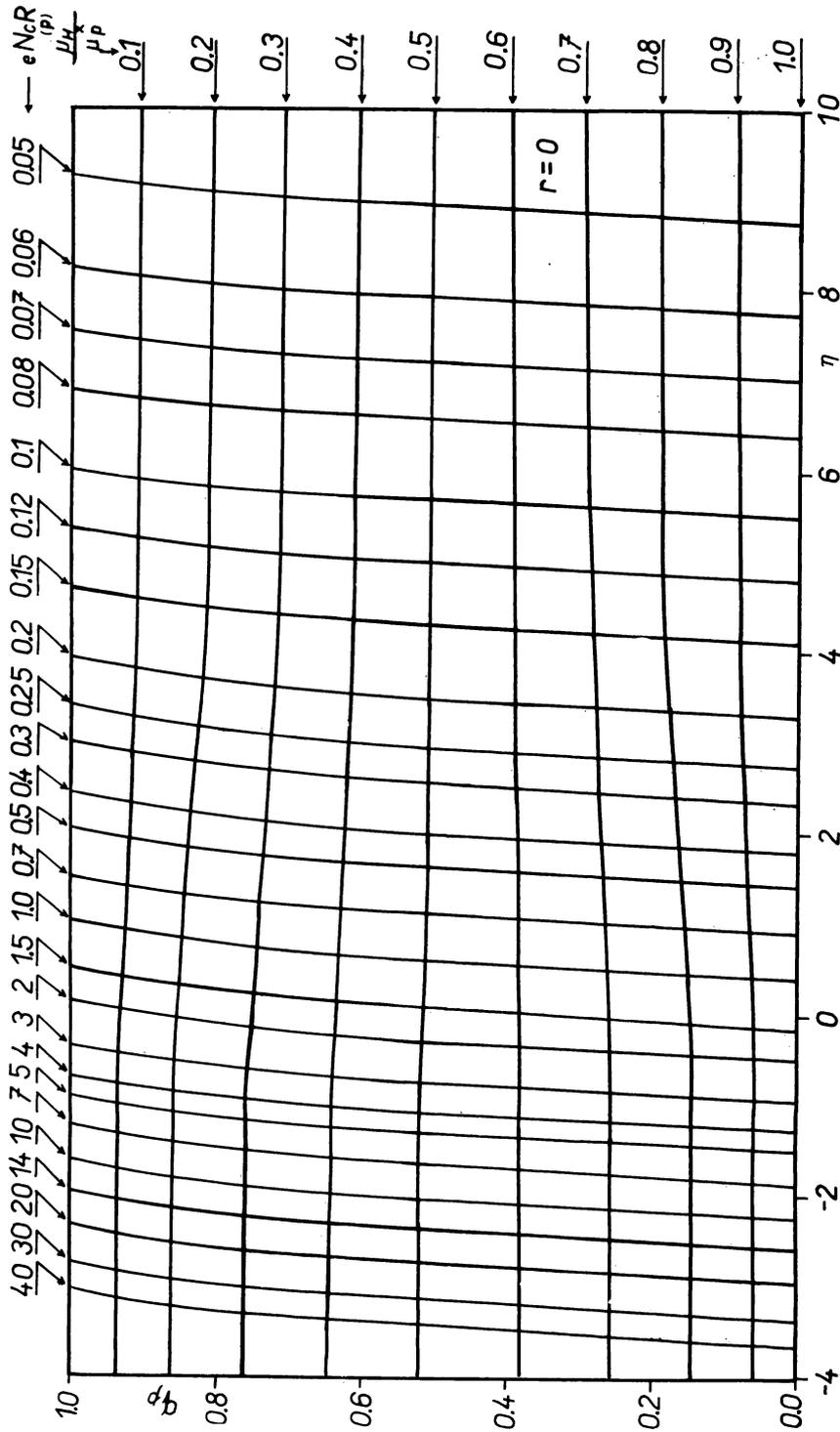
Fig. 15a. The Dependence of q_p on the Reduced Fermi Energy η. Parameters are eN_cR and the Mobility Ratio $\mu_H^{(p)}/\mu_p^*$ ($r = 0$)
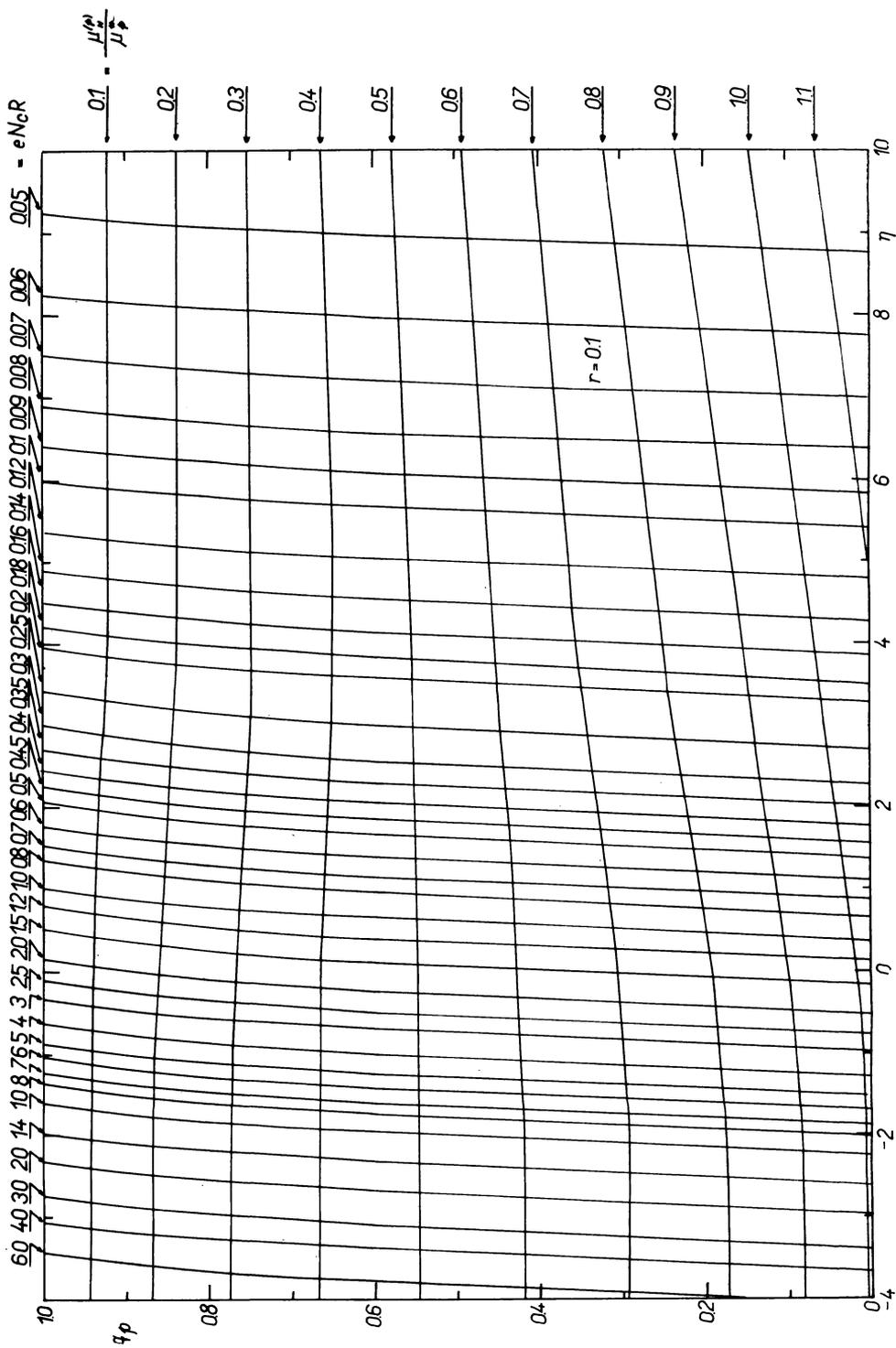
Fig. 15b ditto (r = 0.1)

Fig. 15c ditto ($r = 0.2$)

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Fig. 15d ditto ($r = 0.3$)

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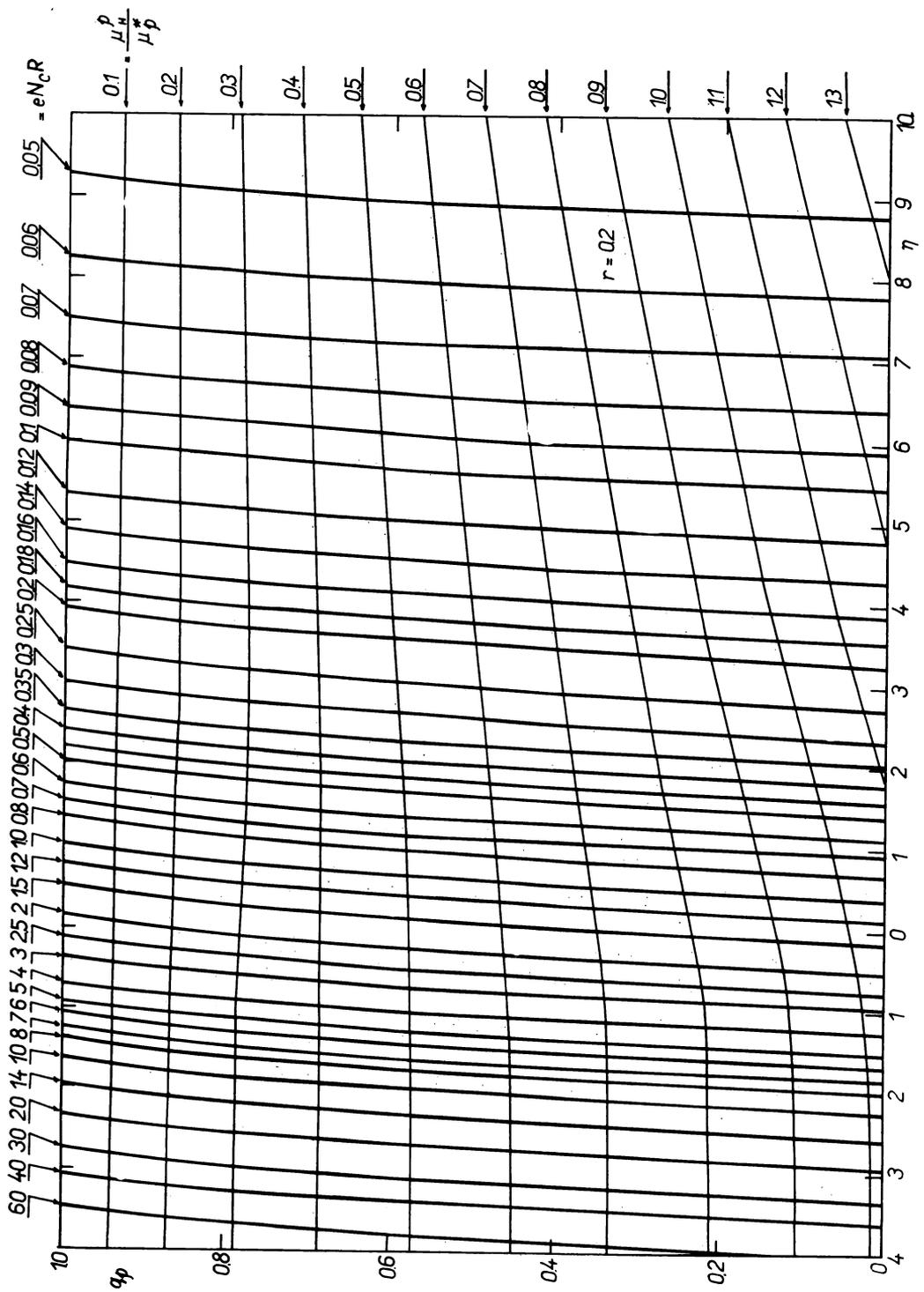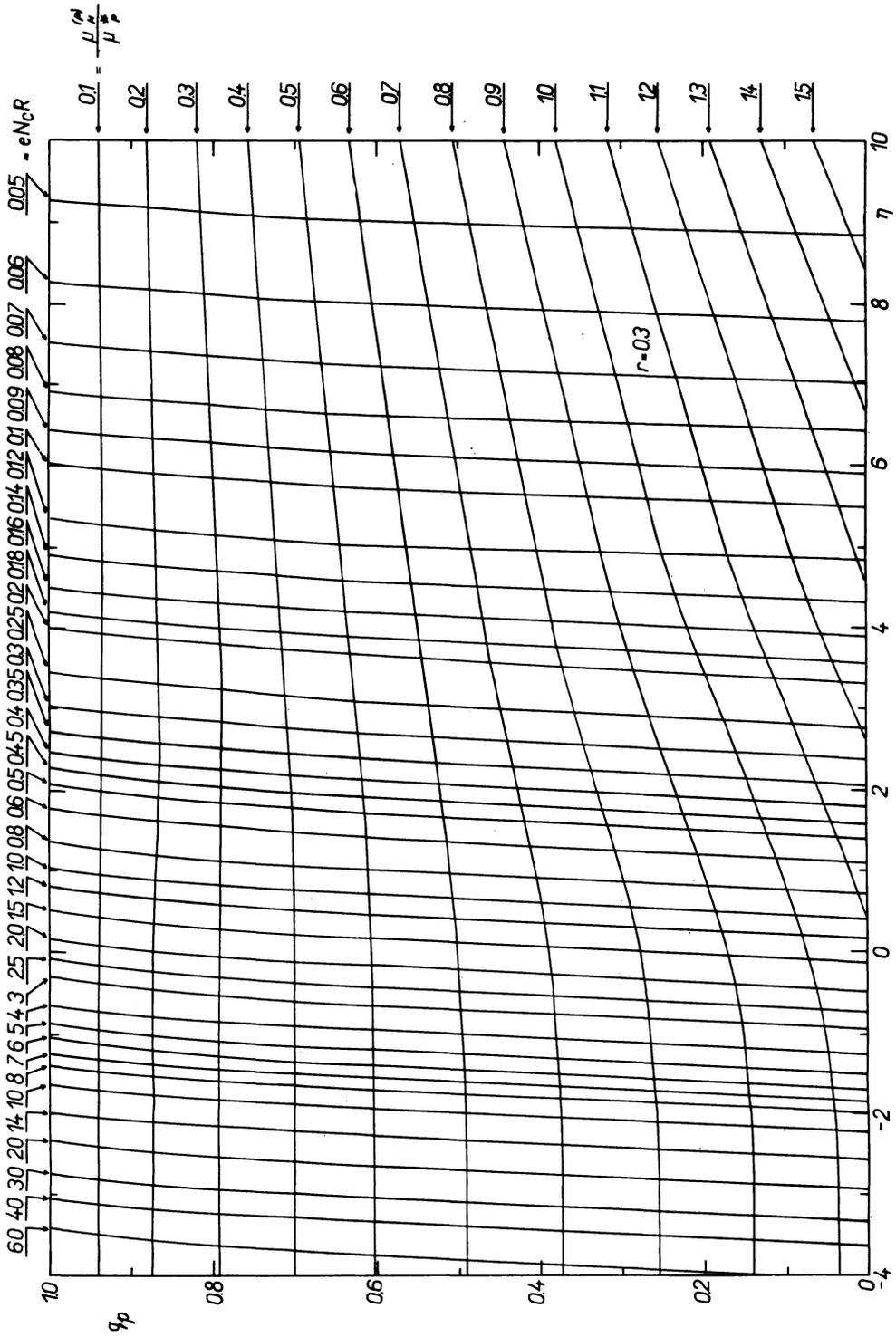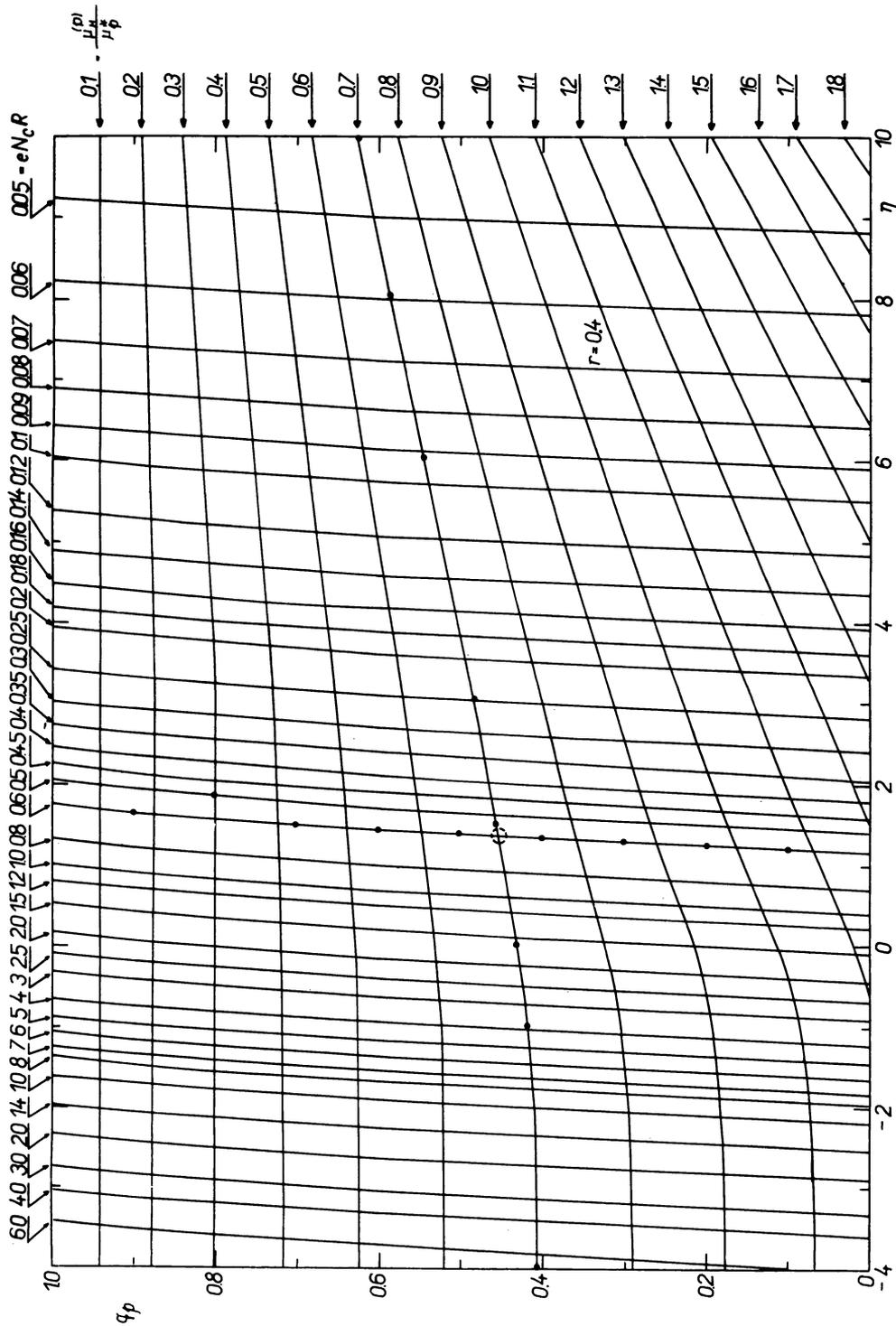
Fig. 15e ditto ($r = 0.4$)

were elaborated only for integer and half-integer orders [23] the values $F_{(1/2+r)}(\eta)$ were estimated from the charts, so that the accuracy of results was about 0.1%.

The parameter q_p and the corresponding scattering factor for non-degenerate semiconductors can be determined as follows: The Hall mobility $\mu_H^{(p)}$ can be calculated from the measurements of electrical conductivity and Hall coefficient, the ratio $\mu_H^{(p)}/\mu_p^*$ will be determined by means of the known mobility μ_p^* and q_p will be estimated from diagram 13. A certain scattering factor $r_H^{(p)}$ (Fig. 13) corresponds to this value, by means of which we can calculate the carrier concentration from the Hall coefficient (7). The factor C_I will be determined from the following relation:

$$(56) \qquad C_I = \frac{(3 + 2r)(1 - q_p)\, C_p \Gamma(3/2 + r)}{12 q_p}$$

and from this the concentration of ionized impurities N_I.

The evaluation is more complicated with regard to degenerate semiconductors, where the ratio $\mu_H^{(p)}/\mu_p^*$ is a function of q_p and η. The parameter eN_cR and the parameter $\mu_H^{(p)}/\mu_p^*$ can be calculated from the measurements of the Hall coefficient and electrical conductivity. The values of q_p and η corresponding to these parameters (e.g. $eN_cR = 0.6$; $\mu_H^{(p)}/\mu_p^* = 0.7$) will be estimated from diagrams 14 and 9 ($r = 0.4$). For these estimated parameters we construct the dependence of q_p on η (Fig. 15e) and from the intersection point of the curves we determine the values of q_p and η. These curves are plotted for some parameters (Fig. 15a, b, c, d, e).

The value of the scattering factor $r_H^{(p)}(q_p, \eta)$ will be calculated from equation (50) and the factor C_I from the following equation:

$$(57) \qquad C_I = \frac{(3 + 2r)(1 - q_p) F_{(1/2+r)}(\eta)}{6 q_p\, F_2(\eta)}$$

When $r = 0$; 0.1; 0.2; 0.3; 0.4, the respective values are mentioned in Table 3 and 5 and in Figs. 15a, b, c, d, e.

The mixed scattering charge carriers by the acoustical lattice vibrations and ionized impurities are determined in the same way.

We may apply relationships (51–57) once more but must exchange the index (p) for (L) and substitute the value $-1/2$ for the exponent r. In order to be able to use the tables computed by Fistul, Omelianovski and Tatarov [21], [24], we shall denote the corresponding factors for degenerate semiconductors by means of the parameter β^2. The ratio of mobilities $\mu_H^{(L)}/\mu_L^*$ will thus be equal

$$(58) \qquad \frac{\mu_H^{(L)}}{\mu_L^*} = \frac{\langle(\tau^{(L)})^2\rangle}{\langle\tau^{(L)}\rangle\langle\tau_L\rangle^\star} = \frac{3\sqrt{\pi}}{4}\; \frac{\Phi_{5/2}(\beta, \eta)}{\Phi_3(\beta, \eta)}$$

where μ_L^* and $\langle\tau_L\rangle^\star$ denote the mobility and average relaxation time values for scattering by acoustical phonons in a non-degenerate semiconductor;

$$(59) \qquad \Phi_3(\beta, \eta) = \int\limits_0^\infty \frac{x^3\, e^{(x-\eta)}\, \mathrm{d}x}{[\beta^2 + x^2]\,[1 + \exp(x - \eta)]^2}$$

$$(60) \qquad \Phi_{5/2}(\beta, \eta) = \int\limits_0^\infty \frac{x^{5/2}\, e^{(x-\eta)}\, \mathrm{d}x}{[\beta^2 + x^2]^2\,[1 + \exp(x - \eta)]^2}$$

The values corresponding to expression equation (58) and integrals (59) and (60) are tabulated in paper [21]. The parameter $q_L = \dfrac{\mu_L}{\mu_L + \mu_I}$ and its corresponding scattering factor $r_H^{(L)}$ for the non-degenerate semiconductor are determined from Fig. 16. The ratio

Table 6. *The ratio $\mu_H^{(L)}/\mu_L^*$ as a function of q_L or β for non-degenerate semiconductor*

β	q_L	$\dfrac{\mu_H^{(L)}}{\mu_L^*}$	β	q_L	$\dfrac{\mu_H^{(L)}}{\mu_L^*}$
0	0	1.178	3.5	0.6712	0.2961
0.2	0.00662	1.038	4.0	0.7273	0.2604
0.4	0.02597	0.9161	4.5	0.7714	0.2307
0.6	0.05660	0.8205	5.0	0.8064	0.2057
0.8	0.09638	0.7428	6.0	0.8571	0.1661
1.0	0.1428	0.6780	7.0	0.8909	0.1367
1.2	0.1935	0.6224	8.0	0.9143	0.1141
1.4	0.2462	0.5744	9.0	0.9310	0.09655
1.6	0.2990	0.5326	10.0	0.9434	0.08257
1.8	0.3506	0.4954	12.0	0.9600	0.06217
2.0	0.4000	0.4625	16.0	0.9771	0.03847
2.5	0.5102	0.3936	20.0	0.9852	0.0262
3.0	0.6000	0.3397	∞	1.0000	0.0000
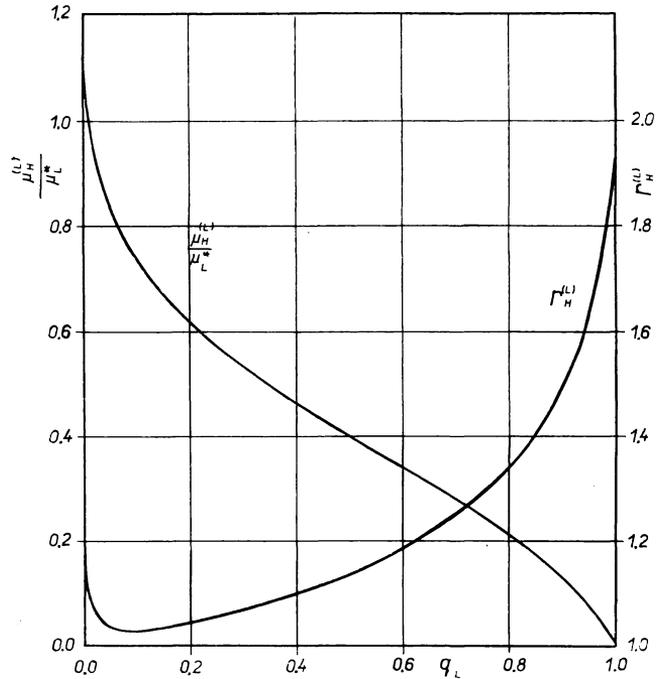

Fig. 16. The Dependence of the Mobility Ratio $\mu_H^{(L)}/\mu_L^*$ and Hall Coefficient Factor $r_H^{(L)}(q_L)$ on Parameter q_L for non-degenerate Semiconductor

η \ β	0	1	2	3	4	5	6	7
—4.0	64.66	56.71	60.19	64.95	69.58	73.99	77.73	81.09
—3.8	52.99	46.49	49.35	53.23	57.03	60.64	63.68	66.41
—3.6	43.45	38.12	40.66	43.63	46.76	49.69	52.18	54.41
—3.4	35.63	31.27	33.18	35.77	38.35	40.73	42.78	44.59
—3.2	29.25	25.67	27.22	29.34	31.46	33.39	35.08	36.55
—3.0	24.01	21.08	22.34	24.09	25.82	27.40	28.78	29.98
—2.8	19.71	17.31	18.35	19.77	21.19	22.49	23.62	24.60
—2.6	16.20	14.24	15.08	16.24	17.41	18.47	19.40	20.20
—2.4	13.32	11.72	12.40	13.35	14.31	15.17	15.94	16.60
—2.2	10.97	9.653	10.21	10.99	11.77	12.48	13.11	13.66
—2.0	9.047	7.964	8.416	9.056	9.692	10.28	10.80	11.25
—1.8	7.461	6.579	6.945	7.467	7.989	8.470	8.899	9.270
—1.6	6.170	5.447	5.740	6.168	6.594	6.990	7.342	7.647
—1.4	5.111	4.517	4.754	5.105	5.453	5.778	6.064	6.318
—1.2	4.241	3.756	3.947	4.234	4.519	4.786	5.020	5.228
—1.0	3.529	3.131	3.285	3.518	3.754	3.971	4.165	4.336
—0.8	2.946	2.619	2.741	2.932	3.126	3.302	3.463	3.604
—0.5	2.260	2.018	2.104	2.243	2.388	2.518	2.640	2.745
0.0	1.483	1.335	1.382	1.465	1.553	1.636	1.711	1.776
+0.5	2.861	0.9122	0.9362	0.9854	1.039	1.091	1.138	1.180
1.0	0.7016	0.6459	0.6563	0.6853	0.7185	0.7510	0.7810	0.8078
2.0	0.3813	0.3605	0.3599	0.3693	0.3821	0.3954	0.4081	0.4198
3.0	0.2349	0.2267	0.2245	0.2271	0.2320	0.2376	0.2434	0.2488
4.0	0.1595	0.1562	0.1544	0.1548	0.1565	0.1589	0.1616	0.1643
5.0	0.1162	0.1149	0.1137	0.1134	0.1140	0.1150	0.1163	0.1176
6.0	0.08917	0.08859	0.08788	0.08754	0.08763	0.08801	0.08861	0.08932
7.0	0.07108	0.07083	0.07042	0.07014	0.07008	0.07019	0.07046	0.07082
8.0	0.05833	0.05821	0.05798	0.05778	0.05768	0.05769	0.05780	0.05797
9.0	0.04896	0.04890	0.04877	0.04863	0.04855	0.04852	0.04855	0.04863
10.0	0.04186	0.04182	0.04174	0.04165	0.04158	0.04155	0.04154	0.04157

Table 7.

8	9	10	12	14	16	18	20	∞
83.96	86.43	88.58	91.98	94.74	96.62	98.05	99.25	105.92
68.82	70.83	72.58	75.35	77.62	79.14	80.33	81.33	86.63
56.41	58.05	59.49	61.74	63.60	64.84	65.82	66.67	71.13
46.24	47.60	48.77	50.61	52.14	53.14	53.96	54.67	58.31
37.92	39.04	39.99	41.78	42.75	43.56	44.24	44.84	47.81
31.12	32.02	32.81	34.04	35.07	35.73	36.29	36.80	39.22
25.53	26.28	26.92	27.94	28.78	29.31	29.78	30.19	32.18
20.95	21.58	22.10	22.94	23.63	24.07	24.46	24.79	26.42
17.22	17.72	18.15	18.85	19.42	19.77	20.10	20.36	21.70
14.15	14.56	14.91	15.51	15.97	16.26	16.53	16.73	17.83
11.64	11.88	12.26	12.77	13.14	13.38	13.61	13.76	14.67
9.583	9.861	10.10	10.52	10.82	11.02	11.21	11.33	12.08
7.902	8.132	8.328	8.671	8.918	9.080	9.242	9.336	9.960
6.528	6.718	6.881	7.159	7.359	7.496	7.630	7.709	8.220
5.403	5.561	5.696	5.919	6.082	6.197	6.308	6.374	6.798
4.484	4.613	4.725	4.904	5.038	5.132	5.223	5.286	5.632
3.725	3.832	3.924	4.072	4.182	4.260	4.334	4.387	4.676
2.837	2.918	2.987	3.097	3.180	3.241	3.294	3.334	3.555
1.834	1.884	1.927	1.998	2.051	2.093	2.123	2.146	2.286
1.217	1.225	1.277	1.323	1.359	1.384	1.405	1.420	1.507
0.8314	0.8521	0.8702	0.8997	0.9223	0.9396	0.9529	0.9639	1.023
0.4303	0.4396	0.4479	0.4614	0.4719	0.4797	0.4864	0.4973	0.5198
0.2539	0.2584	0.2626	0.2694	0.2748	0.2790	0.2824	0.2850	0.3003
0.1669	0.1693	0.1715	0.1752	0.1783	0.1801	0.1826	0.1842	0.1933
0.1190	0.1203	0.1220	0.1238	0.1256	0.1271	0.1283	0.1292	0.1352
0.09007	0.09082	0.09155	0.09288	0.09403	0.09499	0.09577	0.09643	0.1006
0.07123	0.07167	0.07211	0.07294	0.07368	0.07440	0.07486	0.07531	0.07832
0.05819	0.05845	0.05871	0.05925	0.05974	0.06018	0.06055	0.06088	0.06315
0.04875	0.04889	0.04905	0.04939	0.04973	0.05003	0.05030	0.05054	0.05229
0.04164	0.04172	0.04181	0.04203	0.04226	0.04247	0.04267	0.04284	0.04424

$\mu_H^{(L)}/\mu_L^*$ for various q_L values are shown in Table 6. The Tables of integrals elaborated by Dingle, Arndt and Roy [5] were used for the calculations. In the case of degenerate semi-conductors we again calculate the parameters eN_cR

(61)
$$e\,N_cR = \frac{r_H^{(L)}(\beta,\eta)}{F_{1/2}(\eta)}$$

where

(62)
$$r_H^{(L)}(\beta,\eta) = \frac{3}{2}\,F_{1/2}(\eta)\,\frac{\Phi_{9/2}(\beta,\eta)}{\Phi_3^2(\beta,\eta)}$$

and $\mu_H^{(L)}/\mu_L^*$ [21].
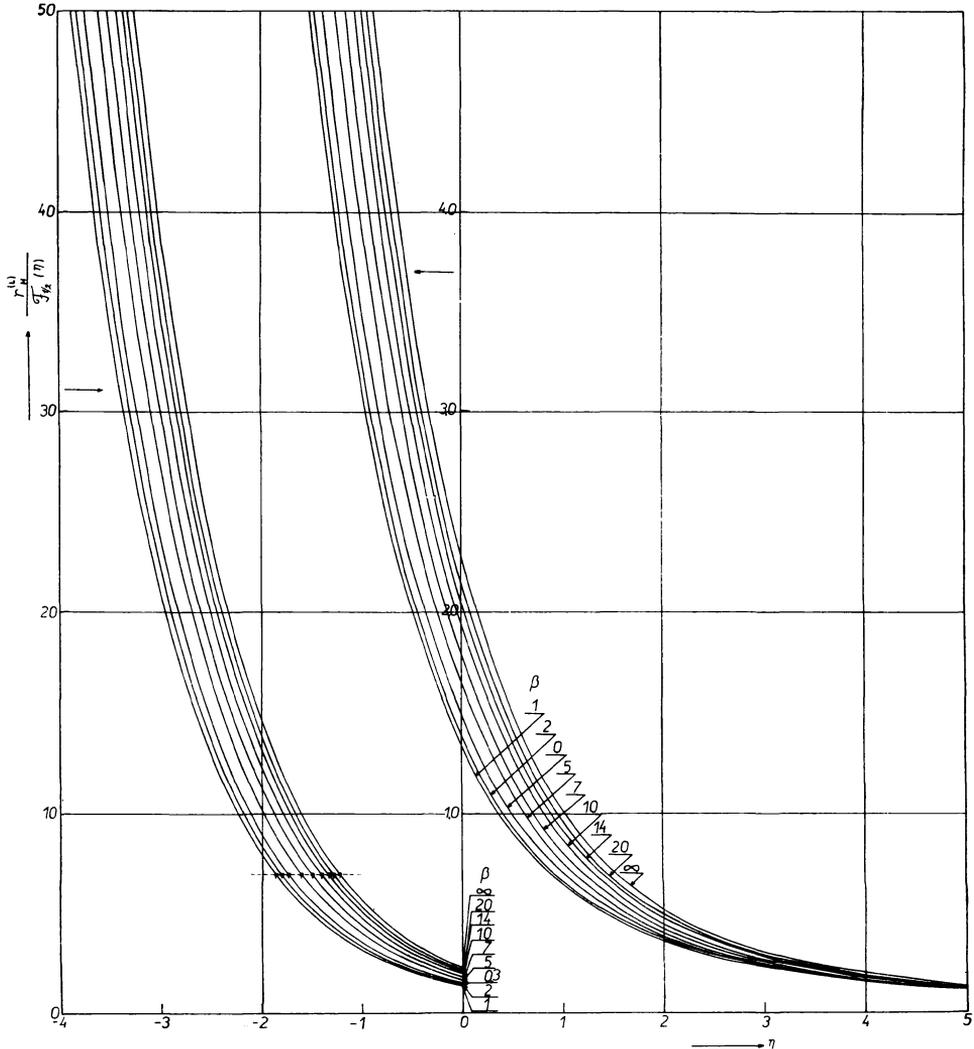
Fig. 17. The Dependence of the Parameter $r_H^{(L)}(\beta,\eta)/F_{1/2}(\eta)$ on the Reduced Fermi Energy for Various Values of β

The values of the right-hand side of eq. (51) for various values of η and β are shown in the Table 7 and Fig. 17.

The values of β and η corresponding to these parameters (e.g. $eN_cR = 7$; $\mu_H^{(L)}/\mu_L^* = 0.2$) are determined from Figs. 17 and 18. From these values we plot the dependence of β on η (Fig. 19) and from the intersection point of the curves determine the values of β and η (—). The curves in the Fig. 19 are plotted for various values of eN_cR and $\mu_H^{(L)}/\mu_L^*$.
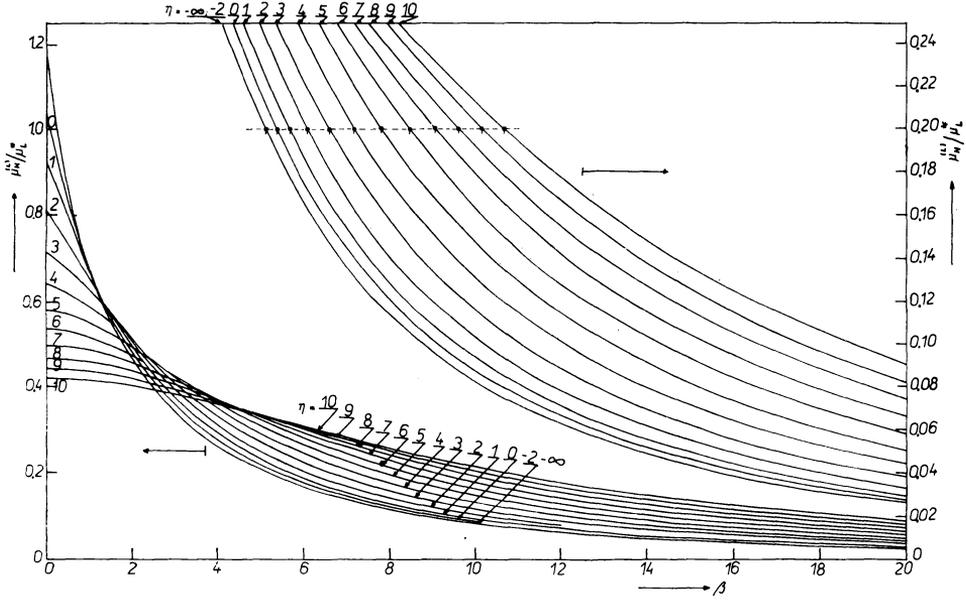
Fig. 18. The Dependence of the Mobility Ratio $\mu_H^{(L)}/\mu_L^*$ on the Parameter β for Various Values of Reduced Fermi Energy η

The calculations of the values of the right-hande sides of equations (58) and (62) were carried out by substituting the same value for β in both the $\Phi_{1/2}(\beta, \eta)$ and $\Phi_3(\beta, \eta)$ functions. We thus, however, introduce a certain error that increases when the value b (29) decreases. According to Mansfield [4, 19] it is necessary to substitute such a value of $\bar{x}$ into the $q(n^*, x, T)$ function for which the integrands of (59) and (60) attain maximum values. Let us denote these values as $\bar{x}_1$ and $\bar{x}_2$. We now substitute β in eq. (59) by β_1 for which the following relationships hold

(63a)
$$\beta_1^2 = \frac{C_L}{\tau_I^0} g(n^*, T, \bar{x}_1)$$

or

(63b)
$$\beta_1^2 = \frac{3F_2(\eta)}{F_0(\eta)} \frac{\mu_L}{\mu_I}$$

Similarly in expression (60) we substitute β by β_2 where

(64)
$$\beta_2^2 = \beta_1^2 \frac{g(n^*, T, \bar{x}_2)}{g(n^*, T, \bar{x}_1)}$$

95
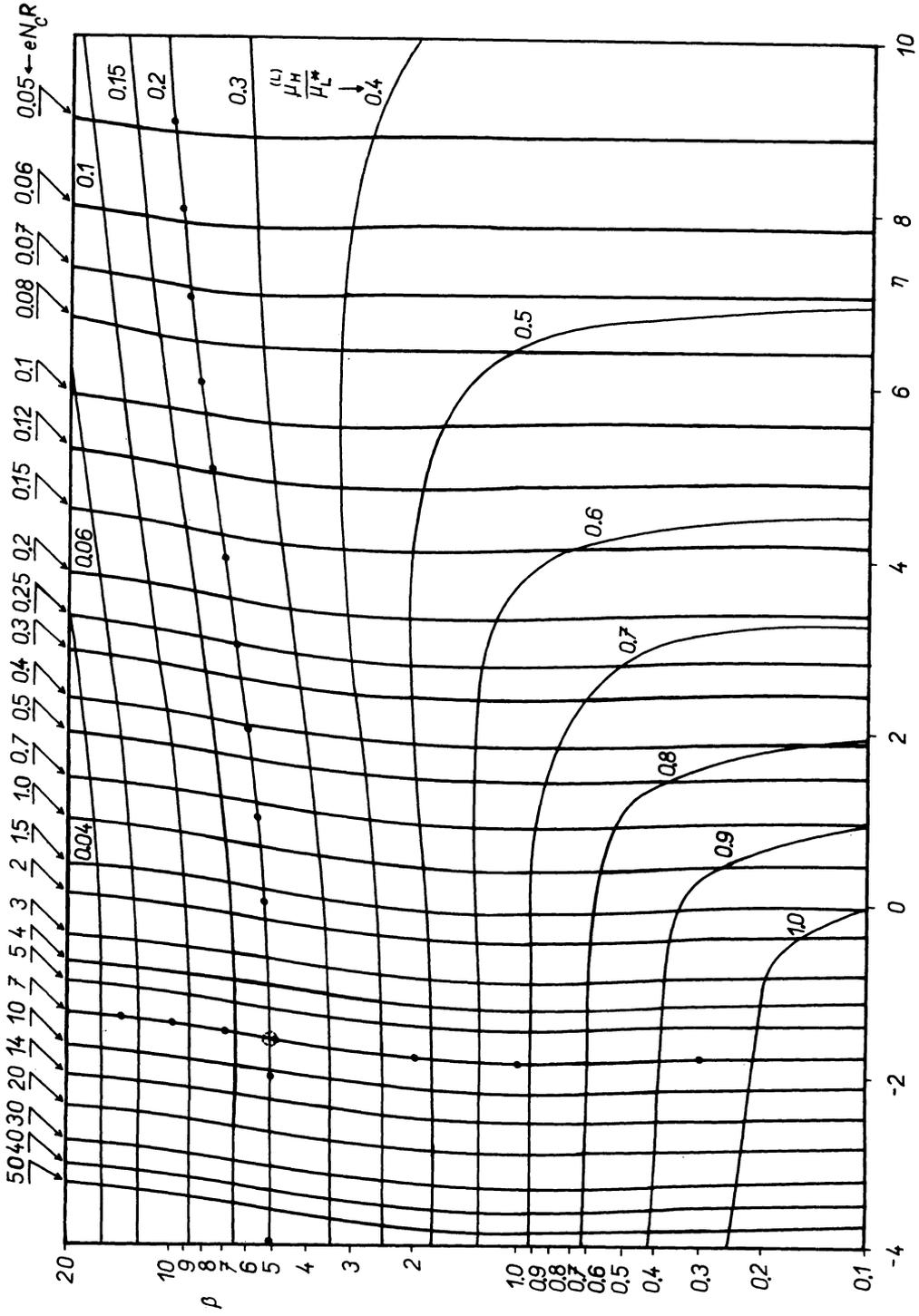
Fig. 19. The Dependence of $\log \beta$ on the Reduced Fermi Energy η. Parameters are eN_cR and $\mu_H^{(L)}/\mu_L^*$.

The mean values x_1 and x_2 depend on the Fermi energy η and on β. These are determined from the following equations:

(65a)

$$\frac{\bar{x}_1^2 - 3\beta^2}{\bar{x}_1^3 + \beta^2 \bar{x}_1} = \tanh \frac{x_1 - \eta}{2}$$

$$\frac{\bar{x}_2^2 - 9\beta^2}{2(\bar{x}_2^3 + \beta^2 \bar{x}_2)} = \tanh \frac{\bar{x}_2 - \eta}{2}$$

or

(65b)

$$\eta = \bar{x}_1 - \ln \frac{\bar{x}_1^3 - \bar{x}_1^2 + \beta^2 \bar{x}_1 + 3\beta^2}{\bar{x}_1^3 + \bar{x}_1^2 + \beta^2 \bar{x}_1 - 3\beta^2}$$

$$\eta = \bar{x}_2 - \ln \frac{2\bar{x}_2^3 + \bar{x}_2^2 + 2\beta^2 \bar{x}_2 - 9\beta^2}{2\bar{x}_2^3 - \bar{x}_2^2 + 2\beta^2 \bar{x}_2 + 9\beta^2}$$

Some of these dependences are mentioned by Mansfield [19].

The evaluation will be rather more complicated when it is desirable to obtain more accurate values of the Fermi-energy and the scattering factor $r_H^{(L)}$; use may, however, be made of the charts and tables mentioned. Let us proceed in the following manner. We first determine the Fermi energy η_1 and the variable $\beta^{(1)}$ by the previously described method and calculate the scattering factor $r_H^{(L)}(\beta^{(1)}, \eta_1)$ and the value of the expression $[\Phi^{\prime}{}_{/_3}(\beta^{(1)}, \eta_1)/\Phi_3(\beta_1^{(1)}\eta_1)]$. We then determine $\bar{x}_1$, $\bar{x}_2$, β_1, β_2 [19] for the found values of η_1 and $\beta^{(1)}$ and once more computate the scattering factors $r_H^{(L)}$ by means of relationship (66):

(66)

$$r_H^{(L)}(\eta_1, \beta_1, \beta_2) = \frac{3}{2} \, F_{1/_2}(\eta_1) \, \frac{\Phi_{\cdot/_2}(\eta_1, \beta_2)}{\Phi_3^2(\eta_1, \beta_1)}$$

ind the value of the fraction

$$\frac{\Phi_{\cdot/_2}(\eta_1, \beta_2)}{\Phi_3(\eta_1, \beta_1)}$$

We substitute the value of the expression $eN_c R$ by the expression

(67)

$$eN_c R \cdot \frac{[r_H^{(L)}(\beta^{(1)}, \eta_1)]}{r_H^{(L)}(\eta_1, \beta_1, \beta_2)}$$

and the ratio $\mu_H^{(L)}/\mu_L^*$ by the expression

(68)

$$\frac{\mu_H^{(L)}}{\mu_L^*} \cdot \frac{\left[\dfrac{\Phi_{\cdot/_2}(\beta^{(1)}, \eta_1)}{\Phi_3(\beta^{(1)}, \eta_1)} \right]}{\dfrac{\Phi_{\cdot/_2}(\beta_2, \eta_1)}{\Phi_3(\beta_1, \eta_1)}}$$

and then by means of the method described determine the Fermi energy η_2 and $\beta^{(2)}$ parameter from Figs. 17 and 18 and compute $r_H^{(L)}(\beta^{(2)}, \eta_2)$ by means of expression (61). If we are not satisfied with the accuracy of these calculations that depend inter alias on the value of b we may proceed further. We determine $\bar{x}_1$, $\bar{x}_2$, and β_1, β_2 and substitute these values and η_2 into the denominators of expressions (67) and (68); we then estimate η_3 and $\beta^{(3)}$ from Figs. 17 and 18. We continue in this manner until the factor $r_H^{(L)}$ is determined with the accuracy demanded. For large values of $b(b > 10)$, the error will be smaller than 5% after the first step.

We proceed in a similar manner when considering non-degenerate semiconductors, the difference being that we need not determine the Fermi energy η. We make use of Fig. 16 but substitute $\mu_H^{(L)}/\mu_L^*$ by expression (68). The values of the $\Phi_{*/_2}(\beta)$ and $\Phi_3(\beta)$ integrals for non-degenerate semiconductors with various β are presented in paper [5].

We may proceed in the same manner when considering combined scattering of charge carriers by optical phonons and ionized impurities. The dependences of the mean values of $\bar{x}_1$ and $\bar{x}_2$ on η and γ have, however, not yet been calculated. They will be dealt with in a separate paper.

8. The Determination of Effective Mass from Measurement of Electrical Conductivity, Hall Coefficient and Thermoelectric Power

In the previous chapter the evaluation of results on the presumption that the effective mass of carriers is known, was described. If the effective mass is unknown, further measurements have to be carried out. The most suitable is the thermoelectric power for which equation (8) is valid.

The effective mass can be calculated for the non-degenerate semiconductors relatively easily. The carrier concentration n are calculated from the Hall coefficient and from the known scattering factor r_H determined by the above method. The value of the scattering factor $B(q_p,\ _L)$ for estimated $q_p,\ _L$ is determined by means of Fig. 20 and the Fermi energy is calculated from the thermoelectric power. (The values of factor $B(q_p,\ _L)$ are mentioned in Tables 8 and 9. Table 8 was taken over from the paper [2] and Table 9 was calculated by means of integrals tabled in the paper [5]). By this means, factors are
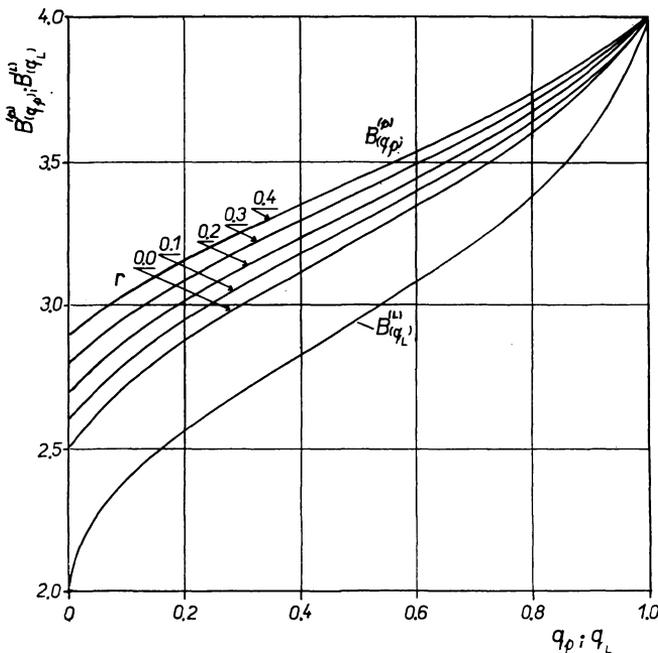

Fig. 20. The Dependence of the Scattering Factors $B^{(p)}(q_p)$ and $B^{(L)}(q_L)$ on the Parameters q_p and q_L, respectively

known which are needed for the computations of the effective mass from equations (2) and (4).

The Fermi energy η and the parameter q_p are determined in the following way for degenerate semiconductors. We calculate the parameters $\mu_H^{(p)}/\mu_p^*$ and $(e/k_0)\alpha$ from the measurements of the electrical conductivity and Hall coefficient and the thermoelectric power. The values q_p and η corresponding to these parameters are estimated from the diagram 14 and 8 when exponent r is known ($r = 0.4$). We plot the dependence of q_p on η (Fig. 21e) and from the intersection point of the curves we estimate the values q_p and η. These curves are plotted for the same parameters in Fig. 21a, b, c, d, e. The scattering factor $r_H^{(p)}(q_p, \eta)$ (50) will be determined from the known q_p and η, as well as the concentration of charge carrier and finally the effective mass m from equation (4). Values of the factor $B(q_L, \eta)$ for the mixed scattering by acoustical lattice vibrations and ionized impurities have to our knowledge not yet been calculated.

Table 8. *The scattering factor $B^{(p)}(q_L)$ as a function of q_p for various values of r* [2]

q_p \ r	0.0	0.1	0.2	0.3	0.4
0	2.50000	2.60000	2.70000	2.80000	2.90000
0.2	2.88163	2.94834	3.01628	3.08539	3.15574
0.4	3.11890	3.17682	3.23469	3.29255	3.35046
0.6	3.34447	3.39445	3.44327	3.49097	3.53757
0.8	3.60405	3.64158	3.67695	3.71026	3.74161
0.9	3.77038	3.79598	3.81932	3.84060	3.86000
1.0	4.00000	4.00000	4.00000	4.00000	4.00000

Table 9. *The scattering factor $B^{(L)}(q_L)$ as a function of q_L or β*

| β	q_L	$B(q_L)$	β	q_L	$B(q_L)$
0.0	0	2.000	3.5	0.6712	3.170
0.2	0.00662	2.077	4.0	0.7273	3.258
0.4	0.02597	2.181	4.5	0.7714	3.330
0.6	0.05660	2.283	5.0	0.8064	3.394
0.8	0.09638	2.380	6	0.8571	3.496
1.0	0.1428	2.470	7	0.8909	3.580
1.2	0.1935	2.552	8	0.9143	3.643
1.4	0.2462	2.628	9	0.9310	3.694
1.6	0.2990	2.700	10	0.9434	3.735
1.8	0.3506	2.766	12	0.9600	3.797
2.0	0.4000	2.826	16	0.9771	3.873
2.5	0.5102	2.961	20	0.9852	3.913
3.0	0.6000	3.077	∞	1.000	4.000
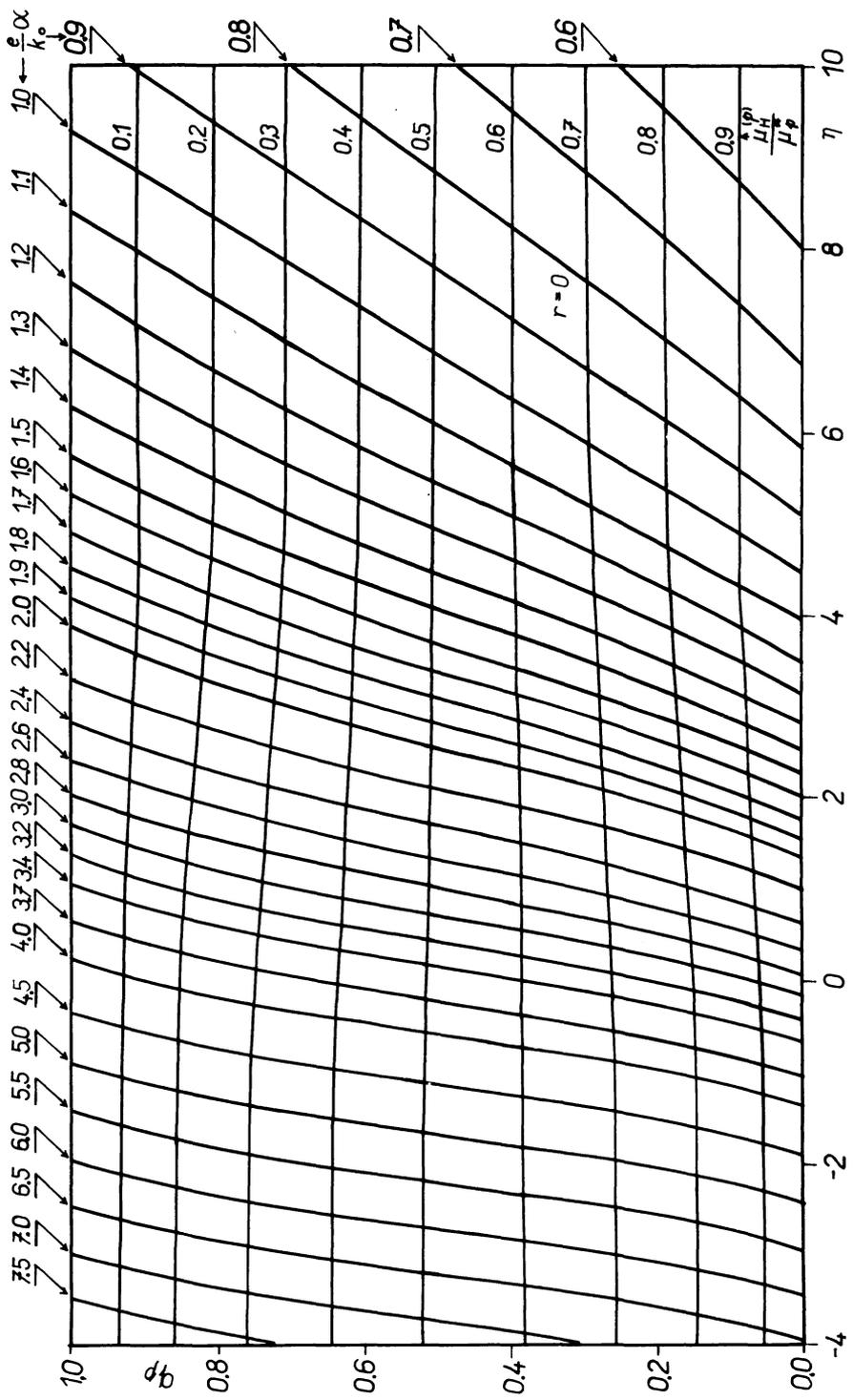
Fig. 21a. The Dependence of q_p on the Reduced Fermi Energy. Parameters are $(e/k_0)\alpha$ and the Mobility Ratio $\mu_H^{(p)}/\mu_p^*$ $(r = 0)$

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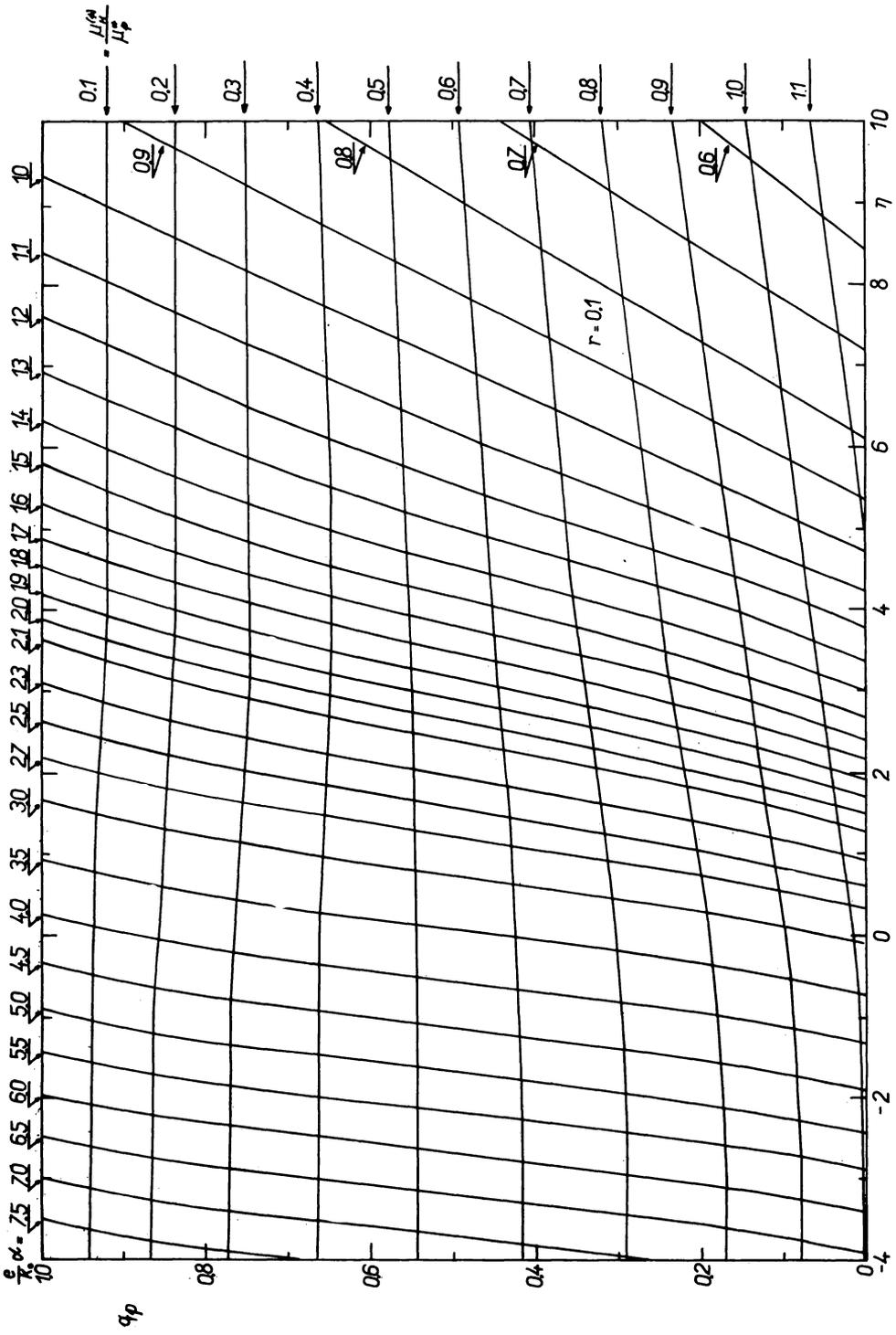
Fig. 21b ditto ($r = 0.1$)
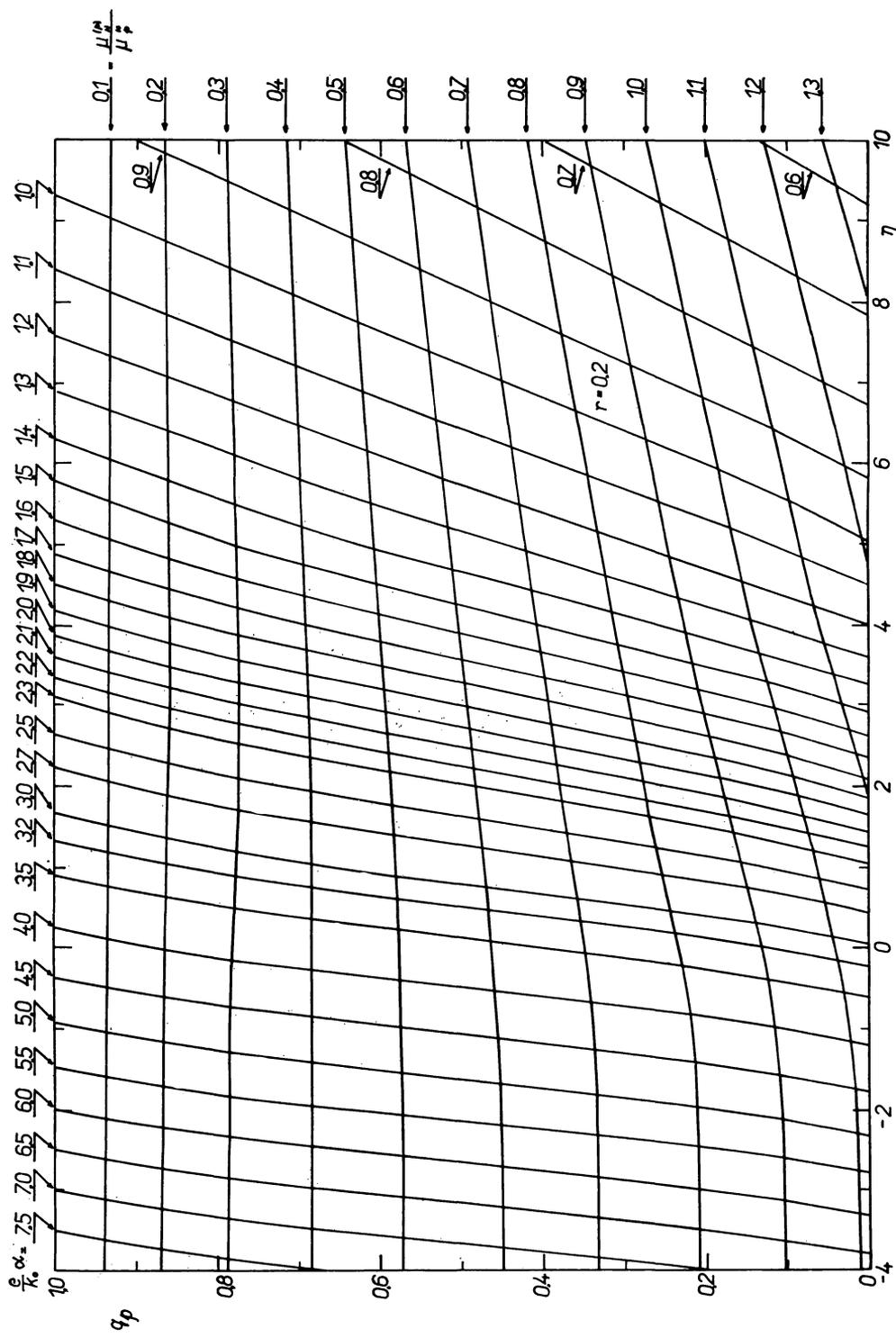
Fig. 21c ditto ($r = 0.2$)
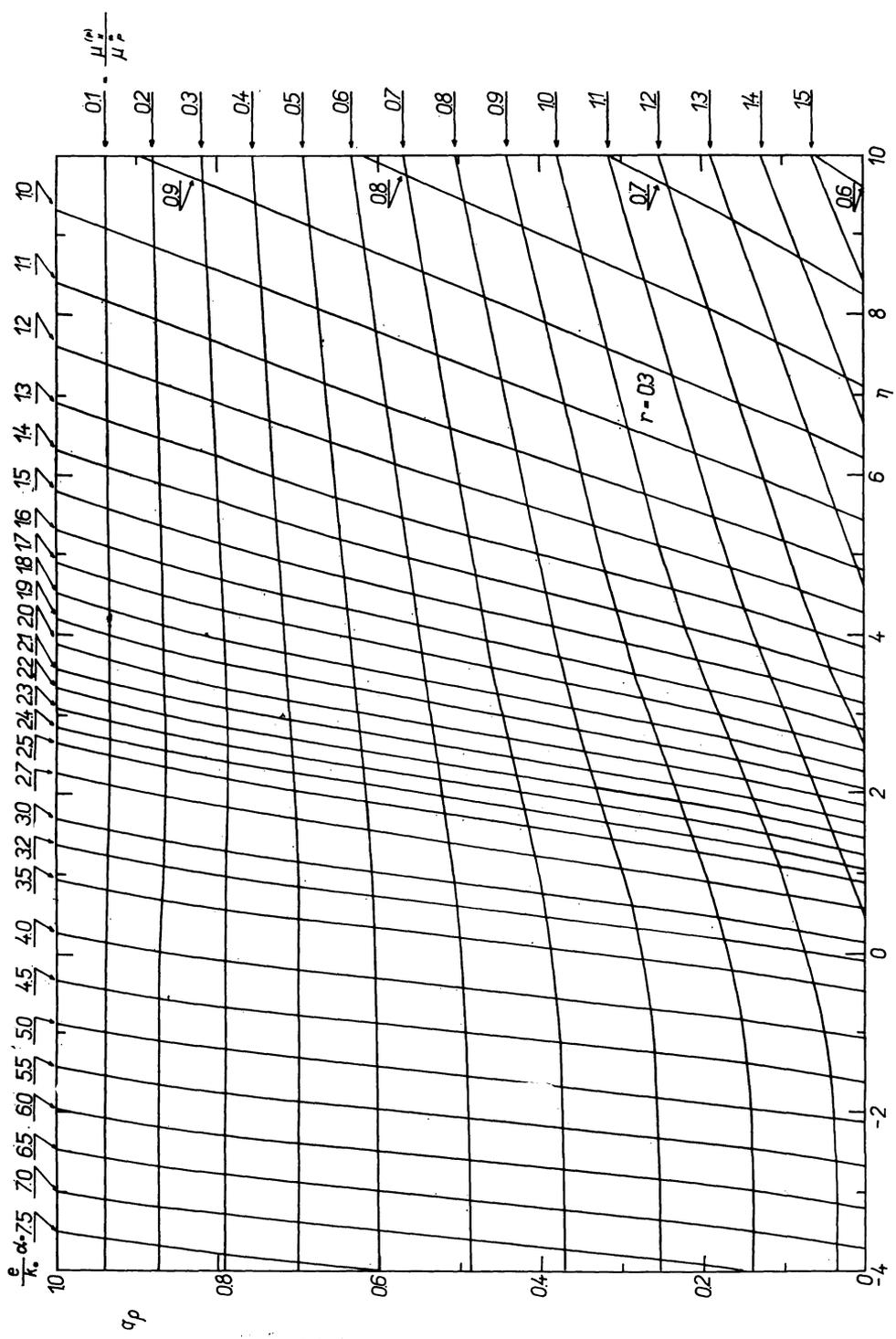
Fig. 21d ditto ($r = 0.3$)

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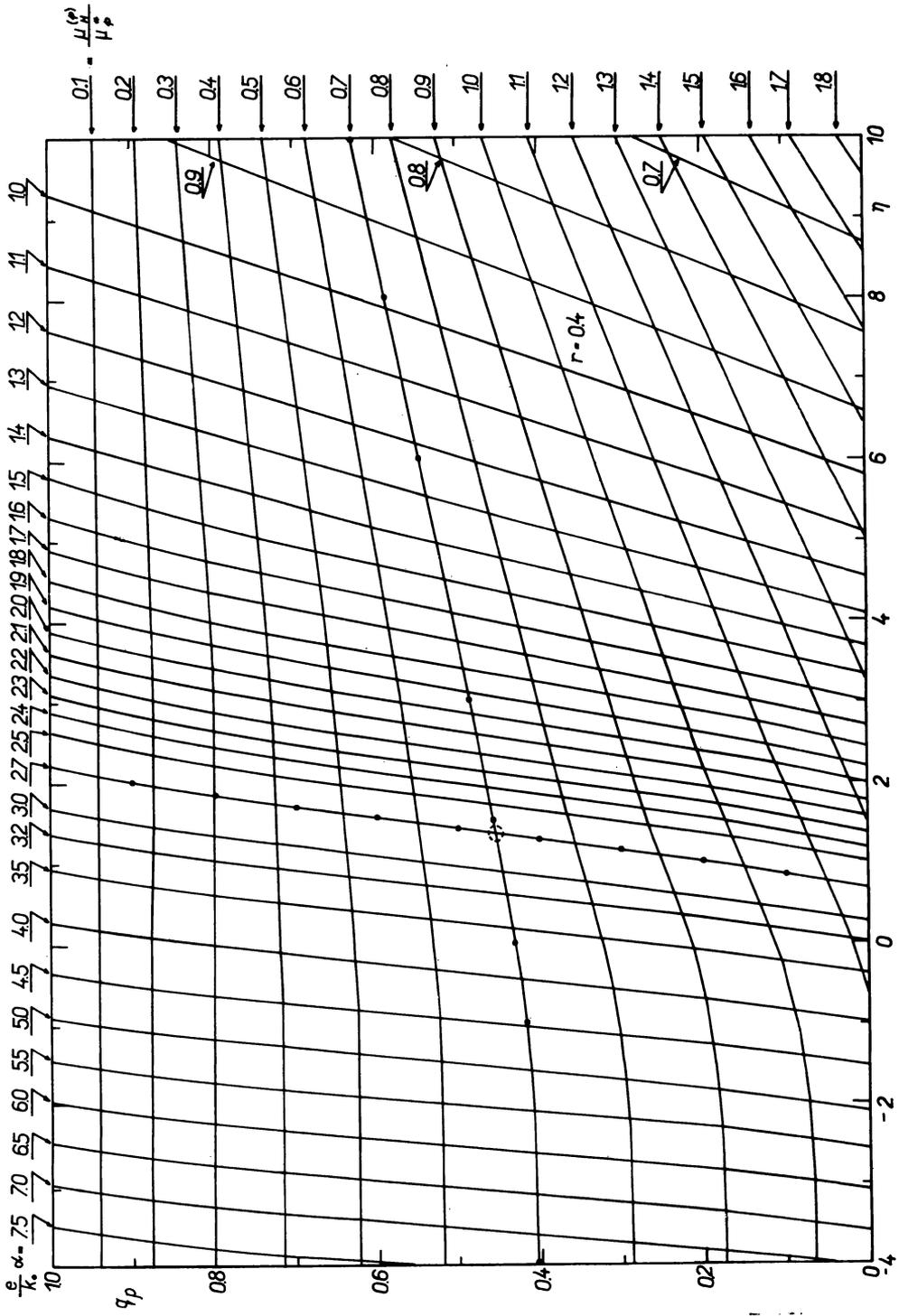
Fig. 21e ditto ($r = 0.4$)

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9. Determination of Transport Coefficients in Semiconductors with a Complex Band Structure

All the relationships mentioned in the preceding chapters hold for an isotropic semiconductor with spherical constant energy surfaces and with a conductivity band minimum or valence band maximum at the centre of the Brillouine zone ($k_e = 0$). The most widely used elements germanium and silicium, however, do not fulfill this assumption. The positions of the energy minima of the conductivity bands are at $k_e \neq 0$ in the directions $\langle 100 \rangle$ for Si and $\langle 111 \rangle$ for Ge. The valence band of both these semiconductors consists of three subbands with energy maxima for $k_e = 0$. For such materials somewhat different relationships hold. We may, however, in some cases use the described method and the charts for evaluation results. The following changes are carried out for n-type Si and Ge. The expression eN_cR is substituted by

$$(69) \qquad eN_cR \, \frac{2(K+1)^2}{3K(K+2)}$$

and

$$\mu_H/\mu^*_{p,L} \quad \text{by}$$

$$(70) \qquad \frac{\mu_H}{\mu^*_{p,L}} \, \frac{2(K+1)^2}{3K(K+2)}$$

where $K = m_L/m_T$, m_L and m_T are the longitudinal transversal effective masses. We further substitute $m_d^{3/2}$ in the relationship for $N_c(4)$ by $v(m_L m_T^2)^{1/2}$ where v signifies the number of equivalent conductivity band minima.

For semiconductors with energy bands consisting of several subbands V_1, $V_2 \ldots$ (e.g. the valence bands of Ge and Si) evaluation is more difficult, as charge carriers with various effective masses m_1, $m_2 \ldots$ and various relaxation times occur in the energy band. We shall assume that these bands have spherical constant energy surfaces and are spin-orbital splitted at $k_e = 0$ by the value of Δ_i (calculated as the distance from the maximum of the highest valence subband). We may also apply this method for the evaluation of non-degenerate semiconductors by substituting in the diagrams the expression eN_cR by

$$(71) \qquad eN_cR \, \frac{[\sum_i \langle \tau_i \rangle \, a_i \, A_i]^2}{[\sum_i \langle \tau_i \rangle^2 \, a_i^{-1} \, A_i] \, [\sum_i a_i^3 \, A_i]}$$

and the expression $(e/k_0)\alpha$ by

$$(72) \qquad \frac{e}{k_0} \, \alpha - \frac{\sum_i \langle \tau_i \rangle \, a_i \, A_i \, \Delta_i/k_0 \, T}{\sum_i \langle \tau_i \rangle \, a_i \, A}$$

where $a_i = (m/m_0)^{1/2}$, $A_i = \exp(-\Delta_i/k_0T)$.

The position of the Fermi level is computed from the maximum of the valence band with the highest energy. The effective mass $m_d^{3/2}$ in the expression for $N_c(4)$ is substituted by $\sum_i m_i^{3/2} A_i$.

The evaluation of the fraction in expression (71) is usually rather difficult as we do not know all the necessary quantities. Further measurements must be carried out in order

to determine them, e.g. the magnetoresistance, cyclotron resonance, respectively Faraday rotation (22).

This method can not be used in general for degenerate semiconductors, except for the case when the spin orbital splitting $\Delta_i = 0$ for all bands V_i or $\Delta_i \gg k_0 T$ holds for some of them. In such cases relationships (71) and (72) are once more valid.

10. Conclusions

In this article the results concerning the scattering of charge carriers by lattice vibrations and ionized impurities are summarized. Methods of determination of some semiconductor parametrs from measurements of electrical conductivity, the Hall coefficient and thermoelectric power are proposed. First of all, the determination of the Fermi level and some of scattering factors (Hall coefficient factor, kinetic term in Seebeck coefficient), eventually effective mass for mixed scattering by lattice vibrations and ionized impurities are discussed here in detail.

If the effective mass and mobility of charge carriers due to scattering by acoustical or optical mode lattice vibrations are known, the measurements of the electrical conductivity and Hall coefficient is sufficient for their calculation. It is necessary to measure the thermoelectric power only if the effective mass or the fundamental mobility due to scattering by lattice vibrations has not been ascertained. The given diagrams enable the necessary parameters to estimate in a relative quick and easy way. Tables are enclosed for construction of the necessary diagrams.

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