Zdeněk Sekanina

Problems of origin and evolution of the Kreutz family of Sun-grazing comets

Acta Universitatis Carolinae. Mathematica et Physica, Vol. 8 (1967), No. 2, 33--84

Persistent URL: `http://dml.cz/dmlcz/142210`

1967 ACTA UNIVERSITATIS CAROLINAE - MATHEMATICA ET PHYSICA NO 2, PAG. 33—84

Problems of Origin and Evolution of the Kreutz Family of Sun-grazing Comets

Z. SEKANINA
Centre for Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, Prague

Received December 15, 1966

In the present paper the distributions of the orbital elements of the Kreutz family comets are studied in detail. Conspicuous empirical relations are found among the respective elements, as well as among their combinations. They can wholly be explained, if the comets are supposed to be evolutionarily interrelated and originated from a single parent body — a protocomet. Methods are proposed, (i) judging of some known processes as possible mechanisms responsible for the protocomet's breakup, and (ii) making it possible to search for new "suspicious" members of this peculiar group among the comets with available orbits. The quantitative solution of the problem of the origin of the Kreutz family is impeded particularly by the sun-grazing character of the orbits and by the related unusual physical behaviour of the comets. A collision of a very massive protocomet with a cosmic projectile of likely much less mass can at present be considered as the most probable process, responsible for the Kreutz family origin. Some non-collisional mechanisms are suspected of taking part in constituting the structure of the family in the course of its evolution.

I. Introduction

The group of sun-grazing comets is one of the greatest peculiarities of the cometary system of our Sun. Most work on the orbits of the comets of this group, observed prior to the beginning of the 20th century was made by KREUTZ (1888, 1891, 1901), and the cometary family is therefore often called the Kreutz group of comets.

There are eight or nine comets of this group known at present: for five comets accurate orbits are available, while only more or less provisional parabolical orbits can have been computed for the remaining, the latter being thus only possible members of the family. In addition, records on a few dozen of ancient comets of perhaps extremely short perihelion distances are available from the period of 371 B.C. to 1702 A.D. and on a few eclipse comets, which are under suspicion of belonging to the Kreutz family. Their definitive incorporation into the group, however, will never be possible because of lack of their orbits.

2. A hypothetical catastrophe

There is no doubt today that more independent comets, obviously evolutionarily interrelated, are moving round the Sun in nearly identical orbits.

Aware of the low interior structure of cometary nuclei, the observed comets can quite naturally be considered decay products of a huge body of similar constitution, or a protocomet, which disintegrated because having met a cosmic catastrophe. Up to now, however, no satisfactory hypothesis has been presented, explaining simultaneously the character of the catastrophe, its time and space positions, the differences in orbital elements of individual "daughter" comets and their connection with the original physical or dynamical conditions.

Neither the writer is able to give any complete picture of the process, originating the present Kreutz group of sun-grazing comets. Thanks to recent apparitions of two other members of this group, however, it is worth to mention and analyze some characteristic features in the distributions of orbital elements, which could be connected with the mechanism of the comets' origin.

The problem is complicated by the comets' extremely short distance of perihelion. Their passages through perihelion are often followed by drastic changes in the interior structure of the comet's nucleus because of extraordinarily high temperatures and tidal forces, which the nucleus is exposed to. A rapid brightness decrease was mentioned for each comet of this group when receding from the Sun, a complete disappearance of the nucleus was reported for one member, splitting of the nucleus into two or more independent fragments for two other members. The nuclear splits are what complicates any analysis of the problem of origin. The total number of the comets increase with time, some of them being, however, products of secondary decay processes, not of the original catastrophe. On the other hand, it is not clear whether these secondary nuclei are able to live as independent comets for a long enough period, say, for one revolution round the Sun at least. The problem of the life-time of sun-grazing comets is generally of high importance. Effects of birth and death are continuously superimposing on one another, but no satisfactory answer can be given what is their interrelation and which is the dominant effect at present.

There are now two classes of possible explanations as for the character of the catastrophe that was met by the protocomet:

1. a breakup at a close vicinity from the Sun — an analogy to the observed splits — produced by solar agents; or,
2. a breakup at larger heliocentric distances the agents having been of non-solar nature, such as a collision with another body, or a disruption due to some interior forces of the comet.

There is a suggestion favouring rather the second possibility; the differences in both angular elements and the perihelion distance among the five indubitable members (Table 1) are too high, requiring splitting velocities of 6.9, 3.5 and 2.3 km/sec for the breakup heliocentric distances 0.1, 0.2 and 0.3 AU, respectively (Section 5). Comet 1965f split into two separate nuclei with the velocity hardly higher than 25 m/sec, hence, more than 100 times lower (Sekanina 1966a). For Comet 1882 II, outstanding in luminosity and probably in dimensions as well, the splitting velocity was likely even less. There is, therefore, no reason for accepting as high values as a few kilometres per second for the splitting velocity of the protocomet, obviously a huge body.

Permitting, on the other hand, a collision of the protocomet with another body of the solar system to produce the breakup process, all possible disruption velocities are acceptable up to a value of a few tens of kilometres per second. Some models of this process will be studied in more detail in the present paper. Several non-collisional mechanisms will be discussed as well. The method of investigation, proposed in a few following sections is independent of the nature of the splitting mechanism.

3. Fundamental equations

Let us assume the protocomet moving round the Sun in a nearly-parabolic orbit. Its orbital elements are T_0, ω_0, Ω_0, i_0, q_0, $p_0 \to 2q_0$, $e_0 \to 1$ and $1/a_0 \to 0$. Given that it meets a cosmic catastrophe at a heliocentric distanc r_0 corresponding with a true anomaly v_0, and breaks up into a number of pieces. A part of its total energy (interior, if an explosion is responsible, or kinetic, if a collision is so) is spent on deformations and breaking up the couplings of the material, on heating the medium and on exerting impulses to its debris. If the motion of the centre of gravity of the disrupted protocomet is changed at the catastrophe, the above elements represent its post-catastrophic orbit.

Each broken block of the protocomet obtains, due to the impulse exerted, a velocity of separation ΔV, the components of which are: $\dot{\xi}$ in the direction of the prolonged radius-vector, $\dot{\eta}$ in the plane of orbit, perpendicular to the former direction and positive with respect to the comet's motion, and $\dot{\zeta}$ perpendicular to the plane of orbit and positive toward the northern pole of the orbit.

Generally, the three components produce certain changes in the orbital elements. If a "daughter" comet has obtained a separation velocity of $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$, the deviations of its from the protocomet's elements will be as follows:

$$(1)\quad \begin{cases} \Delta\omega \equiv \omega - \omega_0 = \dfrac{\sqrt{(p_0)}}{\varkappa e_0}\left[-\dot{\xi}\cos v_0 + \dot{\eta}\left(1 + \dfrac{r_0}{p_0}\right)\sin v_0\right] - \Delta\Omega\cos i_0\,, \\ \Delta\Omega \equiv \Omega - \Omega_0 = \dot{\zeta}\,\dfrac{r_0\sin(\omega_0 + v_0)}{\varkappa\sqrt{(p_0)}\sin i_0}\,, \\ \Delta i \equiv i - i_0 = \dot{\zeta}\,\dfrac{r_0\cos(\omega_0 + v_0)}{\varkappa\sqrt{(p_0)}}\,, \\ \Delta\left(\dfrac{1}{a}\right) \equiv \dfrac{1}{a} - \dfrac{1}{a_0} = -\dfrac{2}{\varkappa\sqrt{(p_0)}}\left[\dot{\xi}\,e_0\sin v_0 + \dot{\eta}\,\dfrac{p_0}{r_0}\right], \\ \Delta e \equiv e - e_0 = \dfrac{\sqrt{(p_0)}}{\varkappa}\left[\dot{\xi}\sin v_0 + \dfrac{\dot{\eta}}{e_0}\left(\dfrac{p_0}{r_0} - \dfrac{r_0}{a_0}\right)\right], \end{cases}$$

where $\varkappa$ is the Gaussian constant of gravitation and the elements without index are those referred to the "daughter" comet. The second and third equations of the set give immediately

$$(2)\quad \Delta i = \sin i_0 \operatorname{ctg}(\omega_0 + v_0)\,\Delta\Omega,$$

or

$$(3)\quad i - \sin i_0 \operatorname{ctg}(\omega_0 + v_0)\,\Omega = \text{const}\,,$$

an equation, called here the first decay condition. The first, fourth and fifth equations of (1) generally give

$$(4)\quad \begin{cases} \Delta\omega = \dfrac{p_0}{2e_0^2}\left[\left(1 - \dfrac{a_0 p_0}{r_0^2}\right)\operatorname{ctg} v_0 - \dfrac{e_0}{1 - e_0^2}\left(1 + \dfrac{p_0}{r_0}\right)\sin v_0\right]\Delta\left(\dfrac{1}{a}\right) - \\ \quad - \dfrac{1}{1 - e_0^2}\left[\dfrac{p_0^2 \operatorname{ctg} v_0}{e_0 r_0^2} + \left(1 + \dfrac{p_0}{r_0}\right)\sin v_0\right]\Delta e - \cos i_0 \,.\, \Delta\Omega \,. \end{cases}$$

For a nearly-parabolic orbit, postulated for the protocomet at the beginning of this section, relation (4) becomes simpler. Since

$$(5)\qquad \frac{\Delta a}{q_0} = \frac{\Delta a}{a_0} - \frac{\Delta e}{1 - e_0},$$

we can write

$$(6)\qquad \Delta\omega = -\cos i_0\, \Delta\Omega + \frac{1}{q_0}\operatorname{ctg}\frac{v_0}{2}\,\Delta q\,,$$

or

$$(7)\qquad \omega + \Omega\cos i_0 - \frac{q}{q_0}\operatorname{ctg}\frac{v_0}{2} = \mathrm{const}\,,$$

which is the second decay condition. Replacing the true anomaly by the corresponding heliocentric distance, it can be written in still another way:

$$(8)\qquad \omega + \Omega\cos i_0 \pm q\,[q_0(r_0 - q_0)]^{-\frac{1}{2}} = \mathrm{const}\,.$$

In this and the following relations the upper sign is always valid for a pre-perihelion period, the below sign for a post-perihelion period.

Later in this paper, the validity of the two decay conditions for the Kreutz family of comets will be investigated in detail.

4. Orbit of the protocomet. Location of the catastrophe in space

Each comet of the family, being a product of the protocomet, should satisfy the two decay conditions. On the other hand, the orbital elements of the protocomet can be derived together with the location of the catastrophe in space. Writing the two relations in the form

$$(9)\quad \begin{cases} i = A + B\Omega\,, \\ \omega = E + F\Omega + Gq\,, \end{cases}$$

we can find the true anomaly from a transcendental equation as follows:

$$(10)\quad v_0 + \operatorname{ctg}\frac{v_0}{2} = \arctan\frac{\sqrt{(1 - F^2)}}{B} \mp \pi\delta + \frac{F}{B}\,[A - \arccos(-F)] - E\,.$$

Function $f(v_0) \equiv v_0 + \operatorname{ctg}\frac{v_0}{2}$ is an odd function, in its absolute value never less than $1 + \frac{\pi}{2}$ Hence, if the right-hand side of equation (10) is greater than $-\left(1 + \frac{\pi}{2}\right)$ and less than $1 + \frac{\pi}{2}$, the equation gives us no real solution for v_0. If the right-hand

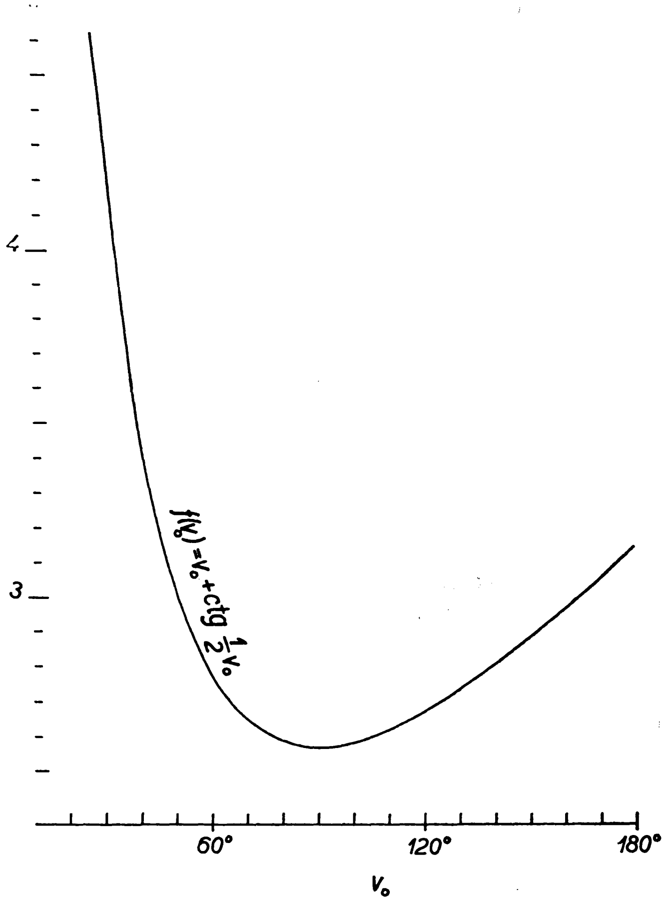

Fig. 1. *Function* f(v_0) = = v_0 + *ctg* $\frac{v_0}{2}$.

side has its absolute value between $1 + \frac{\pi}{2}$ and π, there are two independent solutions, if it is greater than π, there is only one solution. The sign of G determines, whether v_0 is positive or negative:

$$\operatorname{sign} G = \operatorname{sign} v_0\,,$$

while the term of $\pi\delta$ is added (δ equals to zero or to a natural number), because expression $\omega_0 + v_0$ is not determined uniquely from (3). Function $f(v_0)$ is plotted in Fig. 1.

The elements of the protocomet's orbit are then given by the following relations:

$$(11)\quad \left\{\begin{aligned} T_0 &= t_0 - \frac{\sqrt{(2)}}{3\varkappa}\, q^{\frac{3}{2}} \tan\frac{v_0}{2}\left(2 + \sec^2\frac{v_0}{2}\right),\\ \omega_0 &= E + \frac{F}{B}\,[\arccos(-F) - A] + \operatorname{ctg}\frac{v_0}{2},\\ \Omega_0 &= \frac{1}{B}\,[\arccos(-F) - A]\,,\\ i_0 &= \arccos(-F)\,,\\ q_0 &= \frac{1}{G}\operatorname{ctg}\frac{v_0}{2}, \end{aligned}\right.$$

t_0 is the time of catastrophe. The elements should satisfy a condition:

$$(12)\qquad \tan(\omega_0 + v_0) = \frac{1}{B}\sqrt{(1 - F^2)}\,.$$

In addition, the heliocentric distance of the catastrophic event is:

$$r_0 = q_0 \cos^{-2} \frac{v_0}{2}. \tag{13}$$

5. Velocities of separation

The components of the separation velocity, impressed to each comet during the catastrophic process, can be expressed through its elements i and q, the protocomet's elements ω_0, i_0, q_0 and the heliocentric distance of the spot of the catastrophe:

$$(14)\left\{\begin{aligned} \dot{\xi} &= \pm \frac{\varkappa \Delta q}{\sqrt{2}} \cdot \frac{1}{r_0 \sqrt{(r_0 - q_0)}}, \\ \dot{\eta} &= \frac{\varkappa \Delta q}{\sqrt{2}} \cdot \frac{1}{r_0 \sqrt{q_0}}, \\ \dot{\zeta} &= - \frac{\varkappa \Delta i \sqrt{2}}{q_0^{-1/2} (r_0 - 2q_0) \cos \omega_0 \pm 2 \sqrt{(r_0 - q_0)} \sin \omega_0}. \end{aligned}\right.$$

Expressing distances in AU, Δi in radians and asking $\dot{\xi}$, $\dot{\eta}$, and $\dot{\zeta}$ in km/sec, then $\varkappa = 29.77$

Relations (14) can simply be converted to find the respective velocity components nto the three axes of the ecliptical system of co-ordinates:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} P_x & Q_x & R_x \\ P_y & Q_y & R_y \\ P_z & Q_z & R_z \end{pmatrix} \begin{pmatrix} \cos v_0 & -\sin v_0 & 0 \\ \sin v_0 & \cos v_0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

where $P_x, \ldots, R_z$ are the ecliptical vector elements.

6. Indubitable members of the Kreutz family of comets

Up to now, five comets as follows may be considered the indubitable members of this group: 1843 I, 1880 I, 1882 II, 1963 V and 1965f. Their orbits are listed in Table 1; the respective columns indicate: the comet's designation, time of passage through perihelion (E is added, if given in the ephemeris time instead of the Universal Time), angular elements, perihelion distance, eccentricity with the root-mean-square error, orbital period, interval of observation, number of observations, perturbations included, mean positional residual, or mean error of the normal place of unit weight, if an asterisk is added.

Theoretically, the two decay conditions should perfectly be fulfilled (see Section 8), assuming the orbits are:

1. reduced to a uniform equinox;
2. reduced to a common epoch of osculation, identical with the time of the catastrophe;
3. free of non-gravitational effects, which deform the orbits particularly in the neighbourhood of the perihelion passage.

The orbits listed in Table 1, however, fulfil only the first of the three conditions. For fulfilling the two other, no necessary data are available. The accuracy with which the actual comet orbits can satisfy the two decay conditions is, hence, accordingly reduced.

Table 1. Indubitable members of the Kreutz family of comets

Comet	T	Equinox 1950.0			q	e
		ω	Ω	i		
		°	°	°		
1843 I	1843 Feb. 27.91099	82.6374	2.8274	144.3484	0.0055273	0.9999137 ± 0.0000113
1880 I (A)	1880 Jan. 28.09891	85.2350	5.8922	144.5369	0.0055383	0.9999433 ± 0.0000324
1880 I (B)	1880 Jan. 28.11820	86.2462	7.0769	144.6603	0.0054944	1.
1882 II (A)	1882 Sep. 17.72409	69.5865	346.9588	142.0045	0.0077507	0.9999068 ± 0.0000003
1882 II (B)	1882 Sep. 17.72397	69.5721	346.9625	142.0038	0.0077428	0.9999407 ± 0.0000071
1963 V	1963 Aug. 23.93750 *E*	85.8673	6.8772	144.5334	0.0050996	0.9999566 ± (0.00002)
1965f	1965 Oct. 21.18312 *E*	69.0292	346.2520	141.8522	0.0077606	0.9999185 ± 0.0000216

Comet	P	interval of observation		N	Pert.	ε	Note
		range of dates	days				
1843 I	512 ± 105	1843 Mar. 5 – 1843 Apr. 19	45	157	no	± 6.3″*	
1880 I (A)	965 ± 827	1880 Feb. 5 – 1880 Feb. 19	14	29	no	± 1.4″*	most probable ellipse
1880 I (B)	–	1880 Feb. 5 – 1880 Feb. 19	14	29	no	± 1.4″*	most probable parabola
1882 II (A)	758 ± 4	1882 Sep. 7 – 1883 May 26	261	1252	J S[1]	± 1.″5*	orbit fitting all observations
1882 II (B)	1,492 ± 268	1882 Sep. 7 – 1882 Sep. 24	17	30	J S[1]	± 0.″9*	pre-breakup orbit
1963 V	1,274	1963 Sep. 15 – 1963 Oct. 21	36	19	no	± 2.″7	
1965f	929 ± 369	1965 Sep. 21 – 1965 Oct. 16	25	64	no	± 3.″7	pre-perihelion orbit

[1] Date of osculation: 1882 Sep. 21

Table 2. Other sun-grazing comets

Comet	T	Equinox 1950.0			q	e	int. obs.	N	ε	Note
		ω	Ω	i						
		°	°	°						
1872	1872 Dec. 16.616	63.417	48.056	148.441	0.063659	1.	1.0	3	10″**	
1887 I (A)	1887 Jan. 11.754	65.200	340.287	137.961	0.005820	1.	9	22	± 261″*	
1887 I (B)	1887 Jan. 11.631	58.347	325.505	128.472	0.009665	1.	9	22	± 131″*	mean residual:
1893 (ecl.)	1893 Apr. 16.992	271.359	203.917	117.495	0.030637	1.	0.1	3	57″**	± 2880″
1945 VII	1945 Dec. 28.012	50.93	321.69	137.02	0.006305	1.	4	5	?	

For each of the five comets of Table 1 more orbits have been computed. Those given are the best orbits. Some remarks should be added.

The orbit for Comet 1843 I is the most probable orbit as given by KREUTZ (1901).

For Comet 1880 I two orbits are presented. The elliptical orbit was computed by the writer from the normal equations published by KREUTZ (ibid.), while the other orbit is the most probable parabola derived by KREUTZ.

Two sets of elements are given for Comet 1882 II as well. The first listed is the orbit best fitting all the observations available, both prior to and after the perihelion passage, the second is the most probable pre-breakup orbit.

For 1963 V Table 1 involves an orbit, which fits very well 19 observations. This orbit, computed by the writer, has so far been unpublished. It is certainly better than that published earlier and based on 5 positions only (SEKANINA 1964). No mean error was deduced for the orbital elements presented, but that of eccentricity may be estimated at not more than some $\pm$ 0.00002.

For Comet 1965f there are more orbits available, the presented being the most probable one, fitting pre-perihelion observations (SEKANINA 1965).

As seen from Table 1, 1882 II is the only comet, for which planetary perturbations were taken into account. Fortunately, owing to the character of the orbits, the planets can have no essential effect on their forms.

Because of a large dispersion in the orbital periods, their low accuracy and probably non-gravitational effects involved, no discussion is possible of a theoretically required commensurability among the numbers of revolutions.

7. Possible sun-grazing members of the Kreutz family of comets

There exist further retrogradely orbiting comets with extremely small perihelion distances and inclinations about 130° to 140°. For these comets, however, only rough orbits are known derived from extremely short arcs under observation. The comets are as follows: 1872, 1887 I, 1893 eclipse, and 1945 VII.

Comet 1872 was under observation not longer than 23.9 hours. The orbit is, hence, very uncertain. In spite of it, however, KREUTZ (1901) inclined to BRUHNS' (1875) earlier conclusion that a sun-grazing character of the orbit and a retrograde motion are more probable than a short-period character of orbit resembling the path of Comet Biela, as assumed originally.

An exceptional behaviour of 1887 I was the reason for a bad determinacy of its orbit. Observers reported no nucleus or central condensation in the head of the comet, and some of them failed to find its head at all. KREUTZ (ibid.) derived three various sets of elements. Table 2 gives two of them: the first orbit is close to the OPPENHEIM (1889) earlier orbit, the other is an improved path, considered by KREUTZ as the definitive orbit. We will see that some properties of the former orbit prefer it to the latter.

An eclipse comet, discovered on April 16, 1893, was under observation only 2.6 hours. KREUTZ stressed that a plenty of orbits can have been fitted through the minute arc, two sets of elements having been published by himself. One orbit indicated a retrograde motion of the comet, but the two other angular elements differed strongly from those of the indubitable members. This fact prevented KREUTZ from inclining to consider the comet a member of the group. At least, its membership is strongly problematic.

For another eclipse comet, observed on May 16, 1882, only one position is available.

The orbit for 1945 VII is based on a 4 days arc only. Though uncertain enough, the orbits of comets 1887 I and 1945 VII are undoubtedly much better establisched than that of Comet 1872. Just the two former comets can be considered the possible members of the Kreutz family.

A list of the afore orbits is included in Table 2, arranged in a similar way to Table1. Two asterisks design the residual of the middle place, if three positions were only applied.

8. Applicability of the proposed method. Discussion

Before applying the formulae of Sections 3, 4 and 5 to the comets of interest, it should be emphasized that relations (1) are applicable only if

1. the process, producing the changes in orbital elements has a sudden character, and
2. the changes themselves are small enough.

While the former of the two conditions may be accepted with a high probability, the latter is—as seen from Tables 1 and 2—far from to be fulfilled perfectly. Therefore only approximate results may be expected when applying the proposed method. After all, this is not the only obstacle preventing me from obtaining more detailed results (see Sections 6 and 7).

To show why—in spite of their approximate character—the forms of the two decay conditions given by (3) and (7) respectively are advantageous for applying to the Kreutz family of comets, let us make use of the well-known fact that the orbit of each "daugther" body must intersect the orbit of the "parent" body at the point of the catastrophe, x_0, y_0, z_0, and each of the respective sets of orbital elements must thus fulfil the following criteria:

$$(16) \quad \begin{cases} x_0 \sin \Omega - y_0 \cos \Omega + z_0 \operatorname{ctg} i = 0 \,, \\ x_0 \cos \Omega + y_0 \sin \Omega - r_0 \cos (\omega + v) = 0 \,, \\ z_0 \operatorname{cosec} i - r_0 \sin (\omega + v) = 0 \,. \end{cases}$$

Writing the first decay condition in its differential form

$$\mathrm{d}i = \sin i \operatorname{ctg} (\omega + v) \, \mathrm{d}\Omega$$

and inserting from the two last equations of (16) for $\operatorname{ctg} (\omega + v)$, the integration gives the first relation of (16), representing the equation of the plane of orbit.

Writing analogously the second decay condition in the form

$$\mathrm{d}\omega = - \cos i \, \mathrm{d}\Omega + \operatorname{ctg} \frac{v}{2} \, q^{-1} \, \mathrm{d}q \,,$$

it is possible—after a little more complicated integration—to come to a result, which may be transcribed as follows:

$$(17) \qquad \omega - \arccos^{\star} \left(\frac{x_0}{r_0} \cos \Omega + \frac{y_0}{r_0} \sin \Omega \right) \mp 2 \arccos \left(\frac{q}{r_0} \right)^{\frac{1}{2}} = 0 \,,$$

where the asterisk means that the angle is to be taken in order that ω should fall within 0° and 360°. Relation (17) is a new form of the second decay condition, giving the dependence $\omega = \omega (\Omega, q)$. In fact it is nothing but the second equation of (16).

While the first decay condition,

$$\operatorname{ctg} i = M \sin \Omega + N \cos \Omega ,$$

is plausible for the method of least squares, the other two equations of (16) are not pertinent to the method. Hence, the only which can easily be derived are two ratios of the co-ordinates of the spot of the catastrophe, x_0/z_0 and y_0/z_0.

The last two equations of (16) can, however, be made use of as two testing criteria, requiring:

$$(18)\quad \begin{cases} C_1(\omega, \Omega, q) \equiv x_0 \cos \Omega + y_0 \sin \Omega + \\ + (r_0 - 2q) \cos \omega \mp 2 \sqrt{q} \sqrt{(r_0 - q)} \sin \omega = 0 , \\ C_2(\omega, i, q) \equiv z_0 \operatorname{cosec} i + \\ + (r_0 - 2q) \sin \omega \pm 2 \sqrt{q} \sqrt{(r_0 - q)} \cos \omega = 0. \end{cases}$$

Expressions (14) of Section 5 for the components of the separation velocities will accordingly be changed. The first two components, $\dot{\xi}$ and $\dot{\eta}$, can easily be integrated to give

$$(19)\quad \begin{cases} \dot{\xi} = \mp \dfrac{\varkappa \sqrt{2}}{r_0} \left[(r_0 - q)^{\frac{1}{2}} - (r_0 - q_0)^{\frac{1}{2}}\right] , \\ \dot{\eta} = \dfrac{\varkappa \sqrt{2}}{r_0} \left[\sqrt{q} - \sqrt{q_0}\right] , \end{cases}$$

while for the ζ component only limits can be established. The reason is that any infinitesimal change in both Ω and i depends on a product of

$$\frac{1}{\sqrt{q}} \cdot \frac{d}{dt} (\dot{\zeta}) ,$$

where $q(t)$ and $\dot{\zeta}(t)$ change continuously during the cosmic catastrophe, $q(t)$ from the original q_0 to the observed q, while $\dot{\zeta}(t)$ from zero to the resulting $\dot{\zeta}$. Since the two quantities cannot be separated from one another, the expression for the third velocity component should be intergrated as follows:

$$\frac{\varkappa \sqrt{2}}{z_0} \int_{\Omega_0}^{\Omega} \sin^2 i \, d\Omega = F(q, \dot{\zeta}) - F(q_0, 0)$$

where

$$\frac{dF}{dt} = \frac{1}{\sqrt{q}} \frac{d\dot{\zeta}}{dt} .$$

Inserting from the first decay condition we find

$$I(\Omega) = \frac{r_0}{z_0} \int_{\Omega_0}^{\Omega} \sin^2 i \, d\Omega =$$

$$= \arctan \frac{(x_0^2 + z_0^2) \tan \Omega - x_0 y_0}{r_0 z_0} - \arctan \frac{(x_0^2 + z_0^2) \tan \Omega_0 - x_0 y_0}{r_0 z_0} ,$$

and the $\dot{\zeta}$ component is ranged between two limits:

$$(20) \quad \begin{cases} \dot{\zeta}_1 = \dfrac{\varkappa \sqrt{2}}{r_0} I(\Omega) \,.\, \min\left(\sqrt{q}, \sqrt{q_0}\right), \\ \dot{\zeta}_2 = \dfrac{\varkappa \sqrt{2}}{r_0} I(\Omega) \,.\, \max\left(\sqrt{q}, \sqrt{q_0}\right). \end{cases}$$

The arithmetic mean of (20) can be considered the most probable value.

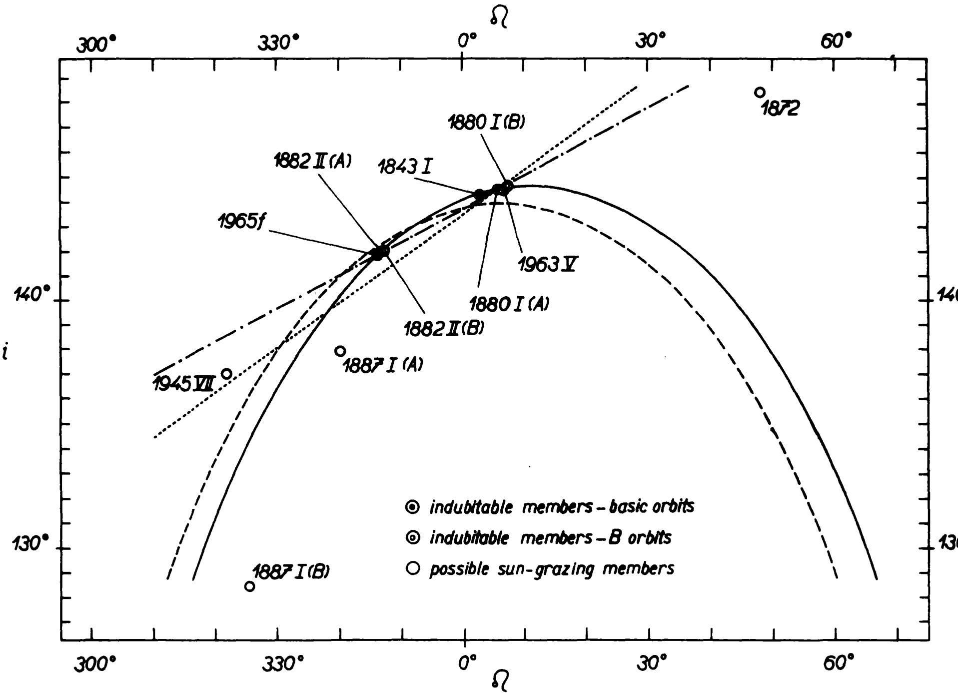

Fig. 2. *The first decay condition for indubitable and possible members of the Kreutz family.*

Only if the differences $\Delta q = q - q_0$ and $\Delta i = i - i_0$ are small enough, expressions (19) and (20) come with a sufficient accuracy to their approximate forms (14).

The results of this section will hereinafter be applied both to testing the membership of comets suspected of belonging to the Kreutz family, and to searching for so far unknown members of this peculiar group.

9. Sun-grazing comets and the decay conditions

The first decay condition is graphically represented in Fig. 2. Based on five couples of Ω and i of the orbits of the indubitable members, making use of the A-orbits only for 1880 I and 1882 II (see Table 1), the exact form of the first decay condition can be written as follows:

$$\text{(21)} \qquad \operatorname{ctg} i = -\,(0.28686 \sin \Omega + 1.38080 \cos \Omega)\,, \\ \pm\,0.00348 \qquad \pm\,0.00058 \text{ m.e.}$$

the mean residual of i being only $\pm$ 1'.0. Approximating (21) by the first equation of (9), I find

$$\text{(22)} \qquad i = 143^\circ.757 + 0.13506\,\Omega\,, \\ \pm\,0^\circ.070 \pm 0.00741 \text{ m.e.}$$

Ω and i are expressed in degrees. The mean residual is now $\pm$ 7'.1, more than seventimes higher. If real, this fact would speak in favour of a small dispersion of the nodes and inclinations that has occurred since the time of the catastrophe, and accordingly, in favour of a low age of the Kreutz family. The individual residuals are listed in Table 3.

As for the possible members, Fig. 2 shows that the residuals are much higher, slightly more than 2° for Orbit A of 1887 I, about 6° for its B-orbit, nearly 5° for 1945 VII, and even 10° for 1872. The Eclipse comet of 1893 is obviously not a member of the family.

The method of least squares gives the following numerical form of the second decay condition ($\alpha = 57^\circ.29578$):

$$\text{(23)} \qquad \omega = 78^\circ.621 + 0.85629\,\Omega + 4.87\,\alpha q\,, \\ \pm\,1^\circ.323 \pm 0.02778 \quad \pm\,4.81 \text{ m.e.}$$

indicating a nearly 80 per cent error of the G coefficient, though for no indubitable member the residual is in excess of 4' (Table 3).

Table 3. Residuals from the decay conditions for the indubitable members

Comet	$O - C$			
	in inclination		in perihelion	
	from (21)	from (22)	from (23)	from (25)
	'	'	'	'
1843 I	+ 1.0	+ 12.6	+ 3.2	+ 0.4
1880 I (A)	+ 1.1	− 1.0	+ 1.4	+ 5.3
1882 II (A)	− 0.4	+ 0.5	− 1.7	− 0.9
1963 V	− 1.8	− 9.2	− 3.9	− 5.3
1965f	+ 0.1	− 2.9	+ 1.0	+ 0.5

Applying the formulae of Section 4 I find two angular elements of the protocomet's orbit, namely

$$i_0 = 148^\circ.9 \pm 3^\circ.1\,, \\ \Omega_0 = 38^\circ.1 \pm 22^\circ.9\,,$$

but I am able to find no other element because

$$f(v_0) = \left\{ \begin{array}{c} -\,0.6266 \\ \text{or} \\ +\,2.5150 \end{array} \right\} < 1 + \frac{\pi}{2}\,,$$

and equation (10) gives no solution. It can be shown that the solutions with $\delta \geqq 2$ are also false.

Fig. 3 shows what is the reason that the method of least squares yielded false values of the three coefficients of (23). Designing

$$\chi = \omega + \Omega \cos i_0 ,$$

the second decay condition may simply be written:

$$\chi = E' + Gq . \tag{24}$$

In Fig. 3 the denependce of $\chi = \chi(q)$ is represented for a few values of cos i_0. It is evident that four of the five indubitable members, 1843 I, 1882 II, 1963 V and 1965f, nearly equally well keep the linear dependence for any i_0. Solution (23) is almost exclusively determined by the position of 1880 I in the graph. Any slight error in either ω or Ω or q of the orbit of 1880 I is able to produce drastic changes in the value of coefficient G of the second decay condition.

I can now follow three different ways to evade the trouble with a bad determinacy of coefficient G:

1. to assume that G is small; then, with respect to the negligible dispersion in the perihelion distances of the indubitable members, term $G \, . \, q$ is nearly constant besides small enough, and it may be incorporated into coefficient E;
2. to make use of the data on some further members of the Kreutz family, either to apply the sets of orbital elements of some of the possible members, or

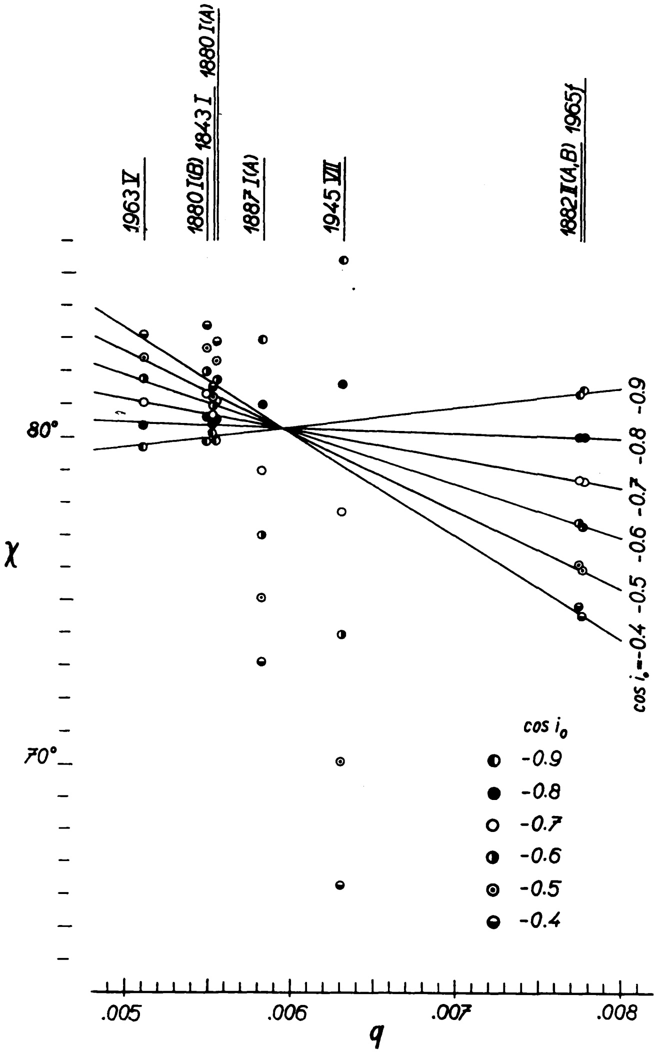

Fig. 3. *The second decay condition for sun-grazing comets.*

3. to search for and detect new members of the family, unknown so far.

Hereinafter I follow each of the three possibilities.

10. A simplified modification of the second decay condition

Fig. 4 represents the dependence of the argument of perihelion on the ascending node, which, based on five couples of ω and Ω (1843 I, 1880 I Orbit A, 1882 II Orbit A, 1963 V and 1965f), may numerically be expressed:

$$\omega = 80^\circ.309 + 0.82109\,\Omega \qquad (25)$$
$$\pm\, 0^\circ{\cdot}034 \pm 0.00363 \text{ m.e.}$$

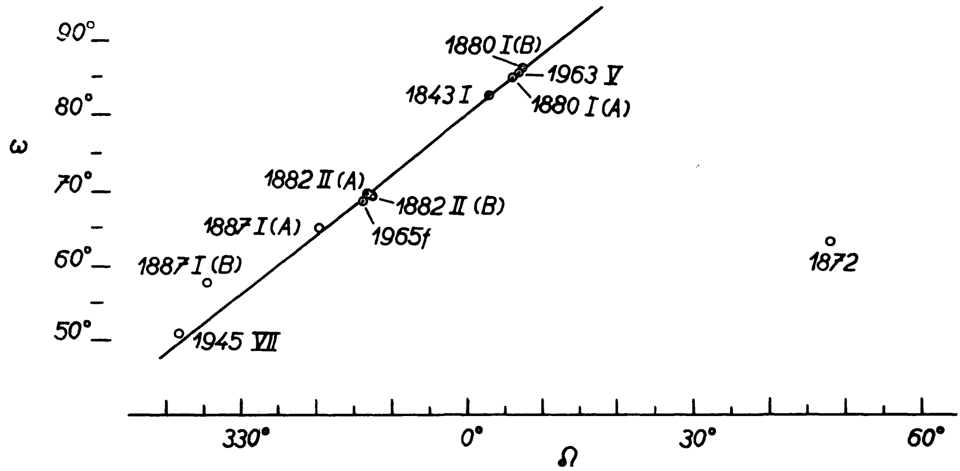

Fig. 4. *Argument of perihelion in dependence on the node.*

leaving a mean residual of only $\pm$ 3'.5. This indicates that the assumption $G \approx 0$ may be acceptable. It is, moreover, noticeable that the residuals from (25) are for each of the possible members smaller than from the general solution of (23).

The validity of (25) would mean that the supposed cosmic catastrophe of the protocomet should have occurred at very great heliocentric distances. The protocoment's angular elements should have been (eq. 1950.0):

$$(26)\quad \begin{cases} i_0 = 145^\circ.19 \pm 0^\circ.36\,, \\ \Omega_0 = 10^\circ.64 \pm 2^\circ.82, \\ \omega_0 = 89^\circ.04 \pm 2^\circ.31\,, \end{cases}$$

where an additional uncertainty of less than $0^\circ.46\ G$ is included in the value of ω_0 because of the neglection of the term with G. The protocomet's perihelion distance remains, however, unknown.

This is one of the possible solutions of the origin of the Kreutz group of sun-grazing comets. Unfortunately, no data can be given on the impulses impressed to the individual fragments of the protocomet without knowledge of q_0 and r_0.

11. Possible members and the decay conditions

Discussing the determinacy of the orbits of the possible members, 1887 I and 1945 VII were preferred to the two other comets. Let us add the two comets to the five indubitable members and solve the two decay conditions anew. The determinacy of the first decay condition is impaired by the inclusion of the possible members, which could

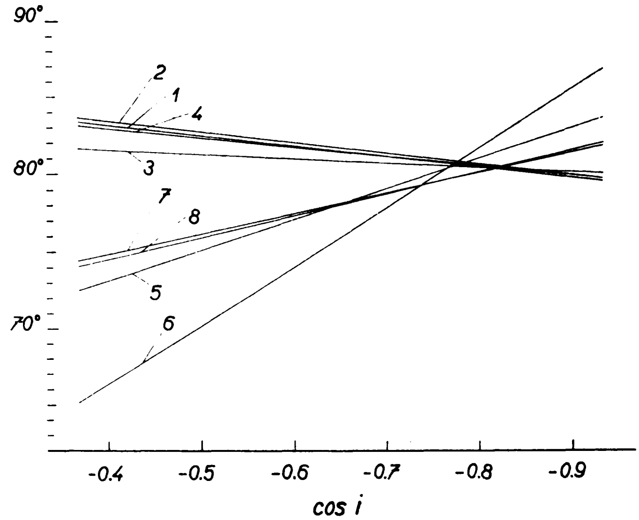

Fig. 5. *Function* $\chi = \omega + \Omega . \cos i_0$ *in dependence on* $\cos i_0$: *1 – 1963 V, 2 – 1880 I (Orbit B), 3 – 1843 I, 4 – 1880 I (Orbit A), 5 – 1887 I (Orbit A), 6 – 1945 VII, 7 – 1882 II (Orbits A, B), 8 – 1965f.*

be expected from a view on Fig. 2. On the other hand, both the $\chi = \chi(q)$ relation in Fig. 3 and the course of χ with cos i_0, plotted in Fig. 5, show that 1887 I and 1945 VII improve the determinacy of the second decay condition. Compared with (23) the relative error of G is reduced from 78 to 13 per cent:

$$\begin{aligned} \omega &= 82^\circ.849 + 0.76577\,\Omega - 7.35\,\alpha q \\ &\pm 0^\circ.340 \pm 0.00371 \quad \pm 0.98 \text{ m.e.} \end{aligned} \tag{27}$$

with the mean residual $\pm$ 6'.

The first decay condition in its exact form is

$$\begin{aligned} \text{ctg}\, i &= -(0.1506 \sin \Omega + 1.3665 \cos \Omega), \\ &\pm 0.1073 \qquad \pm 0.0338 \text{ m.e.} \end{aligned} \tag{28}$$

and in its approximate form:

$$\begin{aligned} i &= 143^\circ.545 + 0.18147\,\Omega, \\ &\pm 0^\circ.481 \pm 0.02650 \text{ m.e.} \end{aligned} \tag{29}$$

with the mean residuals $\pm$ 1°.6 and $\pm$ 0°.9, respectively. The residuals from (27) to (29) are for each of the seven comets listed in Table 4.

Applying now the formulae of Section 4 the first value of $f(v_0)$ equals to $+$ 0.1127 and gives no real solution. But its second value, $-$ 3.0289, leads to two independent

Table 4. Residuals from the decay conditions for the indubitable and possible members

Comet	O — C		
	in inclination		in perihelion
	from (28)	from (29)	from (27)
	°	°	°
1843 I	+ 0.43	+ 0.29	— 0.05
1880 I (A)	+ 0.57	— 0.08	+ 0.21
1882 II (A)	— 0.37	+ 0.83	— 0.01
1887 I (A)	— 3.06	— 2.01	— 0.10
1945 VII	+ 2.63	+ 0.43	+ 0.07
1963 V	+ 0.57	— 0.26	— 0.10
1965f	— 0.40	+ 0.80	— 0.02

solutions. One of them is of high interest. It gives a set of the orbital elements of the protocomet as follows:

$$(30)\quad \left\{ \begin{array}{l} T_0 = 37.3 \text{ days following the catastrophe} \\ \left.\begin{array}{l} \omega_0 = 61^\circ.27 \pm 2^\circ.19 \\ \Omega_0 = 340^\circ.33 \pm 4^\circ.31 \\ i_0 = 139^\circ.98 \pm 0^\circ.33 \end{array}\right\} \; 1950.0 \\ q_0 = 0.01546 \pm 0.00206 \text{ AU}, \end{array} \right.$$

with the data on the location of the catastrophe in space:

$$(31)\quad \left\{ \begin{array}{l} v_0 = - 167^\circ.03, \\ r_0 = 1.212 \text{ AU}, \\ x_0 = - 0.009 \text{ AU}, \\ y_0 = + 0.952 \text{ AU}, \\ z_0 = - 0.750 \text{ AU}, \\ \dot{x}_0 = + 4.51 \text{ km/s}, \\ \dot{y}_0 = - 30.40 \text{ km/s}, \\ \dot{z}_0 = + 22.77 \text{ km/s}. \end{array} \right.$$

The vector-element matrix is of the form:

$$(32)\quad \begin{pmatrix} P \\ Q \\ R \end{pmatrix} = \begin{pmatrix} +0.2265 & -0.7941 & +0.5640 \\ -0.9496 & -0.0514 & +0.3091 \\ -0.2165 & -0.6056 & -0.7658 \end{pmatrix}$$

The separation velocities and the values of the two criteria of Section 8 together with the „plane-of-orbit" criterion,

$$(33)\quad C_0(\Omega, i) \equiv x_0 \sin \Omega - y_0 \cos \Omega + z_0 \operatorname{ctg} i = 0\,,$$

are for each of the seven comets given in Table 5. Since (33) is identical with the first

Table 5. Separation velocities and checking criteria for 7 members of the Kreutz family

Comet	Separation velocity, km/s							Checking criteria, AU		
	$\dot{\xi}$	$\dot{\eta}$	$\dot{\zeta}$[1]	$\dot{x}$	$\dot{y}$	$\dot{z}$	ΔV	C_0	C_1	C_2
1843 I	—0.16	—1.74	—0.86	—1.51	+ 0.62	+ 1.06	1.94	+ 0.095	+ 0.030	—0.075
1880 I (A)	—0.16	—1.73	—0.97	—1.48	+ 0.69	+ 1.15	2.00	+ 0.105	+ 0.025	—0.083
1882 II (A)	—0.12	—1.26	—0.28	—1.17	+ 0.23	+ 0.51	1.30	+ 0.035	+ 0.012	—0.030
1887 I (A)	—0.15	—1.67	0.00	—1.63	+ 0.09	+ 0.38	1.68	—0.061	+ 0.022	+ 0.040
1945 VII	—0.14	—1.56	+ 0.84	—1.70	—0.42	—0.28	1.78	+ 0.064	+ 0.023	—0.059
1963 V	—0.16	—1.84	—1.00	—1.58	+ 0.71	+ 1.18	2.10	+ 0.107	+ 0.035	—0.083
1965f	—0.12	—1.26	—0.25	—1.17	+ 0.22	+ 0.49	1.29	+ 0.033	+ 0.012	—0.028

[1]) The value given is the average from the two extreme values $\dot{\zeta}_1$ and $\dot{\zeta}_2$.

decay condition, and the C_0 values of Table 5 are consequently its residuals expressed in AU, it is of interest to compare these with the residuals of *i* from (28), expressed in degrees, as listed in Table 4. The other solutions are of no interest.

12. Ancient comets

So far only comets observed since the forties of the 19th century have been investigated. Further members of the Kreutz group of comets can above all be found among ancient comets. Out of a large number of ancient comets recorded, only for some of them at least approximate orbits are available.

Ancient members were looked for by KREUTZ and his predecessors (KREUTZ 1901)

Table 6. Dependence of the $\dot{\eta}$ component of velocity on r_0, q_0 and q

q	$\dot{\eta}$ (km/s)								
	$r_0 = 0.2$			$r_0 = 1.0$			$r_0 = 5.0$		
	$q_0 = 0.005$	$q_0 = 0.02$	$q_0 = 0.1$	$q_0 = 0.005$	$q_0 = 0.02$	$q_0 = 0.1$	$q_0 = 0.005$	$q_0 = 0.02$	$q_0 = 0.1$
0.005	0.0	— 14.9	—51.7	0.0	—3.0	—10.3	0.00	—0.60	—2.1
0.01	+ 6.2	—8.7	—45.5	+ 1.2	—1.7	—9.1	+ 0.25	—0.35	—1.8
0.02	+ 14.9	0.0	—36.8	+ 3.0	0.0	—7.4	+ 0.60	0.00	—1.5
0.05	+ 32.2	+ 17.3	— 19.5	+ 6.4	+ 3.5	—3.9	+ 1.3	+ 0.69	—0.78
0.1	+ 51.7	+ 36.8	0.0	+ 10.3	+ 7.4	0.0	+ 2.1	+ 1.5	0.00
0.2	+ 79.3	+ 64.4	+ 27.6	+ 15.9	+ 12.9	+ 5.5	+ 3.2	+ 2.6	+ 1.1
0.5	+ 134	+ 119	+ 82.3	+ 26.8	+ 23.8	+ 16.5	+ 5.4	+ 4.8	+ 3.3
1.0	+ 196	+ 181	+ 144	+ 39.1	+ 36.1	+ 28.8	+ 7.8	+ 7.2	+ 5.8
1.5	+ 243	+ 228	+ 191	+ 48.6	+ 45.6	+ 38.2	+ 9.7	+ 9.1	+ 7.6
2.0	+ 213	+ 268	+ 231	+ 56.6	+ 53.6	+ 46.2	+ 11.3	+ 10.7	+ 9.2
3.0	+ 350	+ 335	+ 298	+ 69.9	+ 67.0	+ 59.6	+ 14.0	+ 13.4	+ 11.9

particularly according to their extremely short perihelion distances. The two decay conditions show, however, that the scale of orbits may be much more extended than assumed so far, if high enough velocities of separation ($\approx$ 10—30 km/s) are permitted. Table 6 gives the values of $\dot{\eta}$, mostly the dominant component, to be impressed to a "daughter" comet in order that the original perihelion distance, q_0, should have changed to the observed value, q, on various assumptions concerning r_0.

Table 7. Ancient comets

Comet	T	Equinox 1950.0			q	e	Classification
		ω	Ω	i			
		°	°	°			
—371	—371 Winter	120	≈330	<150	≪1	1.	estim.
240	240 Nov. 10	82	213	44	0.371	1.	C1
574	574 Apr. 7.78	15.49	147.34	46.36	0.9629	1.	C1
1402	1402 (Mar. 21)	91	125	55	0.38	1.	D1
1556	1556 Apr. 22.6846	100.868	180.736	32.377	0.49082	1.	A1
1668	1668 Feb. 28.0795	109.811	2.515	144.375	0.066604	1.	B1
1684	1684 June 8.763	330.3067	271.8963	65.4230	0.95827	1.	B1
1695 (A)	1695 Oct. 23.768	59.124	285.306	93.587	0.042297	1.	B1
1695 (B)	1695 Oct. 21.123	118.585	341.345	145.394	0.154175	1.	B1

Table 7 lists all the ancient comets of known orbits satisfying the first decay condition (21) with the accuracy of their own. For each comet are given the orbital elements referred to 1950.0 and a classification parameter in the system used by PORTER (1961) in his Catalogue of Cometary Orbits. The orbits are taken over from this Catalogue except for Comet —371, which is taken from the Catalogue by BALDET and DE OBALDIA (1952), assuming the original equinox referring to — 371.0, and except for the B-orbit of 1695 taken directly from the third part of the monograph by KREUTZ (1901).

Fig. 6 shows all the ancient comets, represented by crosses in circlets, satisfying the first decay condition of (21), irrespective of their perihelion distances and the sense of their motion about the Sun.

As shown in Section 8, the first decay condition is only one of the necessary conditions, which have to be fulfilled by all members of the family with respect to their "parent" body. The stronger condition, requiring an intersection of the respective orbits in space, is expressed by (18) and will be discussed below, the third one, requiring an encounter of the respective bodies in the past, can hardly be tested because of the lack of orbital periods for many comets.

Let us now follow this consideration:

The numerical form of the first decay condition is in its general version determined by two independent constants, ratios x_0/z_0 and y_0/z_0. The position in space of the spot of the hypothetical catastrophe is determined by three independent constants, which may be chosen as $x_0/z_0, y_0/z_0$ and z_0, Assuming that the form of the first decay condition, established from the sets of orbital elements of the five indubitable members, covering an arc

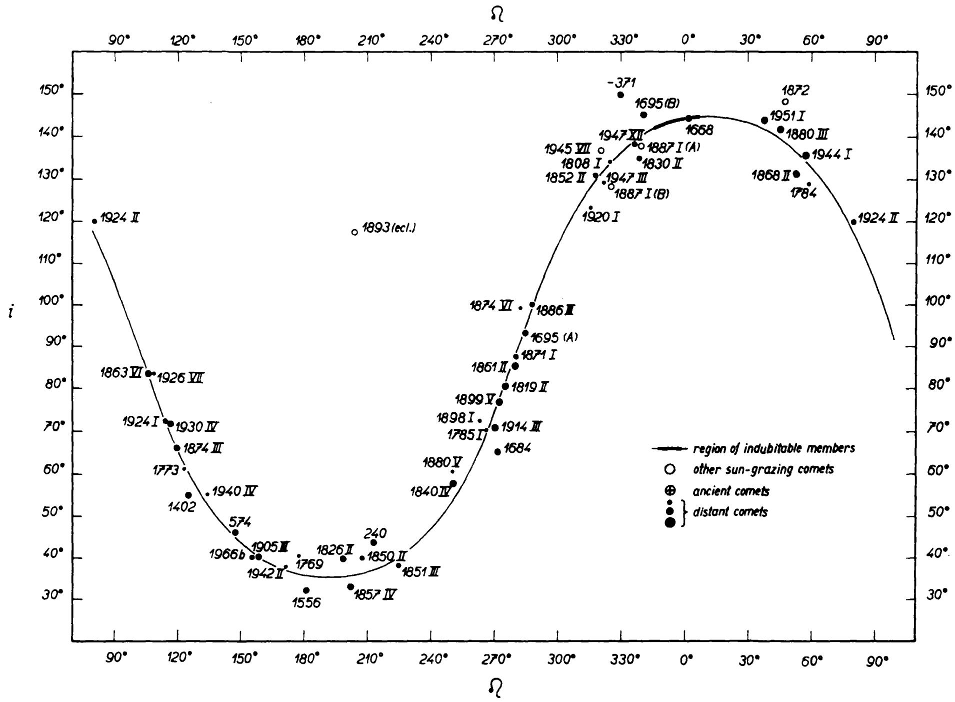

Fig. 6. *The first decay condition for ancient and distant comets.*

of more than 20° in Ω, is more or less perfectly fixed, the two remaining criteria depend on the space location of the catastrophe only through its distance from the plane of ecliptic, z_0. Studying the dependences of $C_1(z_0)$ and $C_2(z_0)$ on z_0 for a number of comets, suspected of belonging to the Kreutz family in accordance with Criterion C_0, the most probable z_0 co-ordinate can be found as the abscissa of the minimum dispersion of C_1 and C_2 from their zero levels. Actually, denoting

$$a = \frac{x_0}{z_0}, \quad b = \frac{y_0}{z_0}, \quad c = +(1 + a^2 + b^2)^{\frac{1}{2}},$$

the three criteria may be transcribed to the form:

$$(34)\quad \left\{ \begin{aligned} C_0 &= z_0 (a \sin \Omega - b \cos \Omega + \operatorname{ctg} i), \\ C_1 &= z_0 (a \cos \Omega + b \sin \Omega) - \\ &\quad - c \mid z_0 \mid \cos \left[\omega + 2 \operatorname{sign} v \arccos \sqrt{\left(\frac{q}{c \mid z_0 \mid} \right)} \right], \\ C_2 &= z_0 \operatorname{cosec} i - c \mid z_0 \mid \sin \left[\omega + 2 \operatorname{sign} v \arccos \sqrt{\left(\frac{q}{c \mid z_0 \mid} \right)} \right]. \end{aligned} \right.$$

For a certain comet, selected pre- or post-perihelion arc of its orbit and known values of a, b, c, (34) are functions of a single variable, z_0, only. Analogously, the distance, Δ, of the point of the co-ordinates x_0, y_0, z_0 from a point $(x, y, z) = f(\omega, \Omega, i, q)$ is on the above assumptions and on that of equal heliocentric distances a function of z_0 only, as follows:

$$(35)\quad \begin{cases} \Delta^2 = 2c^2 z_0^2 + 2\beta z_0 (c \mid z_0 \mid - 2q) - \\ - 4 \sqrt{(q)}\, \gamma z_0 \operatorname{sign} v \sqrt{(c \mid z_0 \mid - q)}\,, \end{cases}$$

where

$$\begin{pmatrix} \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix}.$$

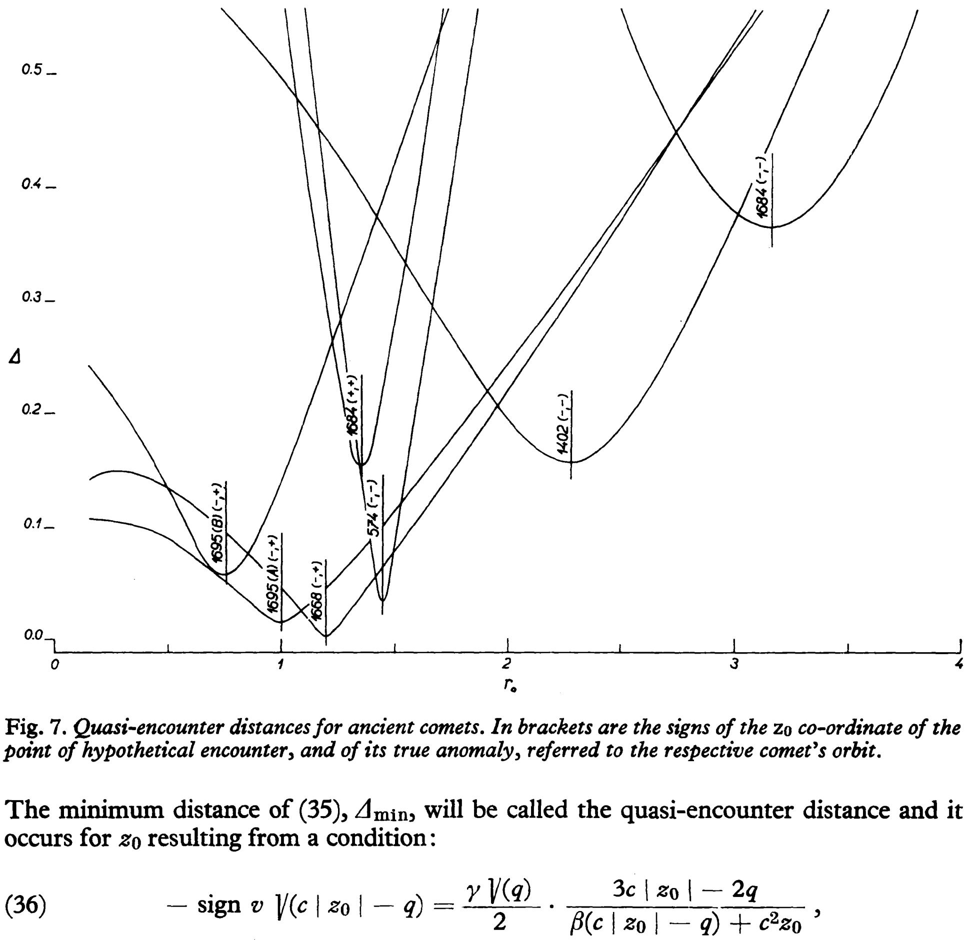

Fig. 7. *Quasi-encounter distances for ancient comets. In brackets are the signs of the* z_0 *co-ordinate of the point of hypothetical encounter, and of its true anomaly, referred to the respective comet's orbit.*

The minimum distance of (35), $\Delta_{\min}$, will be called the quasi-encounter distance and it occurs for z_0 resulting from a condition:

$$(36)\qquad - \operatorname{sign} v \sqrt{(c \mid z_0 \mid - q)} = \frac{\gamma \sqrt{(q)}}{2} \cdot \frac{3c \mid z_0 \mid - 2q}{\beta(c \mid z_0 \mid - q) + c^2 z_0}\,,$$

which generally leads to a cubic equation for z_0. If, however, the perihelion distance, q, is completely negligible compared to the heliocentric distance of the catastrophe, r_0, the solution gives:

$$(37)\qquad \mid z_0 \mid = \frac{9 \gamma^2 q}{4c} \cdot \frac{1}{(\beta + c \operatorname{sign} z_0)^2}\,,$$

and

$$\Delta_{\min} = \frac{3}{4} \sqrt{(42)}\, \gamma^2 q (c + \beta \operatorname{sign} z_0)^{-\frac{3}{2}}. \tag{38}$$

For each ancient comet of Table 7 the values of the three C_i criteria and of the distance Δ have been computed for a long series of heliocentric distances for both positive and negative z_0 and for both pre-perihelion and post-perihelion arcs of orbit, hence, for four combinations.

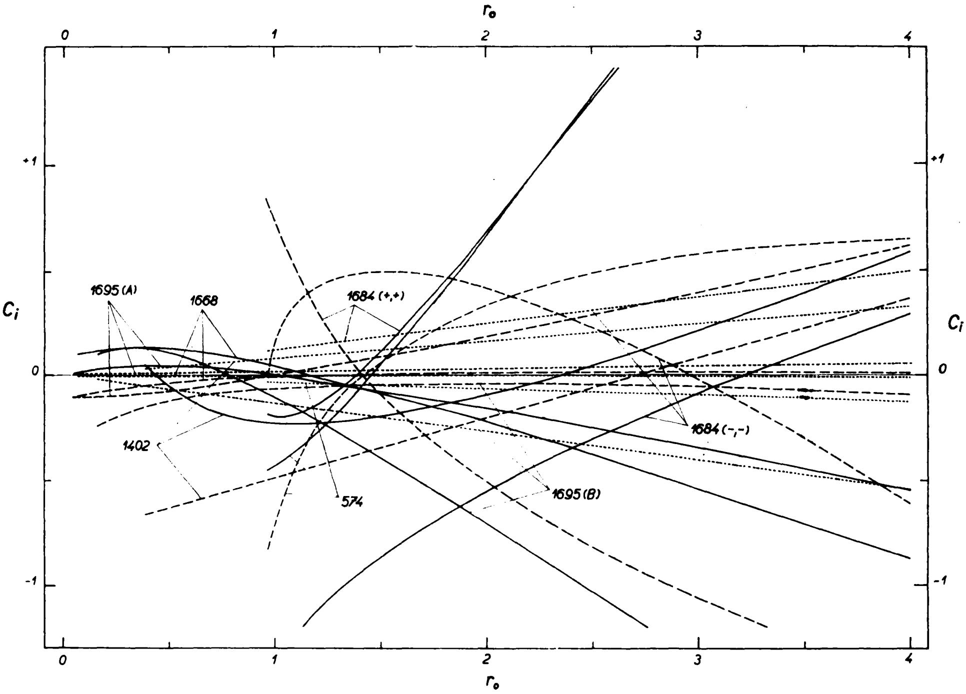

Fig. 8. C_i *criteria for ancient comets. Dotted curves* – C_0, *solid curves* – C_1, *dashed curves* – C_2.

The results are shown in Figs. 7 and 8, where the distance Δ and the C_i criteria, respectively, are plotted against the heliocentric distance for those of the ancient comets, which reveal a local minimum on their $\Delta(r_0)$ curve. The corresponding quasi-encounter distances are with the related data listed in Table 8.

As could be expected, Comet 1668 is strongly suspected of belonging to the Kreutz family, leaving nearly no residuals at $r_0 = 1.2$ AU. It is of interest that this r_0 is in perfect agreement with the value of r_0 derived in Section 11. Another very probable ancient member of the family is Comet 1965. Particularly its A orbit derived by KREUTZ satisfies the criteria very well near $r_0 = 1.0$ AU, close to the above value. Even Comet 574, though distant, could belong to the family, leaving $\Delta_{\min}$ less than 0.1 AU about $r_0 = 1.4$ AU. With respect to its very uncertain orbit, the agreement is surprisingly good. It is of primary importance that the orbits of each of the three comets closely approach the hypothetical protocomet's orbit southwards the plane of ecliptic, and within a compar-

Table 8. Ancient comets as possible members of the Kreutz family

Comet	r_0	Δ_{min}	z_0	arc of orbit	C_0	C_1	C_2
574	1.45	0.033	negative	pre-perihelion	+0.045	0.000	+0.032
1402	2.27	0.156	negative	pre-perihelion	—0.188	—0.020	—0.123
1668	1.19	0.0014	negative	post-perihelion	+0.002	0.000	—0.002
1684	1.35	0.154	positive	post-perihelion	+0.169	—0.013	+0.073
	3.16	0.366	negative	pre-perihelion	—0.395	—0.022	—0.100
1695 (A)	1.00	0.015	negative	post-perihelion	—0.015	+0.001	0.000
1695 (B)	0.76	0.059	negative	post-perihelion	+0.102	+0.009	—0.085

atively narrow interval of solar distances, suggesting thus a possible location of the hypothetical catastrophe. The point of encounter lies on the pre-perihelion arc of the 574 orbit, but on the post-perihelion arcs for 1668 and 1695. This fact could be attributed to the velocity-direction distribution of individual fragments at the time of the hypothetical catastrophe.

Comet 1402 has a larger quasi-encounter distance of some 0.16 AU at a solar distance of more than 2 AU. With respect to its highly uncertain orbit, even its membership to the Kreutz family cannot generally be excluded, but considered as problematic.

On the other hand, 240, 1556 and 1684 are obviously of another origin, having nothing common with the Kreutz family. The first two comets reveal no minimum on their $\Delta(r_0)$ curves up to 5.6 AU; Comet 1684 indicates two independent minima, but the details speak strongly against its membership.

Comet 371 B.C. is suspected of belonging to the family, but the data on its orbit are too scanty to give any decisive evidence for or against its membership.

13. Distant comets

Altogether 38 distant comets between 1769 and 1966 have been collected, being under suspicion of belonging to the Kreutz family according to the first decay condition. They are represented by full circles of various dimensions in Fig. 6. The diameter of each circle characterizes the reliability of the orbit. Because of the large number of the suspicious comets their orbits are not listed at the present paper. The writer refers to PORTER'S (1961) catalogue, from which all the orbits were taken, except for 1944 I, 1951 I and 1966b. The orbits of the three comets were taken from the original sources written by MARSDEN and van BIESBROECK (1963), van HOUTEN-GROENEVELD (1963) and MARSDEN (1966), respectively.

The distant comets have been investigated in the same way as the ancient comets In distinction to the latter, however, osculating or even original periods of revolution are known for some of them. These data make it possible to take into at least rough consideration the time relations of the problem and to reject all the extremely long-period or non-period comets with their original orbits very close to parabola. On the other hand, comets with periods of revolution of the order of hundred to thousand years are under

Table 9. Osculating and original periods of revolution for 19 distant comets

Comet	P_{osc}	P_{orig}
	yrs	yrs
1769	2,090	1,870 :
1840 IV	367	355 :
1857 IV	235	229 :
1861 II	410	394 :
1863 VI	hyperbola	5,300,000
1871 I	5,170	4,250 :
1874 III	13,700	5,490
1886 III	hyperbola	hyperbola
1898 I	417	396
1905 III	226	220 :
1914 III	hyperbola	hyperbola
1924 I	264,000	84,000
1924 II	hyperbola	hyperbola ?
1926 VII	hyperbola	hyperbola ?
1930 IV	hyperbola	82,000
1942 II	85.52	84.4 :
1944 I	hyperbola	hyperbola
1947 XII	hyperbola	420,000 :
1951 I	hyperbola	5,300,000

stronger suspicion than the others. The periods of revolution of the comet osculating and original orbits have been compiled and briefly listed in Table 9, based on the General Catalogue of Original and Future Comet Orbits (SEKANINA 1966b). For the comets, for which only osculating elements are available, a probable original period of revolution has been computed by adding a correction term discussed in the writer's recent paper (SEKANINA ibid.); a colon is then added to the corresponding value of Table 9. The run of Δ is for some distant comets apparent from Fig. 9.

The results of the investigation itself are of high interest. Those comets from Fig. 6, satisfying each of the three C_i criteria are listed in Table 10. It is arranged in the same way as Table 8. Summarizing the results of investigation of both ancient and „recent" comets with respect to their kinematic and physical characteristics, we conclude:

1. There is a comparatively strong concentration of comets with their quasi-encounter distances located at a heliocentric distance of about 1.2 AU: For comets 1668, 1785 I, 1830 II, 1924 II and 1947 III they lie within an interval of 1.15 to 1.19 AU, for 574, 1695 (A-orbit), 1857 IV, 1905 III and 1942 II within 1.00 to 1.54 AU. Taking account of possible effects of the planetary perturbations, acting on the comets' orbits for long periods of time, comparable with, say, 10^3 to 10^5 years, a dispersion in the points of

Table 10. Distant comets as possible members of the Kreutz family

Comet	r_0	Δ_{min}	z_0	arc of orbit	C_0	C_1	C_2
	AU	AU			AU	AU	AU
1769	0.73	0.058	positive	post-perihelion	−0.087	−0.010	−0.070
1785 I	1.16	0.019	negative	post-perihelion	+0.007	+0.012	−0.011
1830 II	1.17	0.090	negative	pre-perihelion	−0.125	−0.017	+0.100
1840 IV	2.85	0.149	positive	pre-perihelion	−0.175	+0.001	−0.100
	3.07	0.160	negative	post-perihelion	+0.189	+0.008	+0.098
1857 IV	1.27	0.066	negative	post-perihelion	−0.123	+0.005	−0.102
	1.76	0.092	positive	pre-perihelion	+0.169	+0.005	+0.139
1861 II	2.82	0.069	negative	pre-perihelion	−0.069	−0.001	−0.003
1863 VI	1.87	0.0070	positive	post-perihelion	+0.007	+0.001	+0.002
1899 V	1.88	0.017	positive	post-perihelion	+0.017	0.000	+0.016
1905 III	1.51	0.0075	negative	pre-perihelion	−0.008	+0.005	−0.003
1920 I	1.69	0.116	positive	post-perihelion	+0.137	−0.015	−0.067
1924 I	2.16	0.012	negative	post-perihelion	−0.012	+0.001	−0.002
1924 II	1.15	0.039	negative	pre-perihelion	+0.045	−0.001	−0.022
1926 VII	1.30	0.049	positive	pre-perihelion	−0.049	0.000	−0.006
	1.80	0.068	negative	post-perihelion	+0.068	−0.001	+0.009
1930 IV	2.85	0.060	negative	pre-perihelion	+0.063	+0.001	+0.020
1940 IV	1.80	0.053	negative	post-perihelion	+0.064	0.000	+0.037
	2.73	0.080	positive	pre-perihelion	−0.098	+0.003	−0.054
1942 II	1.54	0.025	negative	pre-perihelion	+0.040	−0.005	+0.030
1944 I	1.66	0.044	negative	pre-perihelion	+0.063	0.000	−0.045
	1.85	0.049	positive	post-perihelion	−0.071	+0.001	+0.051
1947 III	1.17	0.055	negative	post-perihelion	−0.070	+0.002	+0.044
1947 XII	0.73	0.0063	positive	pre-perihelion	+0.010	0.000	−0.007
1966b	2.63	0.044	positive	pre-perihelion	+0.068	0.000	+0.052

the hypothetical encounter of about 0.5 AU is satisfactory. Altogether 15 comets were found within $r < 2.5$ AU, indicating a Δ_{min}.

2. Locations and values of the quasi-encounter distances for the investigated comets are represented in Fig. 10, separately for southern and northern z_0 co-ordinates. The figure suggests the proposed catastrophe might occur below the ecliptic at a solar distance of about 1.2 AU, the approximate ecliptical co-ordinates having been:

$$(39) \quad \begin{cases} x_0 = -0.20 \text{ AU}, \\ y_0 = +0.96 \text{ AU}, \\ z_0 = -0.69 \text{ AU}. \end{cases}$$

In the y and z co-ordinates the values are in good agreement with the independently obtained result of (31). For a positive z_0 the distribution of Δ_{min} is more or less random in its character.

3. The quasi-encounter distances for the suspected comets are always less than 0.1 AU, and for comets 1668 and 1905 III even less than 0.01 AU.

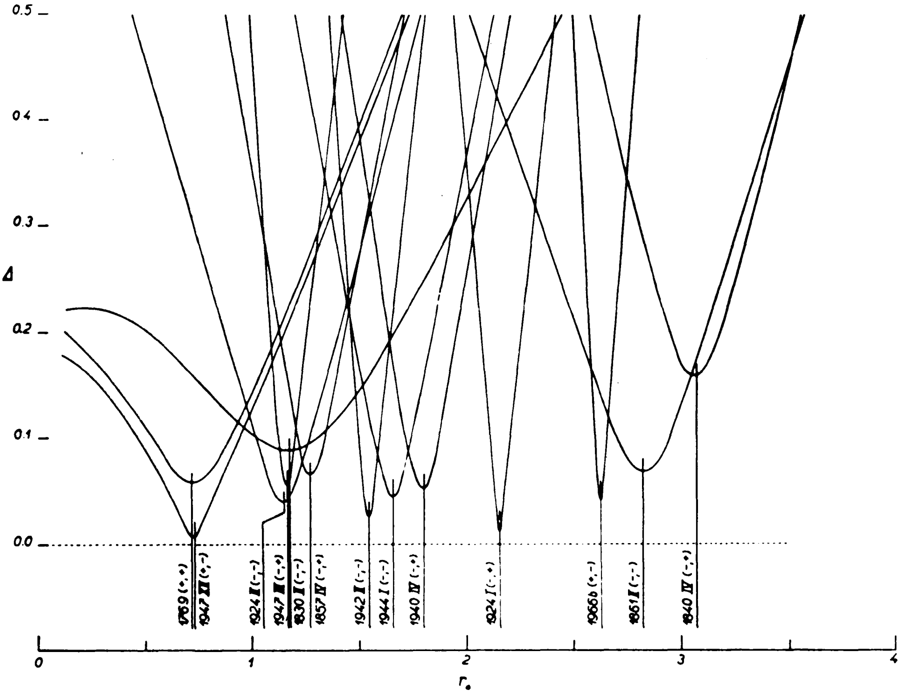

Fig. 9. *Quasi-encounter distances for some distant comets. In brackets are the signs of the* z_0 *co-ordinate of the point of hypothetical encounter and of its true anomaly, refered to the respective comet's orbit.*

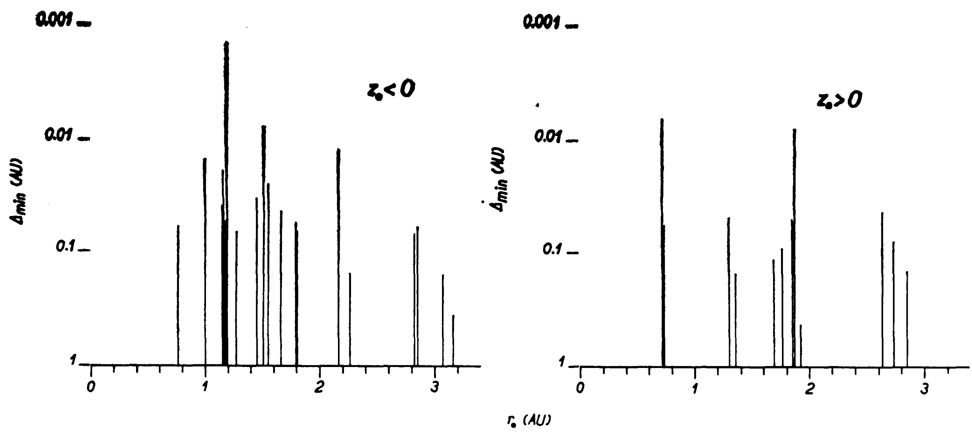

Fig. 10. *Distributions of quasi-encounter distances, assuming* (1) $z_0 < 0$, (2) $z_0 > 0$.

4. The location of the quasi-encounter distances is equally distributed between pre- and post-perihelion arcs of the orbits.
5. For none of the most suspected comets (see point 1.) the original orbit is known, and only for four of them, 1857 IV, 1905 III, 1924 II and 1942 II, the eccentricity of their osculating orbits is available. The first two comets have their periods of revolution about 200 years, consistent in order with those of the sun-grazing comets, the fourth is a short-period comet Väisälä (2), while the third in order, 1924 II has a hyperbolic osculating orbit. It is derived from a monthly arc only and uncertain to some degree, nevertheless, the comet's membership to the investigated group is rather improbable. Of other comets, 1924 I has its original period of revolution equal to 84,000 years.
6. About 50 per cent of suspected comets are of retrograde motion, but only three of them are short-distant; 1668, 1695 and 1830 II. Comet 1924 II has $q = 0.4$ AU, perihelion distances of the other are not far from 1 AU. The orbits of 1668 and 1830 II are similar in character to the orbits of the sun-grazing comets.
7. There seems to be no conspicuous physical resemblance among the suspected comets. Since each comet's physical behaviour is primarily determined by its perihelion distance, this fact is no strong evidence against the supposed evolutionary relationship of the comets. It may be mentionable that two ancient comets, 574 and 1402 are obviously exceptional in its intrinsic luminosity, some others, 1668, 1695, 1785 I, 1830 II, 1924 II, are average, and the remaining, 1857 IV, 1905 III, 1942 II, 1947 III etc., are intrinsically faint comets. Unusual behaviour of a few comets was observed. Comet 1926 VII quickly weakened after its passage through perihelion. Comet 1947 III was observed dropping in luminosity even approaching the Sun. The 1905 III central condensation showed an extension, typical for active comets.
8. A pair of comets, 574 and 1947 III, are of interest still from another point of view, apart from a drastic difference between their luminosities. Table 11 shows that the two comets move about the Sun in similar orbits: Comet 574 in a direct, Comet 1947 III in a retrograde sense. Table 10 confirms this fact by the location of the hypothetical catastrophe relative to their perihelion passages.

Table 11. Comets 574 and 1947 III

	574	retrograde	1947 III
ω_{1950}	15°.49	164°.51	182°.13
Ω_{1950}	147°.34	327°.34	322°.38
i_{1950}	46°.36	133°.64	129°.15
q	0.9629	0.9629	0.9618

14. The C_i criteria and sun-grazing comets

In the preceding sections a concept of the origin of the Kreutz group of comets was being worked out, based on an assumption of a hypothetical cosmic catastrophe, which met the protocomet. The concentration of encounters at a 1.2 AU solar distance (Fig. 10) is noticeable, and at the first sight there seems to be only a low probability that the phenomenon is a product of chance. The hypothesis, hence, appears to be quite hopeful. But a serious obstacle appears when the C_i criteria are applied to the sun-grazing comets: the latter show no quasi-encounter distance, and leave systematic residuals in C_1 and C_2 (Table 12).

Table 12. C_i criteria for sun-grazing comets
($z_0 < 0$, $r_0 = 1.2$ AU)

Comet	C_0	pre-perihelion arc		post-perihelion arc	
		C_1	C_2	C_1	C_2
1843 I	+0.0006	−0.1596	+0.0091	+0.1612	−0.0324
1880 I (A)	+0.0006	−0.1625	+0.0020	+0.1607	−0.0250
1882 II (A)	−0.0002	−0.1766	+0.0495	+0.1825	−0.0841
1887 I (B)	−0.1256	−0.2699	+0.2310	+0.0953	+0.0059
1945 VII	+0.1162	−0.1367	+0.0131	+0.1327	−0.2056
1963 V	−0.0011	−0.1525	+0.0017	+0.1582	−0.0207
1965f	+0.0001	−0.1763	+0.0510	+0.1818	−0.0862

The residuals can be reduced to values of 10^{-3} to 10^{-4} AU for the indubitable members, and to still acceptable values of 10^{-2} AU for 1887 I and 1945 VII, if considerable corrections of about $\pm 8°$ are added to the present values of the argument of perihelion. Hence, a systematic secular apsidal motion should be admitted to keep the observed distribution of orbital elements of the sun-grazing comets consistent with the concept of the hypothetical catastrophe.

The corrected values of ω can simply be found. The quasi-encounter distance is the minimum value of all Δ, which, written through the C_i criteria, are:

(40) $$\Delta^2 = C_0^2 + C_1^2 + C_2^2 - 2C_0C_2 \cos i \,.$$

From their definition, the C_i values are the following functions of ω:

(41) $$\begin{cases} C_1 = \alpha_1 + \beta \cos \omega \mp \gamma \sin \omega \,, \\ C_2 = \alpha_2 \pm \gamma \cos \omega + \beta \sin \omega \,, \end{cases}$$

where

(42) $$\begin{cases} \alpha_1 = x_0 \cos \Omega + y_0 \sin \Omega \,, \\ \alpha_2 = z_0 \operatorname{cosec} i \,, \\ \beta = -r_0 \cos v \,, \\ \gamma = r_0 \,|\sin v| \,. \end{cases}$$

The C_0's are functions od Ω and i only. Looking for the quasi-encounter distance we find a condition for ω:

(43) $$\tan \omega = \frac{\beta(\alpha_2 - C_0 \cos i) \mp \gamma \alpha_1}{\beta \alpha_1 \pm \gamma(\alpha_2 - C_0 \cos i)} \,.$$

The quasi-encounter distance is then given by:

$$\Delta^2_{\min} = C_0^2 \sin^2 i + C_1^2 \frac{\beta^2 + \gamma^2}{(\beta \cos \omega \mp \gamma \sin \omega)^2} \tag{44}$$

with ω from (43).

The required corrections to the argument of perihelion are for the five sun-grazing comets listed in Table 13. For the two remaining comets, 1887 I and 1945 VII, for which only approximate orbits have been available, the solution gave no quasi-encounter distance, since it was not able to remove the continuous growth of Δ with the increasing r_0

Table 13. Corrected ω of the indubitable members of the Kreutz group of comets for $z_0 < 0$ and $r_0 = 1.2$ AU

Comet	ω_{obs}	Δ_{min}	$C_{1,corr}$	$C_{2,corr}$	pre-perihelion arc		post-perihelion arc	
					ω_{corr}	corr	ω_{corr}	corr
	°				°	°	°	°
1843 I	82.64	0.0004	0.0000	—0.0006	74.96	+7.68	90.53	—7.89
1880 I (A)	85.23	0.0003	0.0000	—0.0005	77.44	+7.79	93.03	—7.80
1882 II (A)	69.59	0.0001	0.0000	+0.0001	60.79	+8.80	79.23	—9.64
1963 V	85.87	0.0006	0.0000	+0.0008	78.56	+7.31	93.51	—7.64
1965f	69.03	0.0001	0.0000	0.0000	60.22	+8.81	78.68	—9.65

The residuals of the C_i criteria can essentially be explained in still another way, assuming a secular change of the perihelion distance. Accepting an analogous method to that described above, we find the following condition for q, looking for the quasi-encounter distance:

$$\tan v_0 = \frac{x_0 Q_x + y_0 Q_y + z_0 Q_z}{x_0 P_x + y_0 P_y + z_0 P_z}, \tag{45}$$

where

$$\cos \frac{v_0}{2} = \sqrt{\left(\frac{q}{r_0}\right)}. \tag{46}$$

Applying (45) to the sun-grazing comets, we ascertain perihelion distances of the order of 10^{-5} to 10^{-6} AU, which is absurd. This interpretation should therefore be rejected.

This result and the data of Tables 12 and 13 show that the ascertained spot of the catastrophe, given by (39) lies very close to the major axes of all the five sun-grazing orbits.

Since we do not know at what time the hypothetical catastrophe occurred, and since only approximate data on the periods of revolution are available for the sun-grazing comets, it is not possible to derive the required rate of the apsidal motion, $\dot{\omega}$. Since any strongly suspected comet with its perihelion distance equal to or greater than some 0.05 AU does not show any substantial apsidal motion, analogous to that just mentioned, the above effect should be connected with the sun-grazing character of orbit. Apsidal motions, produced by various known mechanisms, will roughly be compared with the tabulated data in Section 17.

15. Velocity bodies

To conceive the space distribution of the velocities of separation, impressed to the fragments of the hypothetical protocomet, or protocomets, the velocity bodies are represented in Fig. 11 for the suspicious comets at $r_0 = 1.2$ AU. The dispersion in space positions of the comets is apparent from Fig. 12. The group of sun-grazing comets is

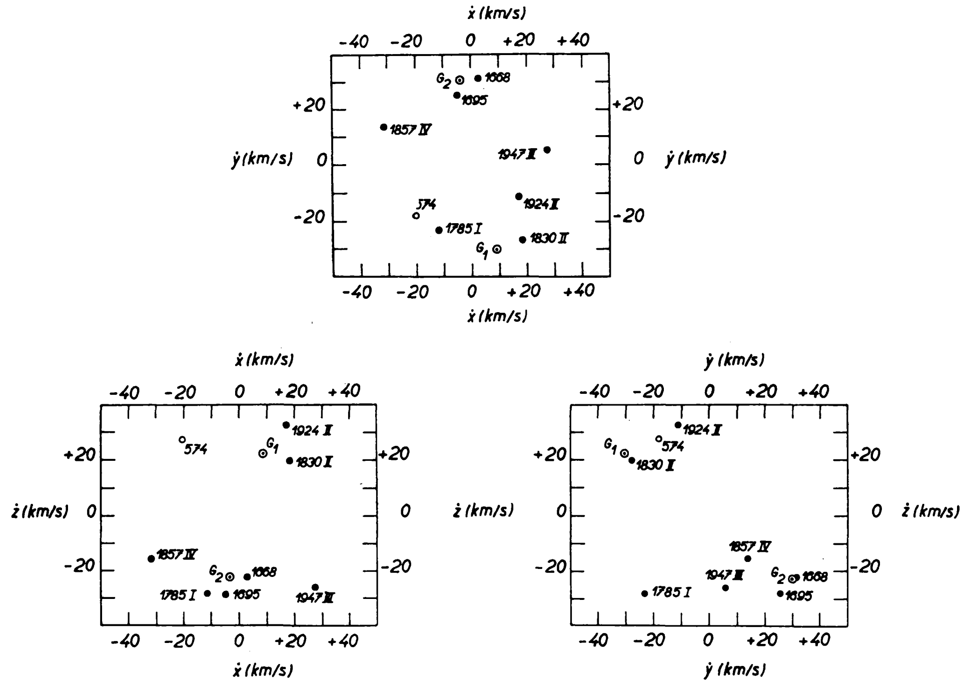

Fig. 11. *Velocity distribution of the debris at a 1.2 AU solar distance. Pre- and post-perihelion velocity vectors of the sun-grazing comets are designed by* G_1 *and* G_2*, respectively.*

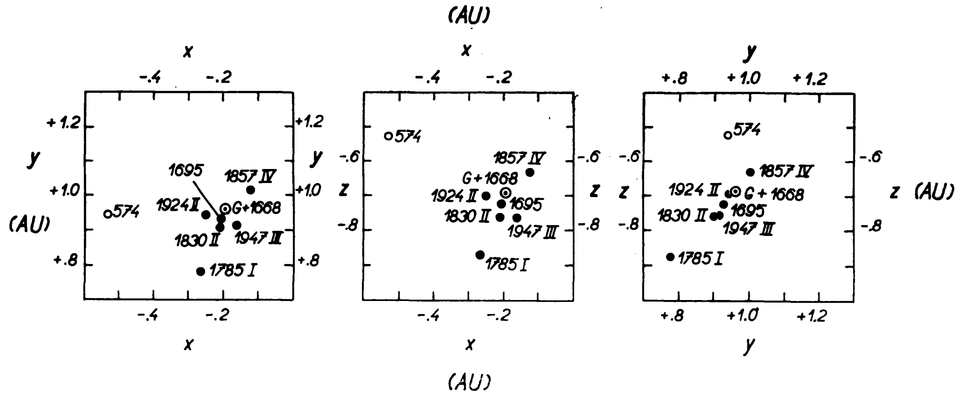

Fig. 12. *Dispersion of the debris in space near to 1.2 a AU solar distance. Position of the sun-grazing comets is designed by* G.

plotted assuming that the catastrophe was located on their pre- or post-perihelion arcs of orbit, respectively. Comet 574 is also included, because its orbit is very uncertain, and the uncertainty may just be responsible for its deviations from the other comets in Fig. 12. The velocity distributions indicate that the vectors of velocity of the investigated comets are essentially concentrated in two separate regions, which are roughly identical with the pre- and post-perihelion velocity vectors of the sun-grazing comets, respectively. The centroids of the two regions correspond to two velocity vectors nearly opposite in direction to one another. The dispersion in the plane of ecliptic is obviously larger than that in the planes perpendicular to the fundamental plane.

The results can be interpreted from the point of view of a collisional hypothesis. The two regions of the velocity space could be attributed to velocity vectors of fragments of two colliding protobodies. To show a possibility of analyzing such velocity distributions in the future, when more data are available on the group, let us consider the following model of the collision. Based on the data available at present, the consideration should be understood as nothing but an example.

There is a suggestion (Section 11) that the sun-grazing comets met the cosmic catastrophe rather on their pre-perihelion arc of orbit. Let us make use of this suggestion and assume that one of the two colliding bodies was the proposed protocomet, hereinafter denoted as Body I, moving in an orbit very close to that of 1882 II. This assumption is based on a high intrinsic luminosity of 1882 II, and on a mass-luminosity relation for comets, according to which the mass of a comet, $\mathfrak{M}$, is in the first approximation an exponential function of its luminosity, expressed by its absolute magnitude, H_0:

$$\mathfrak{M} = 10^{19} \exp(-0.92\, H_0) \qquad [\text{gm}]$$

(ALLEN 1955). The computations show that keeping this relation, the other sun-grazing comets together with the remaining comets in the respective region in Fig. 11 (except Comet 574) represent only about 5 per cent of the mass of 1882 II.

The post-collision orbit of the second colliding body, hereinafter denoted as Body II, can be computed from the concentration of comets 1668, 1695, 1785 I, 1857 IV and 1947 III in Fig. 11. The centre of gravity of these comets is given by the following ecliptic co-ordinates and velocity components (Fig. 11 and 12):

$$(47)\quad \left\{\begin{aligned} x_2 &= -0.2074 \text{ AU},\\ y_2 &= +0.9284 \text{ AU},\\ z_2 &= -0.7268 \text{ AU},\\ \dot{x}_2 &= -\ \ 2.93 \text{ km/s},\\ \dot{y}_2 &= +22.20 \text{ km/s},\\ \dot{z}_2 &= -25.54 \text{ km/s}.\end{aligned}\right.$$

The heliocentric velocity of the centre of gravity is $V_2 = 33.97$ km/s at a solar distance of 1.197 AU. For Comet 1882 II approaching the Sun, we obtain, applying its corrected position of the apsidal line from Table 13:

$$(48)\quad \left\{\begin{aligned} \dot{x}_1 &= +\ \ 9.36 \text{ km/s},\\ \dot{y}_1 &= -30.33 \text{ km/s},\\ \dot{z}_1 &= +21.43 \text{ km/s},\\ V_1 &= \ \ 38.29 \text{ km/s}.\end{aligned}\right.$$

The total mass of the group of the five comets, 1668 to 1947 III, amounts to about 2 per cent of 1882 II. Assuming that the mass of Body II was essentially of the same order, we can conclude that — in the first approximation — the velocity of Body I did not change appreciably during its collision with Body II.

The velocity with which the centre of gravity of disintegrated Body II was moving after the collision relative to Body I was very high:

$$
(49) \quad \left\{
\begin{aligned}
\dot{x}_2 - \dot{x}_1 &= -12.29 \text{ km/s},\\
\dot{y}_2 - \dot{y}_1 &= +52.53 \text{ km/s},\\
\dot{z}_2 - \dot{z}_1 &= -46.97 \text{ km/s},\\
\Delta V &= 71.53 \text{ km/s}.
\end{aligned}
\right.
$$

The centre of gravity of the disintegrated Body II was after the collision moving round the Sun in a short-period orbit as follows.

Post-collision orbit of the centre of gravity of disintegrated Body II

$$
(50) \quad \left\{
\begin{aligned}
T &= \text{ 41 days prior to the collision}\\
\omega &= 57^\circ.2 \\
\Omega &= 292^\circ.7 \quad \text{1950.0}\\
i &= 103^\circ.0 \\
q &= 0.0404 \text{ AU}\\
e &= 0.985\\
P &= 4.5 \text{ years}
\end{aligned}
\right.
$$

Assuming that Body II was originally moving in a nearly parabolic orbit, the short-period character of its post-collision orbit can be interpreted as a consequence of a loss of its kinetic energy spent for its disruption into a number of pieces. The topic of interest is then: What was the orbit of Body II prior to the catastrophe in the proposed model collision process, and to what degree was the process elastic, i.e. what was the coefficient of elasticity of the two colliding bodies? The problems are discussed on the basis of elementary collisional dynamics in the following section.

16. Application of elementary collision dynamics to the hypothetical catastrophe

As known, the analysis of a collision process under general conditions is extremely complicated. The character of the collision depends on the following circumstances:

1. On the mass distribution between the colliding bodies.
2. On the relative motion of the two bodies just before colliding.
3. On the physical, mainly elastic properties of the bodies.
4. On the character of their surfaces.

Two simple examples of a centric collision of semi-elastic bodies are discussed. The collision is characterized by a coefficient of elasticity, k, giving what part of the kinetic energy of the colliding bodies has been lost for their permanent deformations. In accordance with observations, we will, moreover, assume that the forces producing the deformations are equal to or in excess of forces necessary for the crystal structure of the colliding bodies to be broken up. Hence, the disruption of the two bodies into fragments is then a consequence of the collision process.

Let us assume that two bodies, I and II, of masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$ moving round the Sun, collide at a point of intersection of their orbits. Just before colliding the heliocentric velocity of Body I is in the ecliptic system of co-ordinates $\mathbf{U}_1 = (\dot{u}_1, \dot{v}_1, \dot{w}_1)$, of Body II $\mathbf{U}_2 = (\dot{u}_2, \dot{v}_2, \dot{w}_2)$. The velocity of Body II relative to Body I therefore is equal to $\Delta\mathbf{U} = (\dot{u}_2 - \dot{u}_1, \dot{v}_2 - \dot{v}_1, \dot{w}_2 - \dot{w}_1)$. The corresponding velocities just following the collision are $\mathbf{V}_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)$, $i = 1, 2$, and $\Delta\mathbf{V} = (\dot{x}_2 - \dot{x}_1, \dot{y}_2 - \dot{y}_1, \dot{z}_2 - \dot{z}_1)$, respectively. Finally, the changes in the velocities of Bodies I and II due to collision are $\mathbf{W}_i = (\dot{x}_i - \dot{u}_i, \dot{y}_i - \dot{v}_i, \dot{z}_i - \dot{w}_i)$.

The assumption of a head-on centric collision makes it possible to simplify the co-ordinate system as follows: $\mathbf{U}_i = \dot{u}_i, \mathbf{V}_i = \dot{x}_i, \Delta\mathbf{U} = \dot{u}_2 - \dot{u}_1, \Delta\mathbf{V} = \dot{x}_2 - \dot{x}_1, \mathbf{W}_i = \dot{x}_i - \dot{u}_i$. The working formulae of the collision process are in this case:

$$\mathbf{W}_i = (-1)^{i+1} \frac{1+k}{1 + \dfrac{\mathfrak{M}_i}{\mathfrak{M}_j}} \Delta\mathbf{U}\,, \tag{51}$$

$i=1, 2$ and $j = 3 - i$, giving the changes in velocities of both bodies, and

$$\Delta E = \frac{1}{2}(1-k^2)\frac{\mathfrak{M}_1\mathfrak{M}_2}{\mathfrak{M}_1 + \mathfrak{M}_2}\,|\Delta\mathbf{U}\,|^2 = \frac{1}{2}\,\frac{1-k}{1+k}\,\mathfrak{M}_i\left(1 + \frac{\mathfrak{M}_i}{\mathfrak{M}_j}\right)|\,\mathbf{W}_i\,|^2\,, \tag{52}$$

giving the loss of kinetic energy of the colliding bodies spent for their deformations and the consequences. Assuming that the energy density, $\varepsilon^\star$, necessary for breaking up the crystal lattice of the matter, is of the same value for both colliding bodies, e.g. if they are represented by two comets of similar physical constitutions, the loss of the energy density,

$$\Delta\varepsilon = \frac{1}{2}(1-k^2)\frac{\mathfrak{M}_1\mathfrak{M}_2}{(\mathfrak{M}_1 + \mathfrak{M}_2)^2}\,|\Delta\mathbf{U}\,|^2 = \frac{1}{2}\,\frac{1-k}{1+k}\,\frac{\mathfrak{M}_i}{\mathfrak{M}_j}\,|\,\mathbf{W}_i\,|^2\,, \tag{53}$$

should be in excess of $\varepsilon^\star$. This fact gives the following condition for the coefficient of elasticity:

$$k < \frac{\dfrac{\mathfrak{M}_i}{\mathfrak{M}_j}\,|\,\mathbf{W}_i\,|^2 - 2\varepsilon^\star}{\dfrac{\mathfrak{M}_i}{\mathfrak{M}_j}\,|\,\mathbf{W}_i\,|^2 + 2\varepsilon^\star}\,, \tag{54}$$

or

$$k < \left(1 - \frac{(\mathfrak{M}_1 + \mathfrak{M}_2)^2}{\mathfrak{M}_1\mathfrak{M}_2}\cdot\frac{2\varepsilon^\star}{|\,\Delta\mathbf{U}\,|^2}\right)^{\frac{1}{2}}, \tag{55}$$

and, since $k > 0$ (for semi-elastic bodies), we find two conditions for the change of velocities:

$$|\,\mathbf{W}_i\,| > \left(\frac{2\mathfrak{M}_j}{\mathfrak{M}_i}\,\varepsilon^\star\right)^{\frac{1}{2}} \tag{56}$$

and a condition for the pre-collision relative velocity of the two bodies:

$$|\,\Delta\mathbf{U}\,| > \left(1 + \frac{\mathfrak{M}_2}{\mathfrak{M}_1}\right)\left(\frac{2\varepsilon^\star}{\mathfrak{M}_2/\mathfrak{M}_1}\right)^{\frac{1}{2}}. \tag{57}$$

Table 14. Necessary conditions for disruption of semi-elastic colliding bodies (head-on collision)

$\frac{\mathfrak{M}_2}{\mathfrak{M}_1}$	$\lvert \Delta \mathbf{U} \rvert_{min}$	$\lvert \mathbf{W}_1 \rvert_{min}$	$\lvert \mathbf{W}_2 \rvert_{min}$
	km/s	km/s	km/s
0.001	63	0.06	63
0.01	20	0.20	20
0.1	7.0	0.63	6.3
0.2	5.4	0.89	4.5
0.5	4.2	1.4	2.8
1.	4.0	2.0	2.0

After STANYUKOVICH (1960) $\varepsilon^* \approx 2 \, . \, 10^{10}$ erg/gm. For this value and selected $\mathfrak{M}_2/\mathfrak{M}_1$ the minimum $|\mathbf{W}_1|$ and $|\Delta \mathbf{U}|$ are listed in Table 14. Current cosmic velocities of the order of a few tens kilometres per second are, hence, sufficient for a disintegration of the colliding bodies.

Applying this collision model to the numerical example solved at the end of the foregoing section, we would arrive at the conclusion that before colliding Body II should have been moving round the Sun in a strongly hyperbolic orbit, unlikely in fact. Besides, the probability of a head-on centric collision is generally very low.

We can attack this problem permitting of a side collision process. Since the mass of Body II is accepted small enough to make no appreciable effect on the post-collision motion of Body I, the problem will be solved as a side collision of a sphere (Body II) with a fixed plane (Body I). Excluding effects of coarse surfaces of the two bodies, the tangential post-collision velocity component of Body II does not change, while its normal component is reduced the more the less is the coefficient of elasticity. Body II is assumed (Section 15) moving in a parabolic orbit before colliding Body I, the elements of which could approximately be determined. Fig. 13 gives a picture of the first part of the problem to be settled: to derive the components of $\mathbf{U}_2$ and the coeffiicient of elasticity, when $\mathbf{U}_1 \equiv \mathbf{V}_1$, $\mathbf{V}_2$ and $\Delta \mathbf{V}$ are known vectors, and

$$| \mathbf{U}_2 |^2 = \frac{2K^2}{r},$$

K is the constant of gravitation. The problem is easily solvable in the plane, given by vectors $\mathbf{U}_1$ and $\mathbf{V}_2$. Let us choose a co-ordinate system in this plane with the $+\xi$-axis

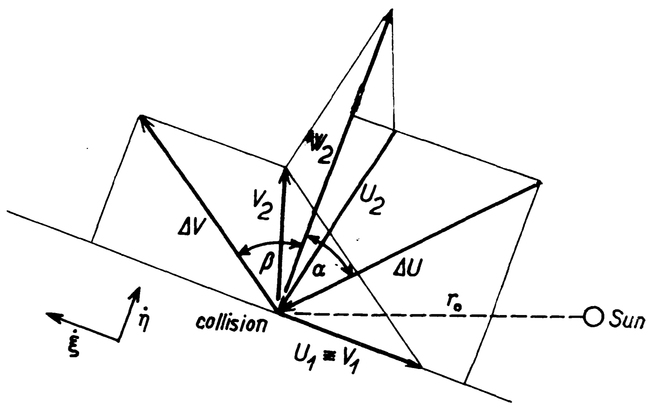

Fig. 13. *Scheme of the assumed side collision.*

identical with the direction of $-\mathbf{U}_1$ and the $\dot{\eta}$-axis perpendicular to the former, positive relative to vector $\Delta\mathbf{V}$. Vectors $\mathbf{U}_1$, $\Delta\mathbf{U}$ and $\Delta\mathbf{V}$ will have the following components in this system:

$$(58)\qquad \begin{cases} \mathbf{U}_1 = (-|\mathbf{U}_1|\,,\ 0) \\ \Delta\mathbf{U} = (|\Delta\mathbf{V}|\ \sin\beta\,,\quad -|\Delta\mathbf{V}|\sin\beta\ \mathrm{ctg}\,\alpha) \\ \Delta\mathbf{V} = (|\Delta\mathbf{V}|\sin\beta\,,\quad |\Delta\mathbf{V}|\cos\beta)\,, \end{cases}$$

where

$$(59)\qquad \sin\beta = -\frac{\dot{u}_1(\dot{x}_2-\dot{x}_1)+\dot{v}_1(\dot{y}_2-\dot{y}_1)+\dot{w}_1(\dot{z}_2-\dot{z}_1)}{|\mathbf{U}_1|\,.\,|\Delta\mathbf{V}|}\,,$$

and

$$(60)\qquad \sin\alpha = \frac{|\Delta\mathbf{V}|\sin\beta}{(|\mathbf{U}_2|^2-|\mathbf{U}_1|^2+2|\mathbf{U}_1|\,.\,|\Delta\mathbf{V}|\sin\beta)^{\frac{1}{2}}}\,.$$

The velocity components $\dot{\xi}$ and $\dot{\eta}$ are converted to the ecliptic components, $\dot{x}$, $\dot{y}$, $\dot{z}$, in the following way:

$$(61)\qquad \begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix} = \begin{pmatrix} -0.24453 & +0.52625 \\ +0.79190 & -0.37628 \\ -0.55954 & -0.76253 \end{pmatrix} \begin{pmatrix}\dot{\xi}\\ \dot{\eta}\end{pmatrix}.$$

Since in the ecliptic system:

$$\mathbf{U}_1 \equiv \mathbf{V}_1 = (+\ 9.36;\ -30.33;\ +21.43)\,,$$
$$\mathbf{V}_2 = (-\ 2.93;\ +22.20;\ -25.54)\,,$$
$$\Delta\mathbf{V} = (-12.29;\ +52.53;\ -46.97)\,,$$

and in the plane system

$$\Delta\mathbf{V} = (+70.88;\ +9.58)$$

(all in km/s), and since:

$$|\mathbf{U}_1| \equiv |\mathbf{V}_1| = 38.29\ \text{km/s}\,,$$
$$|\mathbf{U}_2| = 38.48\ \text{km/s}\,,$$
$$|\mathbf{V}_2| = 33.97\ \text{km/s}\,,$$
$$|\Delta\mathbf{V}| = 71.53\ \text{km/s}\,,$$
$$\sin\beta = +0.9910\,,$$
$$\sin\alpha = +0.9608\,,$$

we find successively:

$$\Delta\mathbf{U} = (+70.88;\ -20.46)\,,$$
$$\Delta\mathbf{U} = (-28.10;\ +63.83;\ -24.06)\,,$$
$$\mathbf{U}_2 = (-18.74;\ +33.50;\ -\ 2.63)\,,$$
$$\mathbf{W}_2 = (+15.81;\ -11.30;\ -22.91)\,,$$

and

$$(62)\qquad \begin{cases} |\Delta\mathbf{U}| = 73.77\ \text{km/s}\,, \\ |\mathbf{W}_2| = 30.04\ \text{km/s}\,, \\ k = 0.47\,. \end{cases}$$

The pre-collision orbital elements of Body II then result as follows.

Table 15. Distribution of motions of five observed comets as debris of the disrupted hypothetical Body II

Comet	$\mathbf{V}_{\underset{\circ}{\Psi}} - \mathbf{U}_2$					$\mathbf{V}_{\underset{\circ}{\Psi}} - \mathbf{V}_2$					
	$\dot{x} - \dot{u}_2$	$\dot{y} - \dot{v}_2$	$\dot{z} - \dot{w}_2$	$\lvert\mathbf{V}_{\underset{\circ}{\Psi}} - \mathbf{U}_2\rvert$	λ	$\dot{x} - \dot{x}_2$	$\dot{y} - \dot{y}_2$	$\dot{z} - \dot{z}_2$	$\lvert\mathbf{V}_{\underset{\circ}{\Psi}} - \mathbf{V}_2\rvert$	λ'	λ''
	km/s	km/s	km/s	km/s	°	km/s	km/s	km/s	km/s	°	°
1668	+21.5	− 2.3	−19.6	29.2	44.6	+ 5.7	+ 9.0	+3.3	11.1	16.3	74.8
1695 (A)	+13.7	− 8.4	−26.0	30.6	46.9	− 2.1	+ 2.9	−3.1	4.7	2.7	21.3
1785 I	+ 7.1	−56.6	−25.8	62.6	109.0	− 8.7	− 0.9	−2.9	9.2	79.1	75.3
1857 IV	−13.1	−19.7	−13.3	27.1	41.5	−28.9	− 8.4	+9.6	31.6	51.6	109.0
1947 III	+45.3	−27.8	−23.5	58.2	100.1	+30.5	−16.5	−0.6	34.7	56.9	111.9

Pre-collision orbit of Body II

$$(63) \quad \begin{cases} T = 49 \text{ days prior to the collision} \\ \omega = 212^\circ.5 \\ \Omega = 120^\circ.8 \quad 1950.0 \\ i = 67^\circ.7 \\ q = 0.429 \text{ AU} \end{cases}$$

Comparing the above elements with the post-collision elements of (50), we find that the orbit of Body II (or, of its centre of gravity) changed drastically. The originally direct sense of motion was replaced by a retrograde sense, the new orientation of the apsidal line has nothing common with the earlier, and the perihelion distance was reduced by nearly 0.4 AU. The centre of gravity of Body II changed its velocity by 30 km per second, and post-collision direction of motion deviated from its pre-collision direction by 23°.8 in the system moving with Body I, and by 48°.5 if referred to the Sun.

Details on the motion of each of the five comets — fragments of Body II — are for the adopted collision model given in Table 15. The respective columns include: the velocity of the comet relative to the pre-collision motion of Body II, i.e. its velocity of separation in a wider sense of the term; its velocity relative to the post-collision motion of the centre of gravity of the disrupted Body II, i.e. its velocity of separation proper; the deviations of the comet's motion from the pre- and post-collision directions of motion of the parent body, λ and λ', respectively; and the deviation of the separation velocity proper from the direction of motion of the centre of gravity, λ''. The angles are defined by:

$$(64) \qquad \cos\lambda = \frac{(\mathbf{V}_{\underset{\circ}{\Psi}}, \mathbf{U}_2)}{\lvert\mathbf{V}_{\underset{\circ}{\Psi}}\rvert \,.\, \lvert\mathbf{U}_2\rvert},$$

$$(65) \qquad \cos\lambda' = \frac{(\mathbf{V}_{\underset{\circ}{\Psi}}, \mathbf{V}_2)}{\lvert\mathbf{V}_{\underset{\circ}{\Psi}}\rvert \,.\, \lvert\mathbf{V}_2\rvert},$$

$$(66) \qquad \cos\lambda'' = \frac{(\mathbf{V}_{\underset{\circ}{\Psi}} - \mathbf{V}_2, \mathbf{V}_2)}{\lvert\mathbf{V}_{\underset{\circ}{\Psi}} - \mathbf{V}_2\rvert \,.\, \lvert\mathbf{V}_2\rvert}.$$

A schematic picture of the described collision model is represented in Fig. 14. In a projection on the plane, given by the velocity vectors of Bodies I and II at the time of collision, it represents (i) heliocentric trajectories of the centres of gravity of the two bodies near to the point of collision; (ii) the two bodies at the moment of collision (dashed contours); (iii) the two bodies a while after colliding and their fragmentation (full contours); (iv) the velocities of separation of the individual fragments of Body II relative to their centre of gravity (+ means: directed above the plane, — below the plane).

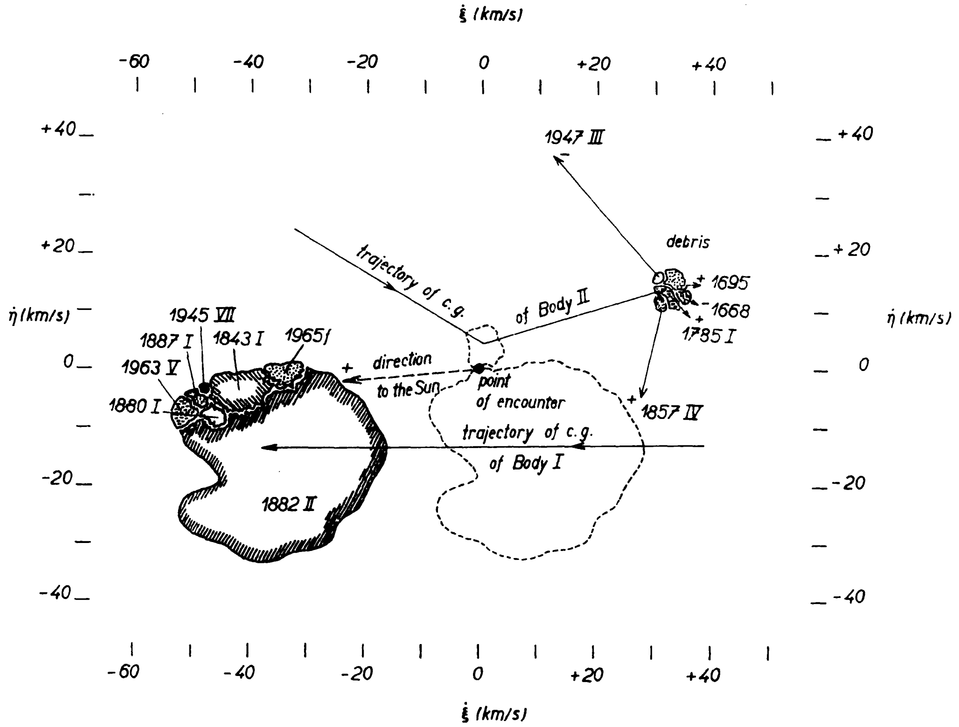

Fig. 14. *Orientation picture of the model collision.*

Qualitatively, we can on the basis of Fig. 14 conceive that due to their relative dimensions Body II was completely destroyed, while Body I was damaged only in its region adjacent to the point of encounter of the two bodies' surfaces. For the same reason, the velocities of separation of the Body II fragments were essentially higher and random in direction, and their resulting orbits accordingly differ from one another more conspicuously than those of Body I, forming a closed system of evidently similar orbits.

The main conclusions from Sections 15 and 16 are summarized as follows:

1. The study of the velocity distribution of the group od comets suspected of being evolutionarily related suggests that the comets observed can be interpreted as remnants of two colliding bodies, not of a single of them.
2. A model of collision is proposed, assuming that the two colliding bodies were incomparable in mass. The huge Body I is suggested to have been a parent body for the present sun-grazing comets, the less massive Body II that for a few distant comets.

3. The sun-grazing comets are included after corrected for their apsidal motion required. If taken uncorrected, they would be in Fig. 11 displaced by slightly more than 5 km/s, and in Fig. 12 by less than 0.2 AU.
4. On some simplifying assumptions, the coefficient of elasticity of the model collision was found close to 0.5. An approximate pre-collision parabolic orbit of Body II was established.
5. Comet 1668, generally assumed as a very probable member of the Kreutz group, may be a product of the other colliding protobody, and need not, hence, be evolutionarily related to the five sun-grazing comets. The same is true about the other comets suspected of belonging to the "generalized" Kreutz family.
6. The proposed collision model is sketched in Fig. 14, from which a rough concept of the process assumed is apparent: a complete destruction of Body II, and a partial decay of the vast Body I.

17. Secular disturbing effects on Keplerian sun-grazing orbits

The collision hypothesis proposed and discussed in the preceding sections is generally promising. It essentially met the only obstacle, consisting in a considerable advance or regression of perihelion of the sun-grazing comets, required to ensure a common point of encounter of the above comets with several other comets of distant-perihelion orbits, the latter having met one another in the same point.

Three effects are considered, which could produce secular motion of the orbital elements of the sun-grazing comets near their perihelion passage: the consequence of the general theory of relativity, a resisting medium (denser parts of the solar corona), and the oblateness of the Sun. Dynamical effects connected with splitting and disintegration processes, observed in the sun-grazing comets, are not considered, because they possibly produce orbital changes of more or less random character; actually, e.g. the splitting of the primary nucleus of Comet Ikeya-Seki yielded a regression of perihelion, while that of 1882 II an advance of perihelion. The resulting effect depends, moreover, on the details of the physical process, and no prediction is possible.

The general relativity gives the following expression for the advance of perihelion:

$$\dot{\omega}_{\text{rev}} = 8''.25 \, \frac{R_\odot}{q} \, (1 + e)^{-1} \text{ per revolution,} \tag{67}$$

$R_\odot$ is the equatorial radius of the Sun. For nearly parabolic orbits with $q = 0.0051$ and 0.0078 AU formula (67) gives for $\dot{\omega}_{\text{rev}}$ only 3.7″ and 2.5″ per revolution, respectively. The ascertained change of 8° requires a total of about 10^4 revolutions round the Sun since the time of the cosmic catastrophe, to be consistent with the Einstein effect. The catastrophe should have occurred on pre-perihelion arcs of the sun-grazing orbits some five to ten million years ago. It is, however, difficult to conceive the interior structure of the nuclei of sun-grazing comets, surviving 10^4 revolutions about the Sun and keeping their absolute luminosities as high as 6^m or even 0^m!

The effect of resisting medium, represented by the solar corona, gives no appreciable changes in the comet's angular elements. For Ω and i it results from a zero component of force perpendicular to the plane of orbit, for the rotation of perihelion the theory gives:

$$\dot{\omega}_{\text{rev}} = -\frac{3 \,.\, c_X \,.\, m}{4 \,.\, R_{\text{☄}}\, \varrho_{\text{☄}}} \int_{-\pi}^{\pi} N \,.\, r \,.\, \sin \frac{v}{2} \, \mathrm{d}v = 0\,, \tag{68}$$

assuming that the space concentration of solar particles in the corona, N, depends only on the solar distance, r. In (68) m is the average mass of a solar particle, $R_{\text{☄}}$ and $\varrho_{\text{☄}}$ the radius and mass density of the cometary nucleus, respectively, and c_X the coefficient of aerodynamical resistence.

The resisting medium, however, affects the comet's orbital energy. The theory gives analogously to (68):

$$\left(\frac{\dot{1}}{a}\right)_{\text{rev}} = \frac{3 \,.\, c_X \,.\, m}{R_{\text{☄}}\, \varrho_{\text{☄}}} \int_{q}^{\infty} \frac{N}{\sqrt{(r(r-q))}} \, \mathrm{d}r\,. \tag{69}$$

To estimate this effect, let us assume the space density distribution in the form:

$$N = N_\odot \left(\frac{R_\odot}{r}\right)^n, \tag{70}$$

where $N_\odot$ is the space concentration of the corona particles near to the surface of the photosphere. Solving (69),

$$\left(\frac{\dot{1}}{a}\right)_{\text{rev}} = \frac{3 \,.\, c_X \,.\, m \,.\, N_\odot}{R_{\text{☄}}\, \varrho_{\text{☄}}} \left(\frac{2R_\odot}{q}\right)^n \frac{(n-1)!}{(2n-1)!!}\,. \tag{71}$$

Adopting $c_X = \frac{2}{3}$, $mN_\odot = 10^{-15}$ g/cm^3, $R_{\text{☄}} = 3$ km, $\varrho_{\text{☄}} = 1$ gm/cm^3, we find:

$$\left(\frac{\dot{1}}{a}\right)_{\text{rev}} = 10^{-7}\, F(q, n)\, (\text{AU})^{-1} \text{ per revolution,} \tag{72}$$

where

$$F(q, n) = \left(\frac{2R_\odot}{q}\right)^n \frac{(n-1)!}{(2n-1)!!}\,. \tag{73}$$

The change in $(1/a)$ produced by the resisting effect of the solar corona can never be in excess of the order of 10^{-7} (AU)$^{-1}$, and is, hence, undetectable. We conclude that the resisting medium produces no essential effect on the character of motion of the sun-grazing comets. Actually, the perihelion distance — the last orbital element — of a sun-grazing nearly-parabolic orbit is lowered due to the resistence in the solar corona by:

$$\dot{q}_{\text{rev}} = -\frac{3 \,.\, c_X \,.\, m \,.\, q}{2 \,.\, R_{\text{☄}}\, \varrho_{\text{☄}}} \int_{q}^{\infty} \frac{N \sqrt{r}}{\sqrt{(r-q)}} \, \mathrm{d}r\,. \tag{74}$$

Substituting (70), we obtain

$$\dot{q}_{\text{rev}} = -\frac{3 \,.\, c_X \,.\, m \,.\, N_\odot}{2\varrho_{\text{☄}}} \,.\, \frac{R_\odot}{R_{\text{☄}}} \,.\, q \,.\, F(q, n-1)\,. \tag{75}$$

Inserting numerical data, we finally find that the drop in the perihelion distance can never be in excess of 10^{-11} AU, or 1 metre, per revolution!

As known, no oblateness of the Sun has optically been found so far. Some authors, however, stress that because of turbulence in the Earth's atmosphere the contours of the solar disc are never perfectly sharp and deviations from a sphere up to 0".1 are undetectable in fact (DICKE 1964). Moreover, spherical contours of the photosphere's surface do not anyway guarantee sphericity of interior, more massive layers of the Sun, which would mainly be responsible for any dynamical effect on the orbits. Various phenomena observed on the Sun, representing what is generally called the solar activity, are concentrated more or less to the Sun's equator and suggest that the interior structure of the Sun is strongly dependent on the heliographic latitude. Since nothing can directly be said about the form of the layers of equal mass density in the Sun's interior, the problem of the effect of the possible solar oblateness on the sun-grazing orbits remains an open question.

Let us analyze the distributions of orbital elements of the sun-grazing comets from the point of view of the oblate Sun. Since the Sun's equator inclines by more than 7° to the ecliptic, it is either possible to convert the formulae for the changes in the „heliographic" elements (with the Sun's equator as the fundamental plane) of comet orbits to those for the corresponding changes in the ecliptical elements, or to convert the ecliptical elements themselves to the heliographic elements and to solve the problem in the heliographic system of co-ordinates. The latter way is here followed. The inclination of the Sun's equator to the ecliptic, $i_{\odot}$, and its ascending node, $\Omega_{\odot}$, on the ecliptic have been adopted as follows:

$$\left.\begin{aligned} i_{\odot} &= 7°15' \\ \Omega_{\odot} &= 75°04' \end{aligned}\right\} \quad 1950.0 \tag{76}$$

and the heliographic orbital elements, ω', Ω', i', have been computed from the respective

Table 16. List of orbital elements of sun-grazing and some other comets, referred to the solar equator

Comet	Equinox 1950.0		
	ω'	Ω'	i'
	°	°	°
1843 I	93.79	296.68	141.60
1880 I (A)	96.14	299.55	141.44
1880 I (B)	97.06	300.67	141.43
1882 II (A)	81.20	281.00	141.19
1882 II (B)	81.18	281.01	141.19
1963 V	96.67	300.45	141.33
1965f	80.63	280.27	141.13
1872	68.74	337.37	141.85
1887 I (A)	76.04	273.29	138.06
1887 I (B)	67.35	256.18	130.56
1893 (ecl.)	264.71	125.55	121.88
1945 VII	61.19	254.29	139.45
1668	121.00	296.40	141.66

ecliptical elements according to:

$$(77) \quad \begin{cases} \operatorname{ctg}(\omega' - \omega) = \operatorname{ctg}(\Omega - \Omega_\odot)\cos i - \operatorname{cosec}(\Omega - \Omega_\odot)\sin i \operatorname{ctg} i_\odot\,, \\ \operatorname{ctg}\Omega' = \operatorname{ctg}(\Omega - \Omega_\odot)\cos i_\odot - \operatorname{cosec}(\Omega - \Omega_\odot)\operatorname{ctg} i \sin i_\odot\,, \\ \cos i' = \cos(\Omega - \Omega_\odot)\sin i \sin i_\odot + \cos i \cos i_\odot\,, \end{cases}$$

regarding the condition

$$(78) \qquad \operatorname{sign}[\sin(\Omega - \Omega_\odot)] = \operatorname{sign}[\sin \Omega'] = -\operatorname{sign}[\sin(\omega' - \omega)]\,.$$

The heliographic orbital elements for the comets from Table 1 and Table 2 and for the ancient comet of 1668 are listed in Table 16.

Oblateness of the central body produces motions of both apsidal and node lines. The theory gives the following formulae in the heliographic system of co-ordinates:

$$(79) \quad \begin{cases} \dot{\omega}'_{\mathrm{rev}} = \dfrac{1}{2}\varkappa(5\cos^2 i' - 1)\,, \\ \dot{\Omega}'_{\mathrm{rev}} = -\varkappa \cos i'\,. \end{cases}$$

If expressed in degrees per revolution, it is

$$(80) \qquad \varkappa = 360° \left(\frac{R_\odot}{q}\right)^2 (1 + e)^{-2} (\alpha - 0.1 \,.\, 10^{-4})\,,$$

$R_\odot$ is the equatorial radius of the Sun, α its visual oblateness. Compared with the relativistic effect, the predicted deviations from the regular motion are now more sensitive to the perihelion distance. Dividing the two expressions of (79) we find the ratio of the two heliographic angular elements depending exclusively on the inclination of orbit to the Sun's equator:

$$(81) \qquad \frac{\dot{\omega}'_{\mathrm{rev}}}{\dot{\Omega}'_{\mathrm{rev}}} = -\frac{5\cos^2 i' - 1}{2\cos i'}\,.$$

The other elements experience periodic changes with time, but their secular changes due to the oblateness are negligible in the first approximation. In this connection it is of interest that the dispersion among the ecliptical inclinations of the indubitable members of about 2°.5 was reduced to less than 0°.5 when referred to the Sun's equator. Considering extremely elongated sun-grazing orbits, the orbital changes produced by oblateness can be attributed to a very short interval of time round the passage through perihelion, and in the second approximation we can write for the motion in inclination:

$$(82) \qquad \dot{i}_{\mathrm{rev}} = \frac{1}{270}\varkappa \,.\, e \,.\, |\tan i'| \,.\, \sin \omega' \,.\, \dot{\omega}'_{\mathrm{rev}}\,,$$

and similarly for the change in perihelion distance:

$$(83) \qquad \dot{q}_{\mathrm{rev}} = 3.23 \,.\, 10^{-5}\varkappa \,.\, q \,.\, e \,.\, \sin^2 i' \,.\, \sin \omega' \,.\, \dot{\omega}'_{\mathrm{rev}}\,.$$

For two comets, 1963V and 1965f the theoretical values of $\dot{\omega}'_{\mathrm{rev}}$, $\dot{\Omega}'_{\mathrm{rev}}$, $\dot{i}'_{\mathrm{rev}}$ and $\dot{q}_{\mathrm{rev}}$ are included in Table 17.

The argument of perihelion should be a conspicuously linear function of the ascending node, as required by (81). Fig. 15 shows that the orbits of the observed comets actually give a linear relation. For $i' \doteq 141°$, however, the ratio $\dot{\omega}'_{\mathrm{rev}}/\dot{\Omega}'_{\mathrm{rev}}$ should amount to + 1.30, while the observed value is only + 0.77.

Table 17. Changes in orbital elements of 1963 V and 1965f due to the oblate Sun

	1963 V			1965f		
	$\alpha = 5 . 10^{-5}$	$\alpha = 10^{-4}$	$\alpha = 5 . 10^{-4}$	$\alpha = 5 . 10^{-5}$	$\alpha = 10^{-4}$	$\alpha = 5 . 10^{-4}$
$\dot{\omega}'_{rev}$	$+10''.8$	$+24''.6$	$+135''.0$	$+4''.6$	$+10''.5$	$+57''.7$
$\dot{\Omega}'_{rev}$	$+\ 8''.3$	$+18''.8$	$+103''.2$	$+3''.6$	$+\ 8''.1$	$+44''.5$
$\dot{i}'_{rev}$	$+0''.0001$	$+0''.0005$	$+0''.0146$	$+0''.00002$	$+0''.0001$	$+0''.0027$
$\dot{q}_{rev}$	8 cm	44 cm	1310 cm	2 cm	12 cm	370 cm

Relation (82) is not satisfactorily represented by the observations. The orbits of the possible members of the Kreutz group are absolutely inconsistent with (82); the orbits of the indubitable members give an absurd value of $\alpha = 0.022 \pm 0.008$, with a low degree of correlation. The group of three comets with $q > 0.006$ AU yield a negative α. Neither (83) comply with the observed distribution of the orbits.

Generally I conclude that there is no decisive effect of the oblateness of the Sun, if any at all, on the orbital elements of the sun-grazing comets during a period, covering less than, say, some 10^2 revolutions round the Sun. Taken by itself, the oblateness cannot explain the observed distributions of the orbital elements of the above comets. Because of the exceptional elongation of the sun-grazing orbits the above effect should occur, in practice, on a very short arc of orbit near to the very passage through perihelion. But no effect like this was observed at Comets 1882 II an 1965f, for which both pre- and perihelion orbits are known. Kreutz found differences $+ 9''$ and $+ 79''$ between the respective post-perihelion orbits of the two main secondary nuclei and the pre-perihelion orbit of the primary nucleus of 1882 II in the ecliptical argument of perihelion, and $- 54''$ and $+ 4''$ in the ecliptical ascending node. The writer found for Comet 1965f $- 190''$ and $- 240''$ in ω, and $- 110''$ and $- 180''$ in Ω from the orbits of nuclei A and B, respectively. Non-gravitational effects connected with the splits of the two comets were obviously dominant among the agents, producing the observed orbital changes.

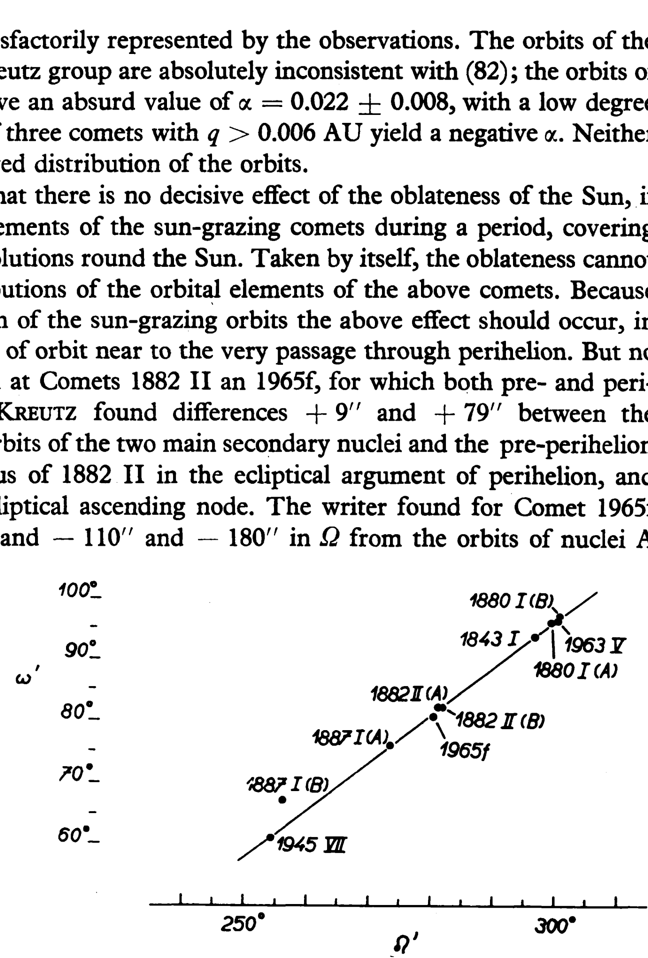

Fig. 15. *Argument of perihelion as a function of the node in the heliographic system of co-ordinates.*

With respect to the main problem of the present paper, the study of the possible factors, which could change the form or space orientation of sun-grazing orbits, leads to the following conclusions:

1. The resistance of the solar corona produces no effect on the angular elements of sun grazing orbits, and completely negligible changes in the comet orbital energy and perihelion distance.

2. The relativistic rotation of the apsidal line of the sun-grazing comets is of the order of a few seconds of arc per revolution.
3. A hypothetical oblateness of the Sun would produce rotations of apsidal and node lines (referred to the plane of the Sun's equator) with rates equal in order. The observed changes in the motion of Comets 1882 II and 1965f confirm no effect of this character, but the results are spoilt by dynamical shocks due to the splitting mechanism having been in action in the two comets.
4. Taking account of the analysis of the motion of Mercury's apsidal and node lines no oblateness of the Sun certainly exists of the order of 10^{-3} or larger, and it is probably less than 10^{-4}, if any. As for the oblateness parameters less than 10^{-3}, they are able to give apsidal and nodal rotation not higher than of the order of a few tens of seconds of arc per revolution.
5. From the point of view of the collision hypothesis of the origin of the Kreutz family, either the relativistic effect or the oblateness effect suggest no satisfactory explanation of the discrepancy between the observed and required position of apsidal lines of the sun-grazing comets.
6. In either interpretation, some 10^3 to 10^4 revolutions of the sun-grazing comets round the Sun must be assumed to be consistent with the required apsidal motion of about $8°$ since the hypothetical catastrophe. Hardly any comet of perihelion distance less than 0.01 AU could survive such a number of revolutions.
7. Besides it, even though the oblateness effect were responsible for the advance of perihelion, simultaneously a nodal motion should be observed according to (81). The numerical form of the first decay condition should then differ from (21), another set of suspected comets should be presented instead of those in Fig. 6 and the identification criteria should be applied anew. A new concept should after all be developed. But no concept can be worked out, lest we are sure of the factors responsible for the supposed motion of the apsidal and node lines.
8. Any concept, which would be able to explain the distributions of the orbital elements of the members of the Kreutz family by secular influence of external effects, would be of high interest from still another point of view. Such a concept, taking the responsibility for considerable differences among the elements of the individual comets of the group over from suddenly acting forces to secular effects, would remove the vast velocities of separation required at solar distances of the order of 0.1 AU. The hypothesis of the origin of the sungrazing comets, suggesting a disruption of the hypothetical protocomet due to the Sun's tidal and high-temperature forces coupled with the protocomet's interior forces would then be acceptable. At the present time, however, there is no mechanism known for complete explaining the strong secular motion of perihelion observed.
9. The remaining possibility, which, however, cannot be analyzed in detail, is an artificial assumption that the resulting effect (after a number of revolutions) of the Sun's forces coupled with the comet's forces, causing splits and similar explosive phenomena at extremely short heliocentric distances (directly observed at comets 1882 II and 1965f), is, on an average, zero on the node line, but yields a systematic motion of perihelion. Then the collision hypothesis would be probable. But there is no evidence for this assumption.

18. Chance of interpretations based on non-collisional mechanisms

Section 14 and 18 show that the proposed collision hypothesis of the origin of the Kreutz comet family is not completely free of contradictious effects, though it is able to explain a series of observed phenomena.

Of other possible mechanisms those connected with any direct solar effects have been excluded in Section 2, because they require high velocities of separation. The remaining mechanisms are those connected with the interior comet processes, which are stimulated by the solar action, in which, however, the latter takes no more essential part: sudden outbursts, or long-term evolutionary changes in the comet nuclei. Intense solar heating is generally not a necessary condition for comet outbursts, as is demonstrated by Comet Schwassmann-Wachmann (1). The explosive phenomena in this anomalous comet have never resulted in its disruption into more pieces, as far as observed. Therefore we have no reason for correlating comet explosions and nuclear breakups, in general.

Of the essentially non-explosive processes, which only in their final stage can lead to some abrupt phenomena, and the existence of which in comet nuclei are likely, the effect of radioactive heating has recently been considered by WHIPPLE and STEFANIK (1966).

In connection with the Kreutz family it should be mentioned:

1. The hypothesis of radioactive heating is an ad hoc hypothesis. No quantitative data are given on the pressures, induced on the brittle surface shell, so that neither we are sure whether they are sufficient for breaking it up, nor, if yes, what are the velocities of separation. Even the existence of the surface shell, formed as described above, is hypothetical. The consequences themselves of the process of radioactive heating are exclusively expected or suggested effects, probable, of course, but not tested, either in laboratory, or mathematically verified. Comet splits can accordingly be considered as only possible consequences of the process discussed.
2. The mechanism is proposed for explaining splits of primarily new comets in Oort's sense. Accordingly the time scales necessary for the conditions of the process to be finished are of the order of 10^8 years. The sun-grazing comets are moving in nearly-parabolic orbits, but they certainly are not new comets, neither as for their physical properties (e.g. central temperature in their nuclei), nor as for their orbital character (period of revolution of about 10^2 to 10^3 years). STRÖMGREN'S (1914) computations led to the original orbit of 1882 II very close to its osculating orbit, which—with respect to the character of its motion round the Sun (no approach to major planets)—could be expected. What were their orbits some 10^1 to 10^2 revolutions ago, remains, of course, absolutely unknown.
3. If most splits, listed by STEFANIK (1966), are actually consequences of the effect of radioactive heating, as supposed by WHIPPLE and STEFANIK, the velocities of separation, induced by the process, lie within at least 2 to 40 metres per second. No split was recorded with the velocity of separation of the order of 0.1 km/s. To explain the deviations among the orbital elements of the sun-grazing comets by the mechanism of radioactive heating with the above velocities of separation, the primary split could be admitted at solar distances of a few tens of AU, which is extremely hard to explain physically. Moreover, no approach of any two members of the Kreutz family results from their orbits at such large solar distances.

There is, however, a good chance for the effect of radioactive heating to explain secondary splits in the Kreutz family. These problems are discussed in the following section.

In spite of the fact that no reasonable process is known at present, explaining the origin of the Kreutz family at very small heliocentric distances ($\ll 0.1$ AU) on the one hand, or at very large heliocentric distances ($\gg 10$ AU) on the other, the form of the first decay condition remains an indisputible fact. It is very well represented (Section 9) and reveals the existence of a common line of intersection of the five orbits, which is given by the following equations:

$$x = a \, . \, z \, , \qquad y = b \, . \, z \, , \tag{84}$$

where $a = +\,0.28686 \pm 0.00348$ (m.e.), $b = -\,1.38080 \pm 0.00058$ (m.e.). We find. that the line of intersection is nearly identical with the major axes of their present orbits. Assuming on the basis of the results of particularly Section 18 that they are essentially independent of time, only two interpretations are consistent with (84): The comets were born either at the very perihelion passage of the protocomet, or at extremely large distances of the order of 10^2 AU (!), somewhere near to its aphelion passage.

Let us analyze the two possibilities apart from the physical mechanisms responsible.

The first version—perihelion region—would require velocities of separation of about 100 km/s, and a common point of intersection of the five comets' orbits should exist at a distance less than 0.01 AU. The actual space orientations of the orbits do not, however, confirm this condition. Their miss distances at a close vicinity to the Sun are up two orders higher than the errors resulting from observations.

The second version—aphelion region—would require very low velocities of separation, of the order of a few metres per second. The difficulties consist in the fact that the motions of the five comets at very large solar distances are not known with sufficient accuracy. Any direct search for a common point of intersection of their orbits from Table 1 at these distances is of no practical sense. The problem may only statistically be settled. The statistically most probable solar distance, at which the split can have occurred, is given by

$$\bar{r}_0 = \frac{\sum q \, . \, (1 + e) \, . \, (1 + e \cos v)}{\sum (1 + e \cos v)^2} \, , \tag{85}$$

the five comets' orbits being summed up. Here

$$\cos v = - \frac{a \, . \, P_x + b \, . \, P_y + P_z}{\sqrt{(1 + a^2 + b^2)}} \, , \tag{86}$$

a, b are the constants of (84) and P_x, P_y, P_z are the vector elements of the five orbits. Inserting the data of Table 1 we find:

$$\bar{r}_0 = 143 \pm 13 \text{ (m.e.) AU} \, . \tag{87}$$

Some related data on each of the five comets are given in Table 18. The respective columns include: the comet, the heliocentric distance of the point of intersection, r_0, resulting from (86), the aphelion distance, q', its mean error, m.e., and the comet's orbital velocities at the point of intersection (85) and at the aphelion, respectively. For each of the comets the point of intersection lies on the pre-perihelion arc of orbit. The dispersion in r_0 is comparable with the mean errors in the aphelion distances.

Table 18. Solar distance of the point of intersection (always pre-perihelion arc of orbit)

Comet	r_0	q'	m.e.	$V(\bar{r}_0)$	$V(q')$
	AU	AU	AU	km/s	km/s
1843 I	125.6	128.1	± 8.4	—	24.4
1880 I	195.3	195.4	±55.8	1.83	16.0
1882 II	128.7	166.3	± 0.3	1.33	22.3
1963 V	212.9	235.0	?	2.21	12.8
1965f	142.7	190.4	±25.2	1.77	19.5

Table 19. Independent positions of the point of intersection

Comet	from P_x, Q_x of (89)		from P_y, Q_y of (89)	
	r_0	arc of orbit	r_0	arc of orbit
	AU		AU	
1843 I	125.5	pre-perihelion	93.1	post-perihelion
1880 I	195.3	pre-perihelion	160.3	post-perihelion
1882 II	128.9	pre-perihelion	134.8	pre-perihelion
1963 V	215.2	pre-perihelion	105.7	pre-perihelion
1965f	132.7	pre-perihelion	140.6	pre-perihelion

To test the accuracy with which the point of intersection can be determined at extremely large solar distances, we apply independent formulae. Looking for the co-ordinates of the point of intersection, and taking account of (84) I find

$$\text{(88)} \qquad \tan v = - \frac{P_x - a \, . \, P_z}{Q_x - a \, . \, Q_z} = - \frac{P_y - b \, . \, P_z}{Q_y - b \, . \, Q_z} ,$$

which gives two independent values of the position of the point of intersection, included in Table 19. As seen, the deviations are rather large, and for two comets, 1843 I and 1880 I, even the arc of orbit is uncertain, where the point of intersection lies. Undoubtedly, both real changes in the orbits with time and observational errors are responsible for the deviations found. Nevertheless, it is of interest that the two comets with the best derived elements, 1882 II and 1965f, reveal the least dispersion in r_0.

The question may arise whether the values of constants a and b applied in (88) are not responsible for the deviations, and whether some other combination of a and b would give less dispersion. Comparing the two expressions of (88) the following equations are obtained for a and b (from each comet):

$$\text{(89)} \qquad a \, . \, R_x + b \, . \, R_y + R_z = 0 ,$$

which is nothing but another form of the first decay condition, identical with the first equation of (16). Consequently, no better result can be obtained from the data on the sun-grazing comets available at present.

Coming back to possible physical interpretations of such a hypothetical split at extremely remote solar distances, we should admit that again a collision of a massive protobody with a light projectile can comparatively best explain the distribution of the orbits. The differentical velocities of separation, obtained at a 140 AU distance and corresponding with the observed deviations in the orbital elements of the sun-grazing comets, would be within 6 metres per second, i.e. 2.5 orders less than the corresponding orbital velocities, and nearly comparable with the aphelion orbital velocities (Table 18). Let us mention that the circular velocity at the above distance is still 2.5 km/s. In the model collision, discussed in Sections 15 and 16, the differential velocities of separation of the sun-grazing comets were within 0.7 km/s, i.e. almost 2 orders lower than the respective orbital velocities.

Consequently, no optimistic chance seems to be of non-collisional mechanisms for explaining the origin of the Kreutz peculiar comet family.

19. Fine structure of the Kreutz family. Couples of comets

At the present time the investigation of the nature of the Kreutz family is more or less near its beginning. The main problems to be studied in the future, necessary for preparing fundamental data on the family for a more detailed analysis, may be summarized as follows:

1. To search for new members of the family, particularly for the sun-grazing ones.
2. To improve the numerical form of the first decay condition as the basis for the further study of possible mechamisms of the Kreutz family origin.
3. To test each discovered comet according to the three C_i criteria.
4. To study carefully orbital periods of the comets suspected of belonging to the family, and to rediscuss this problem for the known members, on the basis of the observational material available.
5. To determine rough limits for the distributions of each of the orbital elements and to estimate the possible range of separation velocities impressed to the "daughter" comets observed.
6. To perform more careful physical studies of the suspected comets.

At present we are certainly not able to study a fine structure of the Kreutz family in general. In spite of it, however, the exceptional character of the distributions of the orbital elements of the indubitable members betrays possibly existing subdivisions in the system of the Kreutz family. The problem is whether the mechanism originating the subdivisions is identical in character with that originating the Kreutz family as a whole, and—if yes—whether the age of the subdivisions is equal to the age of the family, or whether the former are younger.

The first view on Tables 1 and 2 leads to a conclusion that the listed comets are coupled: 1965f is very close to 1882 II, 1963 V to 1880 I, and even to 1843 I, while there is no comet with i between 142°.1 and 144°.3, with Ω between 347° and 2°, with ω between 70° and 82°, and with q between 0.0064 and 0.0077 AU. Time relations are also of interest. Within 7 years, between 1880 and 1887, there were three comets of this family observed, while within nearly 60 years, between 1887, or at least 1893, and 1945, none. Comet 1882 II followed 1880 I in 2.6 years, and Comet 1965f followed 1963 V in 2.2 years.

Table 20. Couples of sun-grazing comets

	1965f – 1882 II	1963 V – 1880 I	1880 I – 1843 I	1965f Nucleus B-Nucleus A
$\Delta\omega$	—33'.4	+37'.9	+155'.9	—0'.7
$\Delta\Omega$	—42'.4	+59'.1	+183'.9	—1'.4
Δi	— 9'.1	— 0'.2	+ 11'.3	—0'.6
$10^5 . \Delta q$, AU	+ 1.0	—43.9	+ 1.1	+1.3
$10^5 . \Delta e$	+ 1.2	+ 1.3	+ 3.0	+1.5
$v + \omega$	—109°.3 or +70°.7	—89°.6 or +90°.4	—96°.0 or +84°.0	+56°.0 or +236°.0
ω	69°.6	85°.2	82°.6	69°.0
v	—178°.9 or +1°.1	—174°.8 or +5°.2	—178°.6 or +1°.4	—13°.0 or +167°.0

The cometary couples are listed in Table 20. For comparison, the differences between the Nucleus A and Nucleus B orbits of 1965f, stimulated by tidal and high-temperature effects acting close to the Sun, are added. Besides the differences in the respective sets of elements, the true anomaly, locating the position in orbit of the secondary breakup for each supposed couple, is given (for the method see Sekanina, 1967).

Several mechanisms responsible for the observed couples of comets can be suggested, namely, tidal and high-temperature splitting, a breakup as a consequence of radioactive heating, a collision process, rapid rotation, or explosion. The following conclusions are arrived at, based on Table 20, and considering respective mechanisms:

1. Comets 1882 II and 1965f form the closest couple.
2. Comet 1880 I forms a close couple with 1963 V as far as the angular elements are considered, but rather with 1843 I, as far as the perihelion distance is considered.
3. Besides their orbital resemblance, 1882 II and 1965f indicate still a striking physical resemblance: these are the only two comets demonstrably split after their perihelion passages, though they passed the surface of the Sun's photosphere at a distance 3.5 times as great as did comets 1843 I and 1880 I, and even 7 times as great as did 1963 V. A split of 1963 V is improbable. Roemer (1965) in her comments on the photographs of this comet obtained on November 9, 1963, when the comet was at a heliocentric distance of 2.0 AU, has written on "a moderately well condensed nucleus of magnitude 17.2 with possibility of secondary nucleus 0'.1 separation from the primary". No secondary nucleus was, however, mentioned on any other of more than 30 photographic plates obtained within the period of September 15 to December 18, corresponding to the interval of solar distances from 0.86 to 2.6 AU.
4. The first of the five suggested mechanisms should take place close to the Sun-Comparing the couples with the 1965f Nucleus A-to-Nucleus B data, we find the deviations in the angular elements of the former one to three orders higher. Velocities of separation of at least a few hundred metres per second are required at about $r = 0.1$ AU, mostly in the direction perpendicular to the plane of orbit.
5. Also the values of true anomaly speak against the tidal or high-temperature breakups. Due to the thermal inertia, this mechanism acts most strongly always after the peri-

helion passage. This is confirmed by the observed breakups of 1965f (SEKANINA 1966a) and of 1882 II (KREUTZ 1891). Even a lag of only 1 day, corresponding to a solar distance of 0.1 AU, leads to true anomalies of about $+ 150^\circ$, which makes the values of $v + 1^\circ$ or $+ 5^\circ$ strongly improbable and artificial, and suggests that the versions with v close to $\pm 180^\circ$ are more likely.

6. Much better agreement with the data of Table 20 is reached if either a mechanism is assumed, inducing high velocities to both comets of the couple, or a low-velocity mechanism is considered, acting, however, at large heliocentric distances, a few astronomical units at least.
7. The true anomalies close to -180° (Table 20) speak in favour of the latter mechanism, which can be perhaps identified with the process of radioactive heating, or, its contribution to the splits can, at least, be admitted. The heliocentric distances, corresponding with the tabulated values of v, are very uncertain but certainly larger than 2 to 3 AU. This is consistent with the data compiled by STEFANIK (1966), who mentioned seven comet splits by non-tidal forces at distances larger than 1 AU, and three at distances larger than 3 AU.
8. Also the deviations $\Delta\omega, \ldots, \Delta e$ speak in favour of a mechanism, impressing low separation velocities to the debris. For a constant velocity of separation, the changes in angular elements linearly increase with the solar distance, while the changes in perihelion distance and eccentricity can but need not increase. This is just the picture we obtain if compare the data for the couples with those for the two nuclei of Comet Ikeya-Seki (Table 20). Permitting velocities of the order of 1 to 10 m/s at heliocentric distances of, say, 5 AU, we obtain for $\Delta\omega$ to Δe values of the order of those given in Table 20.
9. Considering the next version, a collisional mechanism as a possible agent, originating the cometary couples in the Kreutz family, we can refer to the first decay condition, nearly perfectly fulfilled by the five indubitable members, as to a supporting fact. We must, however, point out that the true anomalies from Table 20 are inconsistent with a solar distance of about 1.2 AU, where the collision should have occurred, if the couples of comets originated simultaneously with the whole Kreutz family. Hence, we would have to assume that, in addition to the collision at 1.2 AU, at least two other collisions occurred later at much larger distances, and on pre-perihelion arcs of the orbits, hence, at least, one revolution round the Sun following the "primary" collision. But this assumption is too complicated, and artificial to be acceptable and likely in fact.
10. The discrepancy in the solar distances could possibly be explained. The values of true anomaly in Table 20 may be uncertain to some degree, because the orbits are mostly not corrected for planetary perturbations. They cannot be reduced to the common date of osculation, identical with the date of cosmic catastrophe, since we do not know the latter. It is, however, difficult to estimate the possible effect. Keeping the $\Delta\Omega$ values of Table 20 then the Δi's should be changed from $-9'.1$ to $-5'.2$, from $-0'.2$ to $-1'.8$, and from $+11'.3$ to $-0'.7$, for the three mentioned couples, respectively, to be consistent with a 1.2 AU distance from the Sun. If the deviations of about $2'$ to $12'$ in inclination are due to gravitational instability of the orbits, then the common origin of the couples and of the whole Kreutz family is possible. The residuals from the first decay condition would then however increase a few times.

11. The two last mechanisms, rapid rotation and explosion, are not here discussed in detail. The former because it can itself give the observed effect only with highly improbable periods of rotation of a few minutes (!), the latter because there is no evidence on the existence of comet explosions, resulting in "hurling" the parent body.

20. General conclusions

1. The peculiar group of sun-grazing comets, often called the Kreutz family of comets, consists of five indubitable members, evolutionarily interrelated: 1843 I, 1880 I, 1882 II, 1963 V, 1965f, and of a few possible members, especially 1887 I and 1945 VII, perhaps also 1872. The origin of 1668 is rather indirectly than directly related to the origin of this group.
2. For the five indubitable members comparatively accurate orbits are available, except their periods of revolution. But only the orbit of 1882 II is corrected for planetary perturbations. Fortunately, the character of the sun-grazing orbits excludes any approach of the comets to the major planets. For the possible members only approximate orbits are known, resulting either from a bad determinacy of the comet's nucleus (1887 I), or from a short arc of orbit (1945 VII).
3. The investigations have primarily been based on the orbits of the indubitable members. The deviations among the orbits are comparatively large: up to 20° in the node, nearly 3° in inclination, nearly 20° in the argument of perihelion, and 30 to 50 per cent in the perihelion distance. In spite of it the comets obviously are of common origin. The related character of their orbits is particularly confirmed by the existence of a common line of intersection of the five orbital planes. It is nearly identical with the apsidal lines of the five comets' orbits. The dispersion within the directions of the apsidal lines is less than 0°.5!
4. A striking linear dependence has been found between the nodal longitude and the argument of perihelion of the orbits of the indubitable members. The orbits of some of the possible members comply with the relation with a sufficient accuracy as well.
5. Another striking feature of the Kreutz family is the existence of comet couples: 1965f is moving in an orbit very close to that of 1882 II; 1963 V and 1880 I on the one hand and 1880 I and 1843 I on the other resemble one another in some of their orbital elements. Comets 1882 II and 1965f are the only two members for which nuclear splits were undoubtedly observed.
6. The common origin of the Kreutz family of comets has been investigated essentially in two different ways: (i) on the basis of the distributions of their orbital elements, and (ii) on the basis of the search for their common point of encounter and of the distribution of their orbital velocities near to the point.
7. The first way has lead to conclusions that the orbits actually satisfy the conditions necessary to be fulfilled for comets as debris of a disrupted parent body: both the relation between inclination and nodal longitude, and the dependence of a function of perihelion argument and node on the perihelion distance have been found. An approximate orbit of the parent comet has even been computed, rather close to those of the present sun-grazing comets, particularly of Comet 1882 II. Velocities of separation have been ascertained of about 1 to 2 km/s.

8. The catastrophe should have occurred at a 1.2 AU solar distance, when the protocomet was approaching the Sun. This first way applies to a collisional process equally well as to any other abrupt process, and no conclusion can, hence, be arrived at as for the character of the splitting mechanism.
9. Extrapolating the node-inclination relation, valid for the sun-grazing comets, to all possible values of Ω, 8 ancient comets and 38 distant comets of the years 1769 to 1966 have been found complying with it.
10. A method has been developed, making it possible to find for each of the "suspected" comets the heliocentric distance of its minimum distance from the line of intersection. Of the total of 46 comets, 25 reveal an approach to the line of intersection and for 21 comets the distance of approach is less than 0.1 AU. But the respective solar distances are strongly dispersed. The only exception is a group of at least 7 comets, 1668, 1695, 1785 I, 1830 II, 1857 IV, 1924 II, 1947 III, and possibly even others, 574, 1905 III, etc., for which the approach distances are located at about a 1.2 AU distance from the Sun, southwards the ecliptic, consistent with the result of (8).
11. The study of the distribution of the velocity vectors of the above comets near to the critical point gives an interesting result: They are concentrated in two directions, opposite, essentially, to one another when referred to the Sun. And comets 574 and 1947 III are moving in very similar orbits, but in opposite directions.
12. A model of the catastrophe has been suggested, based on the results mentioned in (11), identifying the splitting mechanism with a side collision of two parent bodies incomparable in mass. A detailed description of the model collision is given in Sections 15 and 16.
13. Comet 1882 II has been suggested as the main fragment of the parent comet because of its high luminosity and, likely, also its big mass. The intrinsic luminosities of the related comets are listed in Bouška's (1966) recent paper.
14. There exists the only discrepancy in the collision model: the orbits of the sun-grazing comets miss the critical points at distances of about 0.15 AU. To correct these deviations, a considerable apsidal motion of the sun-grazing orbits has to be supposed, giving a total change of 8° in the argument of perihelion since the time of the hypothetical collision.
15. Three secular effects have been analyzed, which could have been responsible for the apsidal rotation: the relativistic effect (advance of perihelion), the resisting effect of denser parts of the solar corona, and the effect of the Sun's oblateness. None of them can, however, explain the required value, unless a total of 10^3 to 10^4 revolutions round the Sun elapsed since the time of the catastrophe are admitted. We are not able to conceive a sun-grazing comet, surviving such a number of revolutions with sufficient supplies of gases to keep its luminosity within reasonable limits. Moreover, no effect like this has appeared on the orbits of comets 1882 II and 1965f, for which both pre- and post-perihelion elements are available.
16. Since the results of the method developed in Sections 3 and following are independent of the splitting process, postulating only its abrupt character, a possibility of interpretation of the origin of the Kreutz family on the basis of non-collisional mechanisms has also been considered. But their chances do not appear to be optimistic.
17. The deviations in the orbital elements of the sun-grazing comets are too large, and the resulting velocities of separation too high to be explained by any known mechanism

at the very neighbourhood of the Sun. The required velocities are of about 100 km/s at solar distances less than 0.01 AU, in excess of 7 km/s at distances less than 0.1 AU, in excess of 7 km/s at distances less than 0.1 AU, and in excess of 1 km/s for distances less than 0.7 AU.

18. A breakup of the parent comet at extremely large solar distances close to the aphelia of the present sun-grazing comets is possible. Velocities of separation required are only a few metres per second. No non-collisional mechanism is known, however, able to split a comet nucleus at such remote distances, where any comet is in a completely inactive state.
19. It is not known at present, whether all the sun-grazing comets listed in Table 1 (or in Table 2 as well) are products of a single catastrophe—and are as independent comets, hence, of equal age—, or whether they are debris of a parent body, which met more successive catastrophes. Neither is evident whether the mechanisms originating the present comet couples—see point (5)—are identical with one another in character.
20. Generally, the collisional mechanism is likely to be responsible for the existence of the Kreutz comet family as a whole, while non-collisional mechanisms could partly be responsible for secondary splits and for some features of the family's structure known from our present observations. No conclusive solution of the problem of the Kreutz family can be carried out, unless more orbital and physical data are available on its members.

The computations have been carried out on a Zuse Z23 computer of the ČKD Computing Centre, Prague. I am much obliged to Ing. I. Brand, the head of the Centre, for enabling me to make use of the computer.

Note Added in Proof

During printing the present paper two studies have appeared dealing with the motion of the comets of the Kreutz group. The writer has computed a new orbit of Comet Pereyra 1963 V from observations September 15 to December 18, 1963, including the perturbations by Venus to Neptune (Z. Sekanina, 1967, BAC 18, 229). The orbit is rather close to that included in this paper except for the period of revolution, which amounts to about 850 years.

B. G. Marsden (MS 1967) has presented another explanation for the origin of the Kreutz family, but he has met obstacles, too. From five plates obtained at the Boyden Observatory and measured later by Mowbray, Marsden has derived a new orbit of Comet du Toit 1945 VII and has found it similar to those of 1882 II and 1965f. Hence, this comet can be considered an indubitable member of the family at present. New independent orbits have also been computed by Marsden for Comet Pereyra and Comet Ikeya-Seki, very close to those derived by the writer.

References

ALLEN C. W., 1955, Astrophysical Quantities. London.

BALDET F., de OBALDIA G., 1952, Catalogue Général des Orbites de Comètes de l'an –466 a 1952. Paris.

BOUŠKA J., 1966, Acta Univ. Car. No 2, 67 = Publ. Astr. Inst. Charles Univ. Prague No 49.

Brouwer D., 1952, AJ 56, 9.
Bruhns C., 1875, AN 86, 219.
Dicke R. H., 1964, Nature 202, 432.
Kreutz H., 1888, Publ. Sternwarte Kiel 6, Part 1.
—, 1891, Publ. Sternwarte Kiel 6, Part 2.
—, 1901, Astr. Abh. AN No 1.
Marsden B. G., 1966, IAU Circ No 1972.
Marsden B. G., van Biesbroeck G., 1963, AJ 68, 235.
Oppenheim H., 1889, AN 121, 337.
Porter J. G., 1961, Catalogue of Cometary Orbits. Mem. BAA 39, No 3.
Roemer E., 1965, AJ 70, 397.
Sekanina Z., 1964, IAU Circ No 1859.
—, 1965, IAU Circ No 1947.
—, 1966a, BAC 17, 207.
—, 1966b, Acta Univ. Car. No 2, 3 = Publ. Astr. Inst. Charles Univ. Prague No 48.
—, 1967, BAC 18, 19.
Stanyukovich K. P., 1960, Iskusstv. sputniki Zemli No 4, 86.
Stefanik R., 1966, Nature et Origine des Comètes. Mem. Soc. Roy. Sci. Liège Vol. 12, 29.
Strömgren E., 1914, Publ. Københavns Obs. No 20.
van Houten-Groeneveld I., 1963, BAN 17, 223.
Whipple F. L., Stefanik R., 1966, Nature et Origine des Comètes. Mem. Soc. Roy. Sci. Liège, Vol. 12. 33.