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PRECIPITATION ACTIVITY IN WESTERN AND CENTRAL EUROPE AND IN THE UNITED STATES IN RELATION TO THE SOLAR ACTIVITY

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On the basis of the precipitation data from 44 stations of West and Central Europe the precipitation course is studied in this area. The most important result is the fact that the period of precipitation variations is 6.2 years, varying from 5.4 years to 7.5 years in individual stations. Further, the dependences of the precipitation parameters on the geographical positions of the European stations, the variability of the length of the precipitation period, as well as the long-term variations of precipitation have been ascertained. Also the correlation between the precipitation courses in odd and even cycles is investigated and interpretted by means of the varying length of the precipitation period. The system of the lines of equal correlation degree is then constructed all over West and Central Europe. It is shown that the odd cycles are, as regards the precipitation, more active than the even over West and Central Europe.

A brief investigation of the same character is carried out in the United States. The precipitation data from 10 stations give, on the general, quite analogous results to those obtained from the European stations. The only conspicuous disagreement takes place in the mutual relation between the integrated precipitation activity in the two types of solar cycles.

1. INTRODUCTION

The amount of precipitation is investigated at 44 West and Central European stations and 10 U. S. stations. Statistics of comet discoveries and the problems of cometary climatology, being connected with the former, are the reason why the two areas have been selected (SEKANINA, 1964).

The precipitation course at each station has been studied separately, on the basis of the material published by CLAYTON ET AL. (1927, 1934, 1947). The yearly precipitation data, $A'(i)$, have been smoothed out according to the formula

$$A(i) = \frac{1}{3} \sum_{k=i-1}^{i+1} A'(k), \qquad \text{(I)}$$

reduced to relative units (100 per cent has been put equal to the average of all the observational series) and then plotted in dependence on time. The

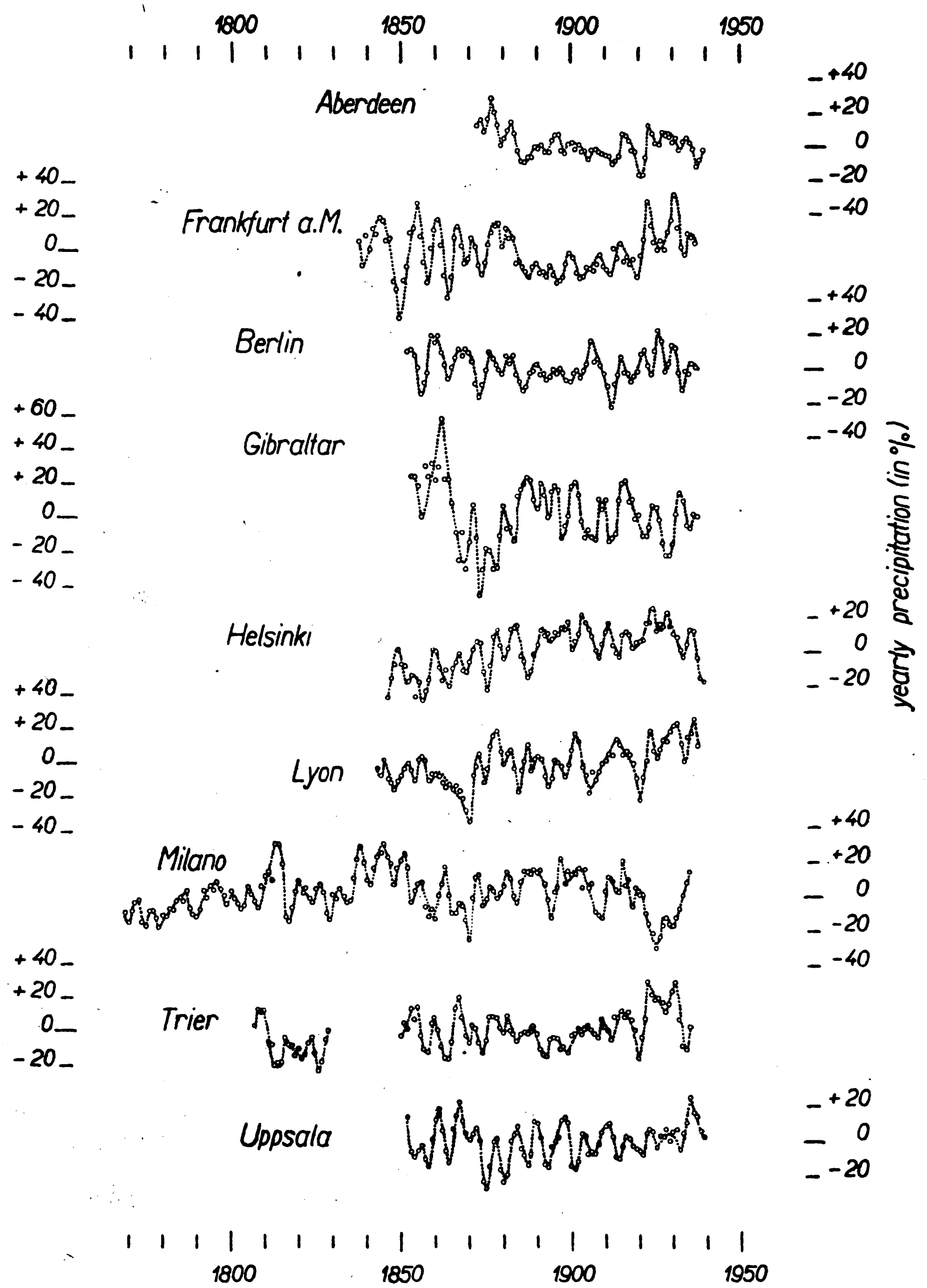

Fig. 1. Precipitation activity at some West and Central European stations.

forms of a few curves obtained in this way are represented in Figure 1 and Figure 15 for European and American stations, respectively. An analysis has indicated that the amount of precipitation fluctuates with the mean period of about 6.2 years, the limits being 5.4 years and 7.5 years. Hence, the period is longer than one half of an eleven-year solar cycle. From this point of view, no real results may be obtained by studying the precipitation activity in dependence on the solar-cycle phase as was done by HELLMANN (1908) and a few other authors. The latter investigation does not lead to a value of actual precipitation period, P, but only to a false period, P':

$$P' = n^{\pm 1} P_{\odot} \tag{2}$$

($P_{\odot}$ is the length of the solar cycle), or, more exactly, to a value of natural number n. As known, the above mentioned investigations revealed a double--wave, i. e. they gave a result of $P' = 0.5\ P_{\odot}$.

The actual form of the dependence of the precipitation activity on the phase of the solar cycle is indicated by some consideration as follows:

Let us assume that the precipitation activity may be expressed through a cosine curve:

$$A(t) = a_0 \cos \frac{2\pi}{P_0} (t - t_0), \tag{3}$$

where t_0 is the time of maximum precipitation, a_0 its mean semiamplitude, and P_0 its mean period, which is, in practice, represented by the average from a number of various stations within a given area. We will make use of the ascertained fact that

$$P_0 \sim 6.2 \text{ years}, \tag{4}$$

then expand function (3) in the Fourier series of

$$A(t) = \sum_i \left[a_i \cos \frac{2\pi i}{P_{\odot}} (t - t_0) + b_i \sin \frac{2\pi i}{P_{\odot}} (t - t_0) \right] \tag{5}$$

and take only $i = 1, 2, 3$, which is owing to (4) quite sufficient. Moreover, the validity of (4) indicates we may write:

$$P_0 = \frac{1}{2} P_{\odot} + \Delta P_0, \tag{6}$$

where ΔP_0 is in its order less than $P_{\odot}$. Introducing (6) into the formulae for coefficients a_i and b_i, and neglecting all the terms with the powers of $\Delta P_0/P_{\odot}$ higher than the first, we obtain:

$$\left. \begin{aligned} a_1 &= \frac{32}{9} a_0 \frac{\Delta P_0}{P_{\odot}}, \\ a_2 &= a_0 \left(1 - \frac{\Delta P_0}{P_{\odot}} \right), \\ a_3 &= -\frac{16}{5} a_0 \frac{\Delta P_0}{P_{\odot}}, \\ b_1 &= b_2 = b_3 = 0. \end{aligned} \right\} \tag{7}$$

Accounting $\Delta P_0 = +0.7$ year and $P_\odot = 11.1$ years the amplitudes are equal to

$$\left.\begin{aligned} a_1 &= 0.22\, a_0, \\ a_2 &= 0.94\, a_0, \\ a_3 &= 0.20\, a_0, \end{aligned}\right\} \quad (8)$$

so that the amplitude of the double-wave strongly prevails over the other two. In the most disadvantageous case, for $t - t_0 = \frac{1}{8} P_\odot$, or $t - t_0 = \frac{5}{8} P_\odot$, the ratio between the sum of the first and third amplitudes and the second amplitude is:

$$\max \left[\frac{a_1 \cos \frac{2\pi}{P_\odot} (t - t_0) + a_3 \cos \frac{6\pi}{P_\odot} (t - t_0)}{a_2} \right] = 32\ \%.$$

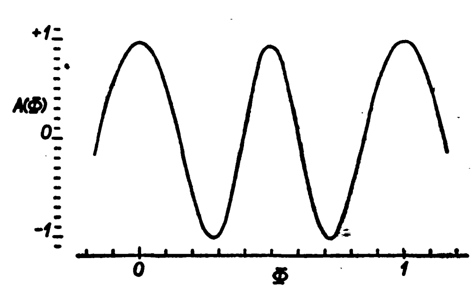

Fig. 2. Expansion in the Fourier series of function $A = A(\Phi)$ with $P_0 = 6.2$ years.

The graphical representation of $A(\Phi)$ with the values of a_1, a_2, a_3 taken from (8) for $a_0 = 1$ is given in Figure 2.

2. PRECIPITATION ACTIVITY AT EUROPEAN STATIONS

First of all, Table 1 gives the most important parameters of the precipitation activity at 44 stations of Western and Central Europe, separately for each station:

Stat.	— observational station;
λ	— geographical longitude (positive westwards from the Greenwich meridian);
φ	— geographical latitude;
int t	— time interval of precipitation measures;
P	— average precipitation period with the resulting probable error;
$P_\odot$	— average length of the solar cycles in the course of the given int t;
k	— ratio $P/P_\odot$ and its resulting probable error;
T_0 (min)	— mean time of the minimum precipitation activity;
T_0 (max)	— mean time of the following maximum of the precipitation activity;
p. e.	— probable error of the respective moments;
φ_0	— the phase distance of the time of the mean maximum relative to that of the mean minimum, and the probable error;
a	— the mean relative semi-amplitude of precipitation.

If t_k and t_l are the moments of the minimum and maximum precipitation activities, respectively, derived in a graphical way from the material, and m, n their respective numbers, the length of the period P is computed according to

Table 1.

Precipitation activity at 44 stations of West and Central Europe

station	λ	φ	int t	P	$P_\odot$	k	T_0 (min)	T_0(max)	p. e.	φ_0	a
	°	°	y	y y	y		y	y	y		%
Aberdeen	+ 2.1	+57.2	1871–1940	6.20 ± 0.23	11.0	0.564 ± 0.021	1899.94	1902.50	±1.03	0.41 ± 0.05	6.7
Athens	—23.7	+38.0	1895–1930	5.78 ± 0.35	11.0	0.526 ± 0.032	1904.66	1907.52	±1.05	0.49 ± 0.09	14.0
Belgrad	—20.4	+44.8	1888–1930	5.92 ± 0.42	11.0	0.538 ± 0.038	1904.58	1907.64	±1.46	0.52 ± 0.11	12.7
Berlin	—13.3	+52.5	1851–1938	6.40 ± 0.23	11.0	0.582 ± 0.021	1904.59	1907.75	±1.13	0.49 ± 0.05	9.2
Bodó	—14.4	+67.3	1872–1940	6.00 ± 0.57	11.0	0.545 ± 0.052	1903.20	1906.41	±2.48	0.54 ± 0.14	8.8
Breslau	—17.0	+51.1	1859–1938	6.33 ± 0.33	11.1	0.570 ± 0.030	1903.08	1906.56	±1.51	0.55 ± 0.08	8.1
Bucuresti	—26.1	+44.4	1865–1940	6.35 ± 0.37	11.0	0.578 ± 0.034	1901.68	1905.30	±1.66	0.57 ± 0.09	10.1
Catania	—15.1	+37.5	1892–1930	6.30 ± 0.37	11.1	0.568 ± 0.033	1899.72	1902.72	±1.17	0.48 ± 0.09	13.3
Copenhagen	—12.6	+55.7	1811–1928	6.20 ± 0.17	11.2	0.554 ± 0.015	1904.10	1906.81	±1.01	0.44 ± 0.04	8.5
Frankfurt a. M.	— 8.7	+50.1	1837–1938	6.64 ± 0.26	11.0	0.604 ± 0.024	1900.78	1904.11	±1.38	0.50 ± 0.06	12.7
Gibraltar	+ 5.4	+36.1	1852–1938	6.48 ± 0.32	11.2	0.579 ± 0.029	1902.29	1905.47	±1.54	0.49 ± 0.07	15.6
Gjesvar	—25.4	+71.1	1884–1926	6.25 ± 0.21	11.0	0.568 ± 0.019	1904.21	1907.79	±0.73	0.57 ± 0.05	12.4
Greenwich	0.0	+51.5	1841–1940	6.32 ± 0.27	11.2	0.564 ± 0.024	1902.14	1905.47	±1.43	0.53 ± 0.06	9.6
Gütersloh	— 8.4	+51.9	1837–1923	6.16 ± 0.27	11.2	0.550 ± 0.024	1905.43	1907.99	±1.35	0.42 ± 0.06	8.7
Haparanda	—24.1	+65.8	1860–1940	6.26 ± 0.35	11.0	0.569 ± 0.032	1902.19	1906.13	±1.68	0.63 ± 0.09	10.0
Helsinki	—24.9	+60.2	1845–1940	6.18 ± 0.24	11.2	0.552 ± 0.021	1900.84	1904.06	±1.22	0.52 ± 0.06	9.2
Hvar	—16.4	+43.2	1867–1917	5.87 ± 0.21	11.3	0.520 ± 0.019	1900.06	1903.46	±0.81	0.58 ± 0.05	11.2
Kaliningrad	—20.5	+54.7	1848–1938	6.72 ± 0.30	11.2	0.600 ± 0.027	1901.84	1905.37	±1.50	0.53 ± 0.07	9.5
Karesuando	—22.5	+68.5	1879–1940	5.45 ± 0.36	10.9	0.500 ± 0.033	1904.87	1907.68	±1.61	0.52 ± 0.10	8.9
Krynica	—20.9	+49.4	1877–1937	6.18 ± 0.27	11.0	0.562 ± 0.025	1904.83	1907.99	±1.11	0.51 ± 0.07	9.8
Lisboa	+ 9.1	+38.7	1864–1940	6.62 ± 0.34	11.1	0.596 ± 0.031	1899.68	1903.23	±1.56	0.54 ± 0.08	14.7

Table 1 (continued)

station	λ	φ	int t	P	$P_{\odot}$	k	T_e(min)	T_e(max)	p.e.	φ_0	a
Lwów	—24.0	+49.8	1876–1937	6.12 ± 0.34	11.0	0.556 ± 0.031	1904.28	1907.26	±1.40	0.49 ± 0.08	9.5
Lyon	— 4.8	+44.0	1842–1938	6.43 ± 0.40	11.2	0.574 ± 0.036	1904.72	1907.92	±2.12	0.50 ± 0.09	10.4
Madrid	+ 3.7	+40.4	1860–1930	6.20 ± 0.26	11.0	0.564 ± 0.024	1901.34	1904.34	±1.16	0.48 ± 0.06	11.6
Marseille	— 5.4	+43.3	1871–1930	6.67 ± 0.32	11.1	0.601 ± 0.029	1903.09	1906.39	±1.24	0.49 ± 0.07	17.8
Milano	— 9.2	+45.5	1768–1936	6.18 ± 0.18	11.1	0.557 ± 0.016	1901.35	1904.57	±1.29	0.52 ± 0.04	9.5
Nantes	+ 1.6	+47.3	1881–1934	6.36 ± 0.39	10.9	0.584 ± 0.036	1902.82	1905.45	±1.46	0.41 ± 0.09	7.9
Obir	—14.5	+46.5	1879–1937	6.69 ± 0.52	10.9	0.614 ± 0.048	1903.16	1906.21	±1.88	0.46 ± 0.12	8.8
Oslo	—10.7	+60.9	1866–1927 1931–1940	5.75 ± 0.36	11.1	0.518 ± 0.032	1902.50	1905.73	±1.61	0.56 ± 0.10	10.0
Östersund	—14.6	+63.2	1874–1940	5.80 ± 0.31	11.0	0.527 ± 0.028	1902.70	1906.05	±1.39	0.58 ± 0.08	7.9
Palma	+ 2.7	+39.6	1866–1925	5.58 ± 0.32	11.0	0.507 ± 0.029	1902.08	1905.07	±1.57	0.54 ± 0.09	12.8
Paris	— 2.5	+48.8	1874–1934	5.94 ± 0.31	11.1	0.535 ± 0.028	1902.06	1905.57	±1.28	0.59 ± 0.08	7.8
Roma	—12.5	+41.9	1782–1930	7.29 ± 0.27	11.3	0.645 ± 0.024	1902.78	1906.19	±1.67	0.47 ± 0.06	12.5
Säntis	— 9.3	+47.3	1883–1940	6.31 ± 0.37	10.9	0.579 ± 0.034	1899.99	1903.32	±1.48	0.53 ± 0.09	9.6
Sassari	— 8.6	+40.7	1883–1930	5.86 ± 0.36	11.0	0.533 ± 0.033	1903.20	1906.20	±1.35	0.51 ± 0.09	11.2
Sonnblick	—12.9	+47.1	1891–1937	5.93 ± 0.56	10.9	0.544 ± 0.051	1902.30	1905.48	±2.10	0.54 ± 0.14	7.6
Sulina	—29.7	+45.2	1868–1940	6.15 ± 0.33	11.0	0.559 ± 0.030	1905.77	1908.83	±1.43	0.50 ± 0.08	12.9
Trier	— 6.6	+49.8	1806–1830 1849–1936	6.34 ± 0.21	11.0	0.577 ± 0.019	1901.41	1904.16	±1.13	0.43 ± 0.05	9.6
Uppsala	—17.6	+59.9	1851–1940	6.04 ± 0.19	11.0	0.549 ± 0.017	1900.92	1903.92	±0.97	0.50 ± 0.05	10.3
Utrecht	— 5.2	+52.1	1849–1940	6.11 ± 0.21	11.2	0.546 ± 0.019	1900.56	1903.28	±1.09	0.45 ± 0.05	8.2
Valentia	+10.3	+51.9	1871–1940	6.47 ± 0.32	11.0	0.588 ± 0.029	1901.47	1904.77	±1.40	0.51 ± 0.07	7.7
Warszawa	—21.0	+52.2	1885–1937	5.81 ± 0.32	10.9	0.533 ± 0.029	1904.91	1907.36	±1.28	0.42 ± 0.08	8.6
Wien	—16.4	+48.3	1851–1938	6.91 ± 0.33	11.1	0.628 ± 0.030	1905.53	1908.86	±1.55	0.48 ± 0.07	9.0
Zürich	— 8.5	+47.4	1864–1940	5.77 ± 0.28	11.0	0.524 ± 0.025	1903.47	1906.41	±1.31	0.51 ± 0.07	8.6

$$P = \frac{1}{m+n-2}\left\{\sum_{k=1}^{m}(t_k - t_{k-1}) + \sum_{l=1}^{n}(t_l - t_{l-1})\right\}, \qquad (9)$$

while the formulae for establishing the mean time of minimum precipitation and that of maximum precipitation are

$$T_0\,(\mathrm{min}) = \frac{1}{m}\sum_{k=1}^{m} t_k \pm P\left(m_0 + \frac{m-1}{2}\right), \qquad (10)$$

and

$$T_0\,(\mathrm{max}) = \frac{1}{n}\sum_{l=1}^{n} t_l \pm P\left(n_0 + \frac{n-1}{2}\right), \qquad (11)$$

respectively, m_0, n_0 being whole numbers.

The three most important statistical causalities have been ascertained in the distribution of individual precipitation characteristics, as follows:

(i) the retardation of the moments T_0(min) and T_0(max) with the geographical longitude, the latter being here a parameter giving the distance from the Azores high-pressure area. Variations in this area together with the position of the Icelandic depression strongly influence the precipitation activity over Western and Central Europe;

(ii) a suggestion of the increase of the phase distance of the time of maximum relative to the time of minimum precipitation with the geographical longitude eastwards;

(iii) the dependence of the amplitude of precipitation on the geographical latitude.

The first relation may be watched only at the stations with the equal precipitation period, otherwise the shift is a function of time. To derive the maximum limits of the dispersion of ratio k, it is enough to require that the mutual phase shift should change at the utmost by a value of Δ int t in the course of the whole interval int t; then the resulting probable error must be less than

$$\Delta k = \frac{k}{\sqrt{N}} \cdot \frac{\Delta\,\mathrm{int}\,t}{|\mathrm{int}\,t|}. \qquad (12)$$

Table 2.

Groups of stations

group	k	Δk	|int t|	N	sign
			years		
1	0.511 ± 0.003	0.004	60	4	△
2	0.530 ± 0.001	0.004	56	6	▽
3	0.543 ± 0.001	0.004	62	4	⊙
4	0.557 ± 0.001	0.002	81	11	●
5	0.574 ± 0.001	0.003	74	9	○
6	0.585 ± 0.001	0.005	70	3	▲
7	0.600 ± 0.001	0.004	82	4	▼

Table 3.

List of stations and their characteristics of the first group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Palma	0.507 ± 0.029	+ 2.7	1902.08	1905.07	±1.57	19
Oslo	0.518 ± 0.032	—10.7	1902.50	1905.73	±1.61	20
Hvar	0.520 ± 0.019	—16.4	1905.93	1909.33	±0.81	15
Karesuando	0.500 ± 0.033	—22.5	1904.87	1907.68	±1.61	20

Table 4.

List of stations and their characteristics of the second group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Paris	0.535 ± 0.028	— 2.5	1902.06	1905.57	±1.28	17
Zürich	0.524 ± 0.025	— 8.5	1903.47	1906.41	±1.31	22
Sassari	0.533 ± 0.033	— 8.6	1903.20	1906.20	±1.35	14
Östersund	0.527 ± 0.028	—14.6	1902.70	1906.05	±1.39	20
Warszawa	0.533 ± 0.029	—21.0	1904.91	1907.36	±1.28	16
Athens	0.526 ± 0.032	—23.7	1904.66	1907.52	±1.05	9

Table 5.

List of stations and their characteristics of the third group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Utrecht	0.546 ± 0.019	— 5.2	1900.56	1903.28	±1.09	27
Sonnblick	0.544 ± 0.051	—12.9	1902.30	1905.48	±2.10	14
Bodó	0.545 ± 0.052	—14.4	1903.20	1906.41	±2.48	19
Belgrad	0.538 ± 0.038	—20.4	1904.58	1907.64	±1.46	12

Table 6.

List of stations and their characteristics of the fourth group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Madrid	0.564 ± 0.024	+ 3.7	1901.34	1904.34	±1.16	20
Aberdeen	0.564 ± 0.021	+ 2.1	1899.94	1902.50	±1.03	20
Greenwich	0.564 ± 0.024	0.0	1902.14	1905.47	±1.43	28
Gütersloh	0.550 ± 0.024	— 8.4	1905.43	1907.99	±1.35	25
Milano	0.557 ± 0.016	— 9.2	1901.35	1904.57	±1.29	51
Copenhagen	0.554 ± 0.015	—12.6	1904.10	1906.81	±1.01	35
Uppsala	0.549 ± 0.017	—17.6	1906.96	1909.96	±0.97	26
Krynica	0.562 ± 0.025	—20.9	1904.83	1907.99	±1.11	17
Lwów	0.556 ± 0.031	—24.0	1904.28	1907.26	±1.40	17
Helsinki	0.552 ± 0.021	—24.9	1907.02	1910.24	±1.22	26
Sulina	0.559 ± 0.030	—29.7	1905.77	1908.83	±1.43	20

Table 7.

List of stations and their characteristics of the fifth group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Gibraltar	0.579 ± 0.029	+ 5.4	1902.29	1905.47	±1.54	23
Lyon	0.574 ± 0.036	— 4.8	1904.72	1907.92	±2.12	28
Trier	0.577 ± 0.019	— 6.6	1907.75	1910.50	±1.13	29
Säntis	0.579 ± 0.034	— 9.3	1906.30	1909.63	±1.48	16
Catania	0.568 ± 0.033	—15.1	1906.02	1909.02	±1.17	10
Breslau	0.570 ± 0.030	—17.0	1909.41	1912.89	±1.51	21
Haparanda	0.569 ± 0.032	—24.1	1908.45	1912.39	±1.68	23
Gjesvar	0.568 ± 0.019	—25.4	1910.46	1914.04	±0.73	12
Bucuresti	0.578 ± 0.034	—26.1	1908.03	1911.65	±1.66	20

Table 8.

List of stations and their characteristics of the sixth group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Valentia	0.588 ± 0.029	+10.3	1901.47	1904.77	±1.40	19
Nantes	0.584 ± 0.036	+ 1.6	1902.82	1905.45	±1.46	14
Berlin	0.582 ± 0.021	—13.3	1904.59	1907.75	±1.13	24

Table 9.

List of stations and their characteristics of the seventh group

station	k	λ	T_0 (min)	T_0 (max)	p. e.	n
		°	y	y	y	
Lisboa	0.596 ± 0.031	+ 9.1	1899.68	1903.23	±1.56	21
Marseille	0.601 ± 0.029	— 5.4	1903.09	1906.39	±1.24	15
Frankfurt	0.604 ± 0.024	— 8.7	1900.78	1904.11	±1.38	28
Kaliningrad	0.600 ± 0.027	—20.5	1901.84	1905.37	±1.50	25

For Δ int $t = \pm 1$ year Table 2 divides all the stations into seven groups and for each of them it gives the maximum deviation Δk, mean value of k, its probable error, mean length of $|\text{int}\, t|$, number of stations N and a mark used in Figure 3. The following Tables 3 to 9 show the distribution of stations into groups, which comply with the condition $\Delta \text{int}\, t \leq \pm 1$ year. For each station are given: its name, ratio k, geographical longitude, mean moments of extremes on the precipitation curve, their probable errors, and the number of extremes, from which factor k has been determined. The dependences $T_0(\text{min}) = f(\lambda)$ and $T_0(\text{max}) = g(\lambda)$ are plotted in Figure 3. Table 10 then includes for each group the retardation of moments of precipitation-curve extremes (in km per day), α_0, the mean geographical latitude, φ, and the number of stations used. Retardation α_0 has been computed from the formula

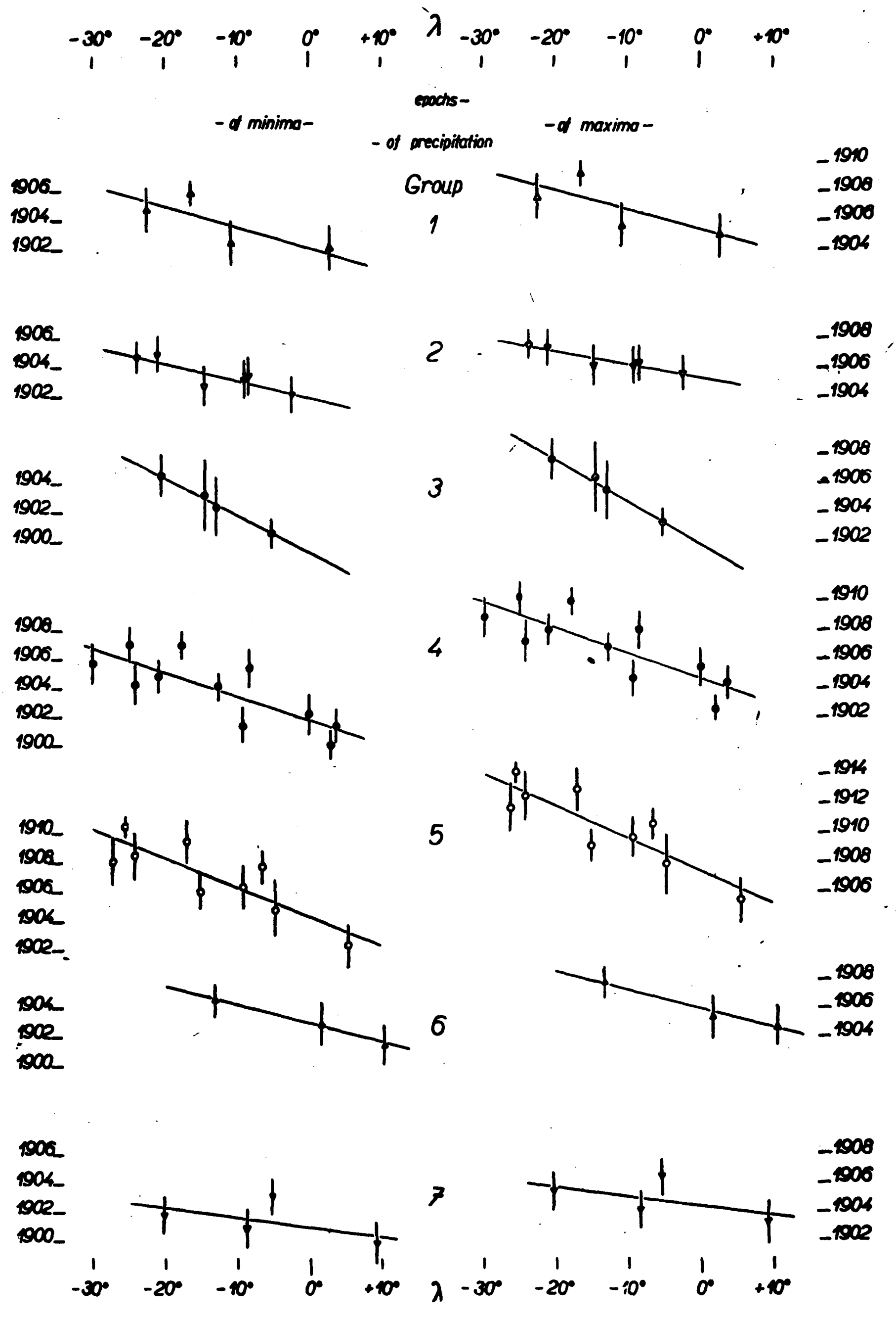

Fig. 3. Shift of the moments of precipitation extremes with the geographical longitude in West and Central Europe.

Table 10.

Retardation of the precipitation extremes in Europe with the geographical longitude

group	α_0		$\bar{\varphi}$	N
	min	max	°	
1	1.33 ± 0.49	1.34 ± 0.55	$+53.1$	4
2	1.71 ± 0.33	2.33 ± 0.35	$+48.4$	6
3	0.69 ± 0.04	0.63 ± 0.04	$+52.8$	4
4	1.14 ± 0.19	1.10 ± 0.18	$+51.5$	11
5	1.00 ± 0.16	0.89 ± 0.12	$+49.7$	9
6	1.48 ± 0.07	1.49 ± 0.15	$+50.6$	3
7	3.1 ± 2.2	3.2 ± 2.2	$+46.7$	4
	1.39 ± 0.18	1.44 ± 0.21	$+50.1$	41

$$\alpha_0 = -0.304 \cos \varphi \left(\frac{\partial T_0(\text{extr})}{\partial \lambda} \right)^{-1}, \tag{13}$$

where the time of extreme precipitation, $T_0(\text{extr})$, is expressed in years. On the average, both the extremes lead to a value of

$$\alpha_0 = 1.41 \pm 0.13 \text{ km per day.}$$

On the basis of HELLMANN's material we arrive at some higher values, about 2 to 3 km per day. DROSDOV (1934) found for the European area of the U.S.S.R. a value of 500 to 700 km per year, i. e. 1.4 to 1.9 km per day.

The second relation, a suggestion of the increase of the phase-distance of the time of maximum precipitation relative to its minimum with the geographical

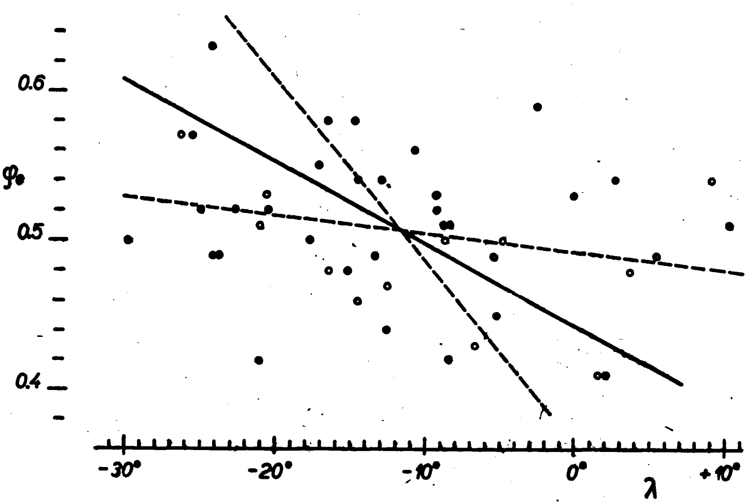

Fig. 4. Increase of the phase distance of the time of maximum precipitation relative to its minimum with the geographical longitude in West and Central Europe.

longitude is represented in Figure 4. All the stations are divided into three groups according to the value of correlation coefficient ψ^* between the courses of precipitation in odd and even solar cycles. This distribution has been introduced as follows:

Group I: correlation coefficient $\psi^* \in \langle -1.0, -0.3 \rangle$;
Group II: correlation coefficient $\psi^* \in (-0.3, +0.3)$;
Group III: correlation coefficient $\psi^* \in \langle +0.3, +1.0 \rangle$.

Table 11.

Correlation degree of the relation between φ_0 and the geographical longitude

group	$\psi(\lambda, \varphi_0)$	N
I	-0.33 ± 0.15	15
II	-0.26 ± 0.16	16
III	-0.09 ± 0.19	13
Σ	-0.24 ± 0.10	44

In Figure 4 the regression lines are represented by dashed lines, the correlation line by a full line. The correlation degree of the relation is, however, comparatively low and in the third group it is completely absent. The values of the correlation coefficient for each group separately as well as together are included in Table 11.

Finally, the third causality, the dependence of the amplitude of precipitation on the geographical latitude, is represented in Figure 5. It shows that the maximum amplitude is attained in low geographical latitudes, it drops rapidly with the increasing latitude down to about 7 per cent near $\varphi = +56^0$, and then it slightly increases again.

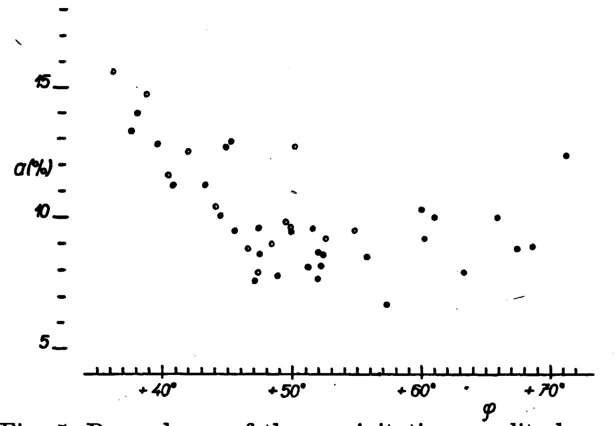

Fig. 5. Dependence of the precipitation amplitude on the geographical latitude in West and Central Europe.

In Figures 4 and 5 the stations of the first group are denoted by the mark ○, those of the second group by ● and those of the third group by ⊙. The distribution of the lengths of precipitation periods at the stations of each of the three groups separately, as well as together is given in Figure 12.

Each of these effects misrepresents the course of precipitation which, when summarized all over West and Central Europe, could give a general characteristic of the precipitation activity in this area.

3. RELATION BETWEEN THE PRECIPITATION ACTIVITY AT EUROPEAN STATIONS IN ODD AND EVEN SOLAR CYCLES. DERIVATION OF THE CORRELATION COEFFICIENT

Křivský (1951) derived the correlation degree between the courses of precipitation in odd and even cycles on the basis of data from 32 stations of Europe and Iceland. He showed that:

(a) the correlation coefficient varies from station to station, obtaining successively all the possible values;

(b) its changes with the geographical latitude and longitude may be understood as a continuous function, and therefore a system of lines of equal correlation degree may be constructed (see Figure 4 of Křivský's paper).

The most plausible explanation of this effect is the assumption of the absence of commensurability between the precipitation period and the length of solar cycle for most stations. Obviously, there are two possibilities: either $P \geq P_\odot$, or $P < P_\odot$.

In the paper of Křivský (1951) the precipitation period of 11 years or 23 years is considered on the basis of Hanzlík's research (Hanzlík, 1936), i. e. a special case (commensurability of P and $P_\odot$) of the first possibility. Of course, Hale's period corresponds to the negative and the eleven-year cycle to the positive values of correlation coefficients.

The latter conception ($P < P_\odot$) has been brought about thanks to the results of the foregoing sections in this paper. We will apply it in the following consideration, where the correlation coefficient between the courses of precipitation in both cycle types will be derived. Let us, first of all, assume that the precipitation amplitude, a_0, as well as the precipitation period do not depend on the cycle type. Further we assume that the precipitation variation may be expressed through a cosine curve; then for the odd cycle we can write:

$$A(o) = a_0 \cos \frac{2\pi}{P} (t - t_1), \tag{14}$$

where t_1 is the time of maximum precipitation. Analogously, for the following or preceding even cycle we have:

$$A(e) = a_0 \cos \frac{2\pi}{P} (t - t_1 \pm P_\odot) \tag{15}$$

Denoting further the outset of the odd cycle of solar activity as t_0, it is

$$\left.\begin{aligned} \Phi &= \frac{t - t_0}{P_\odot}, \\ \Phi_1 &= \frac{t_1 - t_0}{P_\odot}, \end{aligned}\right\} \tag{16}$$

and, hence,

$$A(o) = a_0 \cos \frac{2\pi}{k} \xi, \tag{17}$$

$$A(e) = a_0 \cos \frac{2\pi}{k} (\xi \pm 1), \tag{18}$$

where we denote

$$\left.\begin{aligned} \xi &= \Phi - \Phi_1, \\ k &= \frac{P}{P_\odot}. \end{aligned}\right\} \tag{19}$$

On the basis of what has been said at the beginning of this section, and of what has been found in the foregoing section, for k we will consider the interval

$$0.5 \leqslant k \lessdot 1.0.$$

The coefficient of correlation between the functions $A(o) = F(\Phi)$ and $A(e) = G(\Phi)$ is given by the expression:

$$\psi^*(o, e) = \frac{\int_0^1 d\Phi \int_0^1 A(o) . A(e) d\Phi - \int_0^1 A(o)\, d\Phi \int_0^1 A(e)\, d\Phi}{[\int_0^1 d\Phi \int_0^1 (A(o))^2 d\Phi - (\int_0^1 A(o)\, d\Phi)^2]^{1/2} \cdot [\int_0^1 d\Phi \int_0^1 (A(e))^2 d\Phi - (\int_0^1 A(e)\, d\Phi)^2]^{1/2}} \tag{20}$$

After introducing (17), (18) and the first equation of (19) into (20), we obtain:

$$\left.\begin{aligned} \psi^*(o, e) = \\ = \Bigg[\int_{-\Phi_1}^{1-\Phi_1} \cos\frac{2\pi}{k}\xi . \cos\frac{2\pi}{k}(\xi \pm 1)\, d\xi - \int_{-\Phi_1}^{1-\Phi_1} \cos\frac{2\pi}{k}\xi\, d\xi . \int_{-\Phi_1}^{1-\Phi_1} \cos\frac{2\pi}{k}(\xi \pm 1)\, d\xi\Bigg] \times \\ \times \left[\int_{-\Phi_1}^{1-\Phi_1} \cos^2\frac{2\pi}{k}\xi\, d\xi - \left(\int_{-\Phi_1}^{1-\Phi_1} \cos\frac{2\pi}{k}\xi\, d\xi\right)^2\right]^{-1/2} \times \\ \times \left[\int_{-\Phi_1}^{1-\Phi_1} \cos^2\frac{2\pi}{k}(\xi \pm 1)\, d\xi - \left(\int_{-\Phi_1}^{1-\Phi_1} \cos\frac{2\pi}{k}(\xi \pm 1)\, d\xi\right)^2\right]^{-1/2} \end{aligned}\right\} \tag{21}$$

This formula is valid for two successive cycles. For a series of cycle couples the same formula holds good with a certain average value of Φ. Now we will refer to Section 4, where I draw to a conclusion that the maximum precipitation derived from statistics of European stations falls on the period round the beginning of the odd cycle. To be able to apply relation (21) to the statistical material, we must put

$$\Phi_1 \doteq 0.$$

Then (21) turns into the form of

$$\left.\begin{aligned} \psi^*(o, e) = \Bigg[\cos\frac{2\pi}{k} + \frac{k}{2\pi}\sin\frac{2\pi}{k}\Bigg((1 \pm 1)\cos^2\frac{2\pi}{k} \mp 1 - \\ - \frac{k}{\pi}\sin\frac{2\pi}{k}\left\{(1 \pm 1)\cos\frac{2\pi}{k} \mp 1\right\}\Bigg)\Bigg] \times \\ \times \left[1 + \frac{k}{2\pi}\sin\frac{2\pi}{k}\left(\cos\frac{2\pi}{k} - \frac{k}{\pi}\sin\frac{2\pi}{k}\right)\right]^{-1/2} \times \\ \times \Bigg[1 + \frac{k}{2\pi}\sin\frac{2\pi}{k} . \Bigg\{\cos\frac{2\pi}{k}\left([1 \pm 1]\cos\frac{4\pi}{k} \mp 1\right) - \\ - \frac{k}{\pi}\sin\frac{2\pi}{k}\left(2[1 \pm 1]\cos\frac{2\pi}{k}\left[\cos\frac{2\pi}{k} - 1\right] + 1\right)\Bigg\}\Bigg]^{-1/2}. \end{aligned}\right\} \tag{22}$$

If even cycles follow odd cycles in the material used, i. e. if the positive sign is valid in (15), the correlation coefficient is

$$\left.\begin{aligned}\psi_+^* = &\left[\cos\frac{2\pi}{k} + \frac{k}{2\pi}\sin\frac{2\pi}{k}\left\{2\cos^2\frac{2\pi}{k} - 1 - \frac{k}{\pi}\sin\frac{2\pi}{k}\times\right.\right.\\ &\left.\left.\times\left(2\cos\frac{2\pi}{k} - 1\right)\right\}\right]\cdot\left[1 + \frac{k}{2\pi}\sin\frac{2\pi}{k}\left(\cos\frac{2\pi}{k} - \frac{k}{\pi}\sin\frac{2\pi}{k}\right)\right]^{-1/2}\times\\ &\times\left[1 + \frac{k}{2\pi}\sin\frac{2\pi}{k}\left\{2\cos\frac{2\pi}{k}\left(\cos\frac{4\pi}{k} - \frac{1}{2}\right) - \frac{k}{\pi}\sin\frac{2\pi}{k}\times\right.\right.\\ &\left.\left.\times\left(4\cos\frac{2\pi}{k}\left(\cos\frac{2\pi}{k} - 1\right) + 1\right)\right\}\right]^{-1/2},\end{aligned}\right\} \quad (23)$$

while, if even cycles precede odd cycles, i. e. if the negative sign is valid in (15), we obtain:

$$\begin{aligned}\psi_-^* = &\left[\cos\frac{2\pi}{k} + \frac{k}{2\pi}\sin\frac{2\pi}{k}\left(1 - \frac{k}{\pi}\sin\frac{2\pi}{k}\right)\right]\times\\ &\times\left[1 + \frac{k}{2\pi}\sin\frac{2\pi}{k}\left(\cos\frac{2\pi}{k} - \frac{k}{\pi}\sin\frac{2\pi}{k}\right)\right]^{-1}.\end{aligned} \quad (24)$$

In (23) and (24) all the terms including a factor of $\frac{k}{2\pi}$ have within the considered interval of $k \in \langle 0.5, 1.0\rangle$ the character of correction terms. Hence the two expressions may be expanded in a series, and high enough accuracy will be attained when all terms of higher order than $\left(\frac{k}{2\pi}\right)^2$ are neglected. For the correlation coefficients ψ_+^* and ψ_-^* we obtain then the expressions as follows:

$$\left.\begin{aligned}\psi_+^* &= \sum_{i=0}^{2}\beta_+^{(i)}\left(\frac{k}{2\pi}\right)^i,\\ \psi_-^* &= \sum_{i=0}^{2}\beta_-^{(i)}\left(\frac{k}{2\pi}\right)^i.\end{aligned}\right\} \quad (25)$$

Coefficients $\beta_+^{(i)}$ and $\beta_-^{(i)}$ are the functions of ratio k:

$$\left.\begin{aligned}\beta_+^{(0)} &= \cos\frac{2\pi}{k},\\ \beta_+^{(1)} &= \sin^3\frac{2\pi}{k}\left(1 - \sin^2\frac{2\pi}{k}\right),\\ \beta_+^{(2)} &= -\sin^2\frac{2\pi}{k}\left[6 - 8\sin^2\frac{2\pi}{k} + 2\sin^4\frac{2\pi}{k} - \cos\frac{2\pi}{k}\right)\times\\ &\times\left(\frac{19}{4} - \frac{49}{4}\sin^2\frac{2\pi}{k} + 17\sin^4\frac{2\pi}{k} - 12\sin^6\frac{2\pi}{k}\right)\right],\end{aligned}\right\} \quad (26)$$

and similarly

$$\left.\begin{aligned} \beta_-^{(0)} &= \cos\frac{2\pi}{k}\,, \\ \beta_-^{(1)} &= \sin^3\frac{2\pi}{k}\,, \\ \beta_-^{(2)} &= -2\sin^2\frac{2\pi}{k}\left[1-\frac{5}{2}\cos\frac{2\pi}{k}+\sin^2\frac{2\pi}{k}\cos\frac{2\pi}{k}\right]. \end{aligned}\right\}\quad(27)$$

The course of functions $\beta_+^{(i)}$ and $\beta_-^{(i)}$ within the interval $0.5 \leqslant k \leqslant 1.0$ is represented in Figure 6. The course of the correlation coefficients ψ_+^* and ψ_-^* is included in Figure 7.

The theoretically derived relation is compared with the observations in Figure 8, where the correlation coefficient ψ^* is plotted against ratio k, derived directly from the material, comprising 44 European stations. The curve of ψ_-^* is plotted in a full line, that of ψ_+^* in a dashed line. The probable errors of the precipitation periods are given and individual stations are denoted by: *A* - Athens, *Ab* - Aberdeen, *B* - Berlin, *Bc* - Bucuresti, *Bd* - Bodó, *Be* - Belgrad, *Br* - Breslau, *C* - Catania, *Cp* - Copenhagen, *F* - Frankfurt a. M., *G* - Greenwich, *Gi* - Gibraltar, *Gj* - Gjesvar, *Gü* - Gütersloh, *H* - Haparanda, *He* - Helsinki, *Hv* - Hvar, *K* - Kaliningrad, *Ka* - Karesuando, *Kr* - Krynica, *L* - Lisboa, *Lw* - Lwów, *Ly* - Lyon, *M* - Marseille, *Ma* - Madrid, *Mi* - Milano, *N* - Nantes, *Ob* - Obir, *Os* - Oslo, *Ö* - Östersund, *P* - Paris, *Pa* - Palma, *R*-Roma, *S*-Sassari, *Sn* - Säntis, *So* - Sonnblick, *Su* - Sulina, *T* - Trier, *U* - Uppsala, *Ut* - Utrecht, *V*-Valentia, *W* - Warszawa, *Wi* - Wien, *Z* - Zürich.

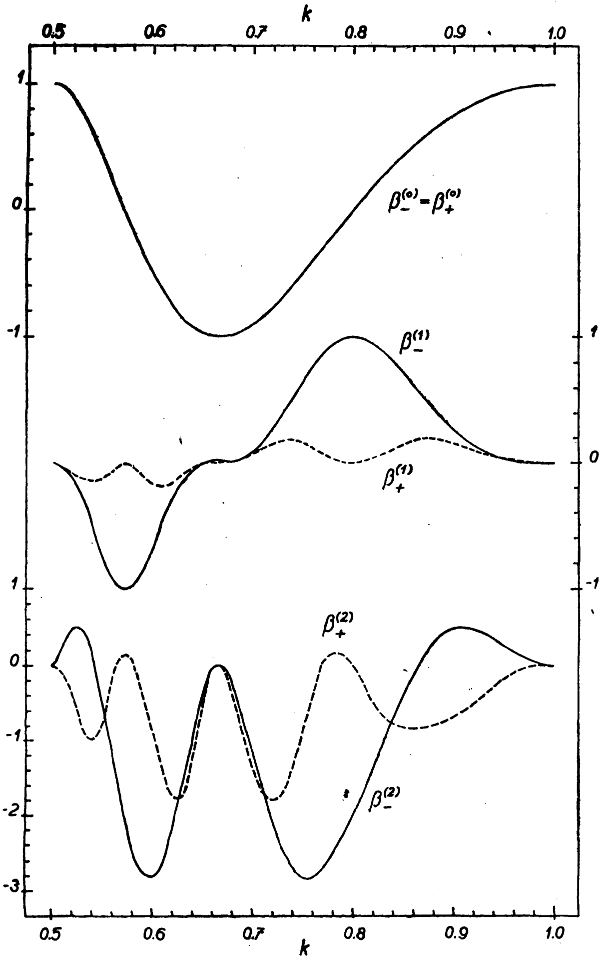

Fig. 6. Coefficients $\beta_+^{(i)}$ and $\beta_-^{(i)}$ in dependence on the reduced precipitation period.

The empirical correlation coefficients, $\psi^*(o, e)$, partly taken over from the paper of KŘIVSKÝ (1951), partly computed by the author, comprise both the dependence of $\psi^* = \psi^*(k)$ and a number of other effects, such as deviations from formulae (14) and (15), dispersion in precipitation amplitudes, long-term variations, random deviations, etc., that, on an average, reduce the value of correlation coeffiicient as computed from observations. In fact, no coefficient has been in its absolute value greater than 0.85; therefore the ψ^*-coefficient has been derived from the "observed" ψ-coefficient according to

$$\psi^* = 1.18\,\psi. \tag{28}$$

With respect to the fact that the particular assumptions, on which the theoretical curves of ψ^*_+ and ψ^*_- have been derived, are in practice fulfilled only to a first approximation, the agreement between the theory and observation is in Figure 8 quite good. The regression lines of the empirical dependence are sketched in Figure 8 in full lines.

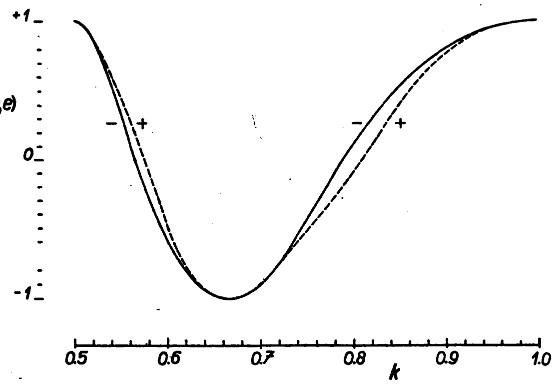

Fig. 7. Theoretical dependence of correlation coefficient ψ^* on the reduced precipitation period.

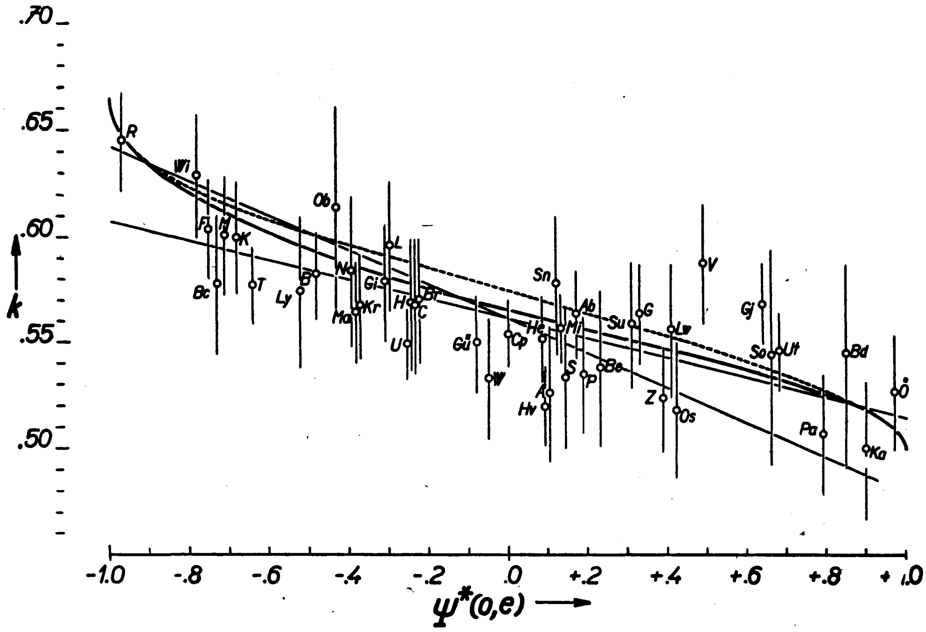

Fig. 8. A comparison of the theoretical relation between the reduced precipitation period and correlation coefficient ψ^* with the material in West and Central Europe.

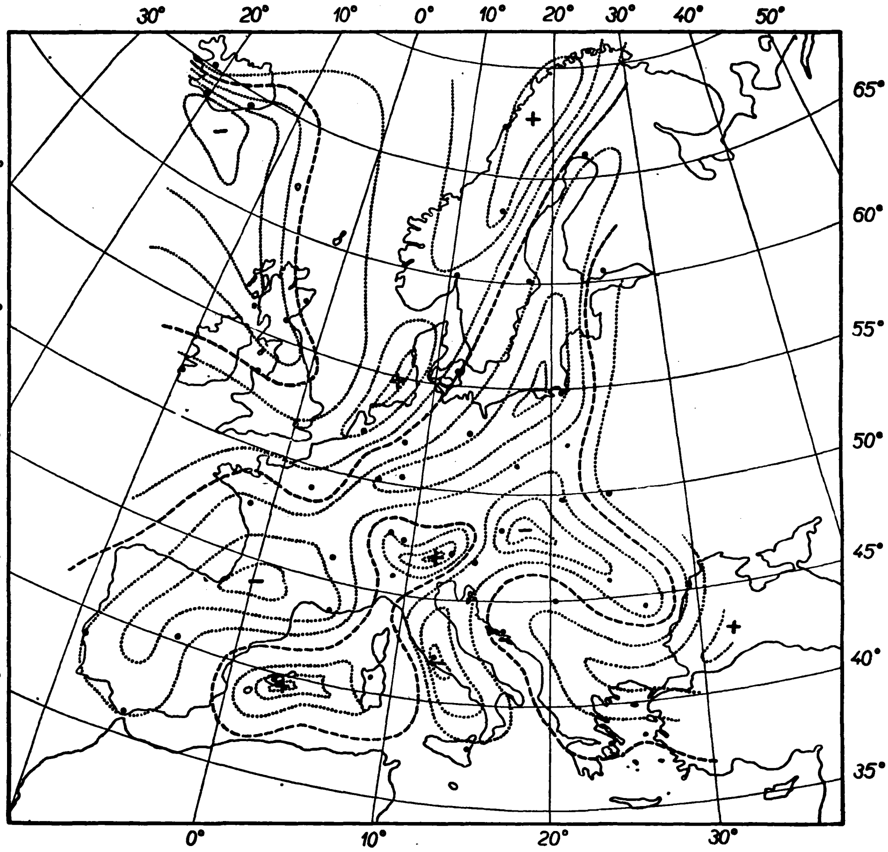

Fig. 9. The system of lines of equal correlation degree in West and Central Europe. Dotted lines stand for the isolines of $\psi^* = \pm 0.3$, ± 0.6 and ± 0.9, dashed lines indicate the zero isolines.

Another confirmation of the theoretical dependence $\psi^* = \psi^*(k)$ is seen from Figure 12, where the mode of the period shifts from 5.5 years for the first group of stations (negative φ^*) to 4 years for the third group (positive ψ^*).

After the supplementation of the former list of correlation coefficients $\psi^*(o, e)$ published by KŘIVSKÝ (1951) with 17 values from other European stations, it has been possible to spread and partly correct the system of lines of equal correlation degree in Figure 4 of KŘIVSKÝ's paper. A map of Europe with a new system of these isolines is in Figure 9. Individual stations are represented by dots. A few causalities may be found: first of all, in many places the zero isoline follows the coast of the continent, as e. g. both the coasts of Spain and France, partly that of Italy and mainly the Balkan Peninsula. The form of the isolines is strongly influenced by mountain chains; the centres of isoline systems are placed in the Pyrenees, Alps, Scandinavian Mountains and in the neighbourhood of the Tatras. The system of negative coefficients $\psi^*(o, e)$ appears to penetrate into Europe through two corridors: partly over Spain and partly over Italy. However, the system of positive coefficients $\psi^*(o, e)$

over the Baleares and the Alps may be connected with the system over the Black Sea, and Italy may thus become something like a "gulf" of the negative system of $\psi^*(o, e)$.

4. AVERAGE PRECIPITATION COURSE IN WESTERN AND CENTRAL EUROPE WITHIN A SOLAR CYCLE

In the foregoing sections and, first of all, in Figure 1 we have seen how the precipitation curves are complicated, and how at the stations comparatively near to each other the conditions are different in this respect. This circumstance is the reason for the assertion that from the meteorological point of view it is unjustifiable to construct the precipitation course for larger areas by summing up the precipitation curves of individual stations. On the other hand, to be able to compare the results of the precipitation investigation with those of comet-discovery statistics, we must consider an area as extensive as possible so that the influence of random deviations relative to the effects of systematic character may be as small as possible. It means that from the view-point of cometary climatology we must define some quantitative characteristic of precipitation all over West and Central Europe.

Considering the system of the 44 European stations to be representative, the most probable course of the average precipitation activity may be with sufficient accuracy derived in this area by means of registering the moments of extreme precipitation at the individual stations for a statistical conception to be made up. Let us put an equal weight to each station and each extreme, and let us further call the expression

$$2\,\frac{N_{\max} - N_{\min}}{N_{\max} + N_{\min}} \tag{29}$$

the "relative precipitation activity" all over the investigated area, $N_{\max}$ and $N_{\min}$ being the numbers of maxima and minima, respectively, ascertained on the precipitation curves of individual stations within the investigated interval of time. The differences between the numbers of precipitation maxima and minima may be formally understood as quantities analogous to random deviations. The value of the resulting mean deviation of the latter depends on the number of the events. Assuming that the mean value, independent of the total number of events (varying with time), is what we are looking for, expression (29) must be multiplied by the square root of their number, so that the relative precipitation activity within the studied area is given by the formula

$$\Delta A = 2\,\frac{N_{\max} - N_{\min}}{\sqrt{N_{\max} + N_{\min}}}\,. \tag{30}$$

If we now divide the whole period, in the course of which the quantities $N_{\max}$ and $N_{\min}$ are investigated, into the intervals corresponding to the phase ranges of the solar cycle Φ from 0.06 to 0.15, 0.16 to 0.25, etc., we obtain the dependence $\Delta A = \Delta A(\Phi)$, which is comprised in Table 12 for both odd and even cycles separately for the station groups I, II, III (see Section 2) as well as for all the European stations together. This table indicates that each of the three groups gives in odd cycles the main precipitation maximum near the solar-

Table 12.

Dependence $\Delta A = \Delta A\ (\Phi)$ in Europe

int Φ	I		II		III		Σ	
	odd	even	odd	even	odd	even	odd	even
0.06—0.15	+3.44	—4.13	+4.22	—0.53	—0.57	—0.30	+5.63	—3.00
0.16—0.25	+0.51	—2.54	+1.82	—0.26	—0.60	0.00	+1.09	—1.68
0.26—0.35	+2.58	+0.29	—0.77	0.00	+0.57	+2.30	+1.38	+1.36
0.36—0.45	—1.66	+1.09	—4.35	+1.35	+1.73	+1.33	—2.58	+2.16
0.46—0.55	+0.26	+0.29	—1.09	0.00	+1.11	0.00	+0.15	+0.18
0.56—0.65	—1.33	—0.82	—0.54	—2.14	—0.30	—1.85	—1.29	—2.77
0.66—0.75	—2.22	+1.11	+0.79	—1.07	—0.63	—0.65	—1.15	—0.33
0.76—0.85	—1.24	+0.53	—1.46	—1.05	+1.04	—1.43	—1.09	—1.09
0.86—0.95	+3.71	—1.43	+1.37	+0.28	+2.19	+0.31	+4.18	—0.51
0.96—0.05	+4.95	—3.61	+1.09	+0.27	+0.92	—2.04	+4.16	—3.16

Table 13.

Correlation coefficient ψ^* (o, e) of the relation $\Delta A = \Delta A\ (\Phi)$

group	ψ^* (o, e)
I	—0.89
II	—0.34
III	+0.31
Σ	—0.60

Table 14.

Precipitation activity in odd and even cycles for individual groups of European stations

group	$\overline{\Delta A}$	
	odd cycles	even cycles
I	+2.19	—1.78
II	+0.29	—0.59
III	+0.99	—0.51

Table 15.

Precipitation activity in individual solar cycles at European stations

solar cycle	$\overline{\Delta A}$
6	—0.55
7	+1.16
8	—1.46
9	+1.41
10	—2.22
11	+2.58
12	—1.45
13	+0.15
14	—1.09
15	+1.06
16	—0.37

-activity minimum, the second maximum then near $\Phi \approx 0.4$ to 0.6, consequently, it follows the maximum of sunspot numbers. In even cycles the maxima and the minima are respectively shifted according to the length of the precipitation period, or, as results from Section 3, in accordance with the size of correlation coefficient ψ^* (o, e), the values of which are included for individual groups of stations as well as together in Table 13.

An approximate view on the total precipitation amount in odd and even cycles is given in Table 14, which includes the average values $\overline{\Delta A}$ in both cycle types for each of the three groups of the stations separately as well as for the total. The table indicates in a telling way that the odd cycles are as to the precipitation much more active. Even more convincing evidence for the increased precipitation activity in odd cycles results from the average values ΔA in individual solar cycles (Table 15).

5. AVERAGE PRECIPITATION COURSE IN WESTERN AND CENTRAL EUROPE AS A FUNCTION OF TIME

If we choose a year as an interval for the investigation of N_{max} and N_{min} from (30), the latter may be used for determining the dependence $\Delta A = \Delta A(t)$, which will be called the "integrated precipitation course" in West and Central Europe. This relation is represented in Figure 10. The average period is

$$P_0 = 6.22 \pm 0.12 \text{ years}, \qquad (31)$$

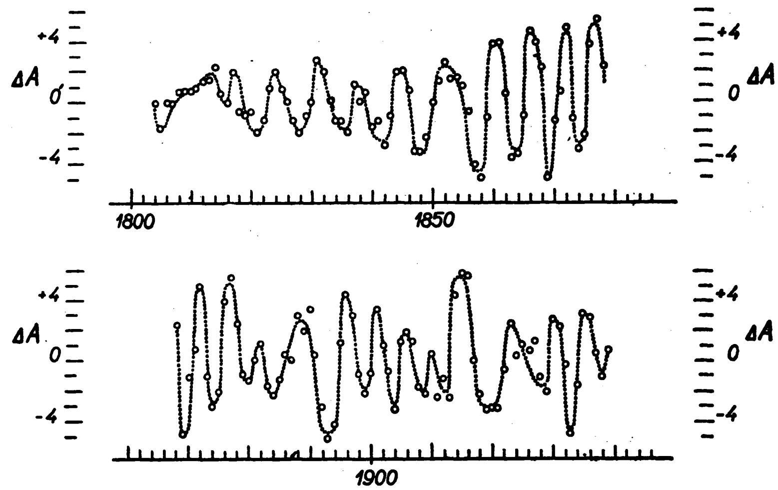

Fig. 10. The integrated precipitation course in West and Central Europe.

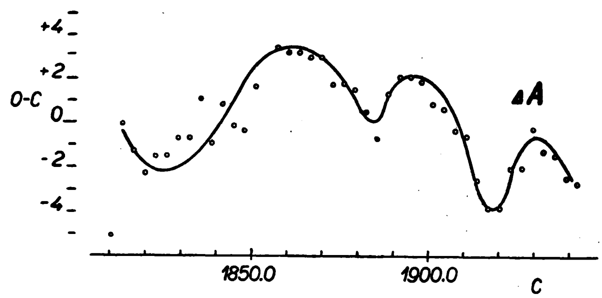

Fig. 11. Variations in the length of the precipitation period in West and Central Europe.

which wonderully agrees with the value computed as the weighted mean from the 44 stations:

$$6.22 \pm 0.04 \text{ years}.$$

The length of the period, however, is not constant, but it varies within the limits from 4 years to 11 years. The moments of the extremes are given by

$$\left.\begin{aligned} T_0(\text{min}) &= 1903.85 \pm 0.33, \\ T_0(\text{max}) &= 1906.84 \pm 0.30. \end{aligned}\right\} \qquad (32)$$

The variation in the length of the period appears very well in Figure 11, where it is a function of the slope of the tangent to the $(O - C)$ curve, the latest expressing the difference between the time of the extreme as observed and that as computed from the formulae

$$T_0(\text{min}) = 1903.85 + 6.22\,i,$$
$$T_0(\text{max}) = 1906.84 + 6.22\,j,$$

where i passes the whole numbers within the interval of $-15 \leqslant i \leqslant +6$, and j those within $-15 \leqslant j \leqslant +5$. There exists a certain suggestion of a forty-year periodicity of variations in the length of the precipitation period.

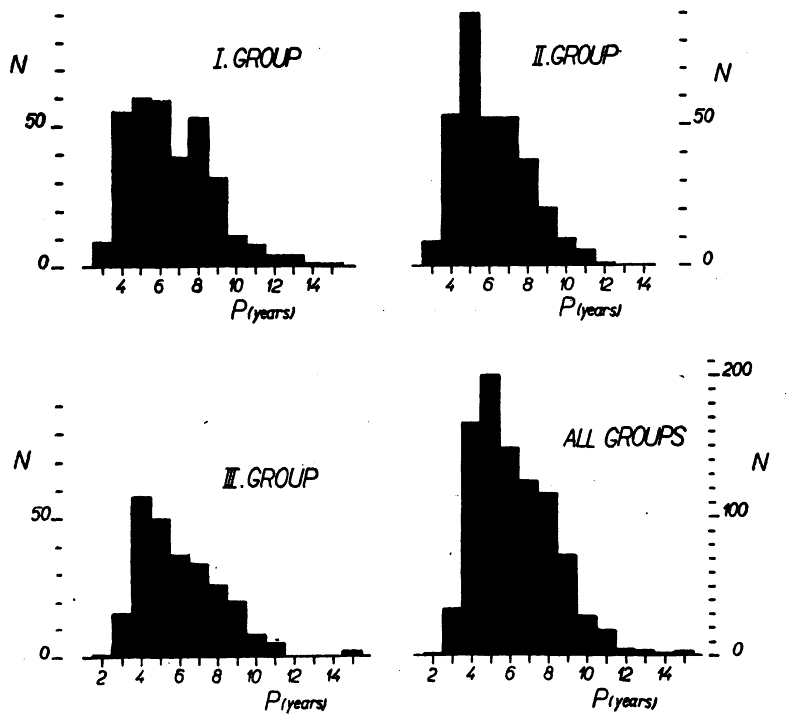

Fig. 12. The distribution of the lengths of precipitation periods at European stations.

In addition to the described way there exists another possibility for determining the moments of the maxima and the minima of the integrated precipitation course all over West and Central Europe. Let us construct the curves of frequency of the moments of both precipitation extremes for each of the 44 stations, included in Table 1 within an interval of a few precipitation periods. In Figure 13 the period from 1897 till 1914 is selected; the curve of frequency of the moments of the precipitation minima is at the top, that of the precipitation maxima at the bottom. The smoothed-out curves are given in dots. It is interesting that just after the maximum a sudden rapid decrease takes place, which disappears with the same quickness. Regardless of this disturbing effect a 6-year course of the frequency curves is well apparent.

Let us now turn our attention to such a part of the curves, in order that one extreme of each station should be included in the respective interval of time at the utmost. In accordance with the lengths of the average precipitation periods at the individual stations the interval of 5.5 years has been chosen and placed in such a manner that the maximum on the frequency curve has lain in its centre.

The precipitation course at each station is assumed again in the form ($i = 1, \ldots, 44$)

$$A_i(T) = a_i \cos \frac{2\pi}{P_i} (T - T_i), \tag{33}$$

where T_i is the time of the precipitation maximum at the i-th station, P_i is the respective precipitation period and a_i the semi-amplitude (see the last column of Table 1). If P_0 is the most probable value of the "average" precipitation period all over the investigated area, then, after putting

$$\Delta P_i = P_i - P_0,$$

and neglecting higher orders of $\frac{\Delta P_i}{P_0}$, we obtain equation (33) in the form:

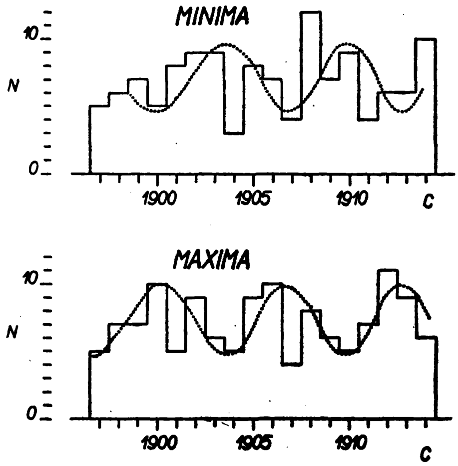

Fig. 13. The frequency curves of the moments of basic precipitation maxima and minima at West and Central European stations.

$$\left.\begin{aligned} A_i(T) = a_i \cos\varphi \left[\cos\varphi_i + \varphi_i \frac{\Delta P_i}{P_0} \sin\varphi_i\right] - a_i\varphi_i \frac{\Delta P_i}{P_0} \cos\varphi \sin\varphi_i + \\ + a_i \sin\varphi \left[\sin\varphi_i - \varphi_i \frac{\Delta P_i}{P_0} \cos\varphi_i\right] + a_i\varphi_i \frac{\Delta P_i}{P_0} \sin\varphi \cos\varphi_i, \end{aligned}\right\} \tag{34}$$

where

$$\varphi = \frac{2\pi}{P_0} T,$$

$$\varphi_i = \frac{2\pi}{P_0} T_i.$$

Equation (34) is valid for any T. For the integrated precipitation course at time T we write (n is the number of the stations):

$$A_0(T) = \frac{1}{n} \sum_{i=1}^{n} A_i(T), \tag{35}$$

where we again put

$$A_0(T) = a_0 \cos \frac{2\pi}{P_0} (T - T_0). \tag{36}$$

In this equation T_0 is the time of the precipitation maximum (in statistical sense) for the whole area and a_0 the precipitation semiamplitude in it. Applying equations (34), (35) and (36) to

$$T_{01} = T_0 \pm \frac{P_0}{4} \tag{37}$$

we obtain for T_{01} a transcendental equation

$$\operatorname{tg} \varphi_{01} = \frac{\varphi_{01} \sum_{i=1}^{n} F_i - \sum_{i=1}^{n} E_i}{\varphi_{01} \sum_{i=1}^{n} H_i + \sum_{i=1}^{n} G_i},$$

where

$$\left.\begin{aligned} \varphi_{01} &= \frac{2\pi}{P_0} T_{01}, \\ E_i &= a_i \left(\cos \varphi_i + \varphi_i \frac{\Delta P_i}{P_0} \sin \varphi_i \right), \\ F_i &= a_i \frac{\Delta P_i}{P_0} \sin \varphi_i, \\ G_i &= a_i \left(\sin \varphi_i - \varphi_i \frac{\Delta P_i}{P_0} \cos \varphi_i \right), \\ H_i &= a_i \frac{\Delta P_i}{P_0} \cos \varphi_i. \end{aligned}\right\} \tag{39}$$

Equation (37) yields, of course, a double solution of time T_0, but the two values differ from each other for more than three years, so that the mistake is excluded by comparing them with their graphical representation in Figure 13.

If we write equation (35) for T_0, the semi-amplitude of the integrated precipitation course may be obtained from

$$a_0 = \frac{1}{n} \cos \varphi_0 \left[\sum_{i=1}^{n} E_i - \varphi_0 \sum_{i=1}^{n} F_i + \operatorname{tg} \varphi_0 \left(\sum_{i=1}^{n} G_i + \varphi_0 \sum_{i=1}^{n} H_i \right) \right]. \tag{40}$$

The disadvantage of this method when applied to the precipitation data is a low accuracy of the period of the integrated precipitation course as results from Figure 13. For the most probable value it has been accepted

$$P_0 = 6.3 \text{ years.}$$

However, from how the P_0-period appears in (39) it follows that no high accuracy is required.

The treatment of the precipitation activity at the European stations has given in this way the results as follows:

$$\sum_{i=1}^{40} E_i = -0.4958,$$

$$\sum_{i=1}^{40} F_i = +0.0318,$$

$$\sum_{i=1}^{40} G_i = -0.0658,$$

$$\sum_{i=1}^{40} H_i = +0.0121,$$

when the minima have been investigated in the period of 1901.0 to 1906.5, and

$$\sum_{i=1}^{40} E_i = +0.6609,$$

$$\sum_{i=1}^{40} F_i = -0.0242,$$

$$\sum_{i=1}^{40} G_i = +0.2454,$$

$$\sum_{i=1}^{40} H_i = -0.0112,$$

when the maxima have been investigated in the period of 1904.5 to 1910.0 (see Figure 13). The resulting moments of the extremes are then

$$\left.\begin{aligned} T_0(\text{min}) &= 1903.18 \pm 0.18, \\ T_0(\text{max}) &= 1906.50 \pm 0.17. \end{aligned}\right\} \quad (41)$$

These values are in good agreement with those obtained earlier in this paper. Inserting (41) into (40) the semi-amplitudes of the integrated precipitation course may be established:

$$\left.\begin{aligned} a_0(\text{min}) &= 1.5 \pm 0.1 \text{ per cent}, \\ a_0(\text{max}) &= 2.1 \pm 0.1 \text{ per cent}. \end{aligned}\right\} \quad (42)$$

6. LONG-TERM PRECIPITATION VARIATIONS IN WESTERN AND CENTRAL EUROPE

The precipitation course at the 44 European stations makes it possible to compare its long-term course with that of the eighty-year solar period, too. A momentous obstacle is the fact that at most European stations the precipitation has been measured since seventies of the 19th century, and only at a few of them since the end of the 18th century. Hence, the investigation of the long-term precipitation variation all over West and Central Europe may be carried out over a period less than one eighty-year solar period.

Comparing the precipitation courses at various European stations the following may be ascertained: the long-term precipitation periods are different at various stations as to their length as well as to the positions of the extremes, the latter being, above all, a function of the geographical latitude. The differences are considerable, as to their size comparable with the length of the

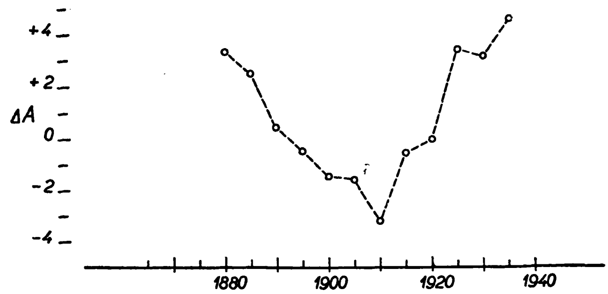

Fig. 14. The long-term integrated course of precipitation in West and Central Europe.

Table 16.
Long-term precipitation course at European stations

station	min	max
Aberdeen	1905	1931
Berlin	1888	(1859), 1925
Catania		1902
Frankfurt a. M.	1902	(1865), 1934
Gibraltar	1873	1902
Gjesvar	1898	
Greenwich	1885	(1853), 1921
Haparanda	(1868), 1917	1897
Helsinki	(1862)	1919
Kaliningrad	(1858), 1914	1891
Karesuando	1912	1890, 1937
Krynica	1896	1929
Lisboa	1926	1885
Lwow		1894
Lyon	1909	1880, 1939
Madrid	1871, 1917	1887
Marseille		1909
Milano	(1779), (1828), 1870, 1926	(1802), (1845), 1900
Nantes	1895, 1935	1912
Obir		1910
Oslo	1901	(1869), 1935
Ostersund	1890	1933
Paris	1894	1925
Roma	(1843)	(1805), 1897
Säntis		1921
Sassari	1912	1886
Sonnblick	1924	1896
Sulina	1906	1875
Trier	(1824), 1894	(1856), 1930
Uppsala	1910	1875
Valentia	1896	1925
Wien	(1858)	1913
Zürich	1917	1880

eighty-year period of solar activity. It may be even asserted that the curves for the 65°-latitude stations resemble the mirror pictures of those for the 40°-latitude stations. Inside a 55°-latitude zone, on the other hand, nearly no long-term variations appear. Similar results have been arrived at by WILLETT (1951), who considers a 50° to 60°-latitude zone to be the border between two areas of mutually contrary courses of long-term precipitation variations.

The integrated course of the long-term precipitation variations may be, on the general, investigated by the same method as the short-term precipitation variations, i. e. by means of formula (30). However, when applying it to the numerical data an obstacle arises, consisting in their unsufficient number available. This effect may partly be reduced by taking a longer interval for establishing one value of ΔA. Figure 14 indicates the integrated long-term precipitation course in Western and Central Europe, where each dot represents five years. The material used is comprised in Table 16. The extremes before 1870 (in Table 16 in brackets) have not been comprised in statistics owing to a small number of them. In Figure 14 a conspicuous minimum near 1910 is apparent, which corresponds very well with the minimum of the eighty-year solar period.

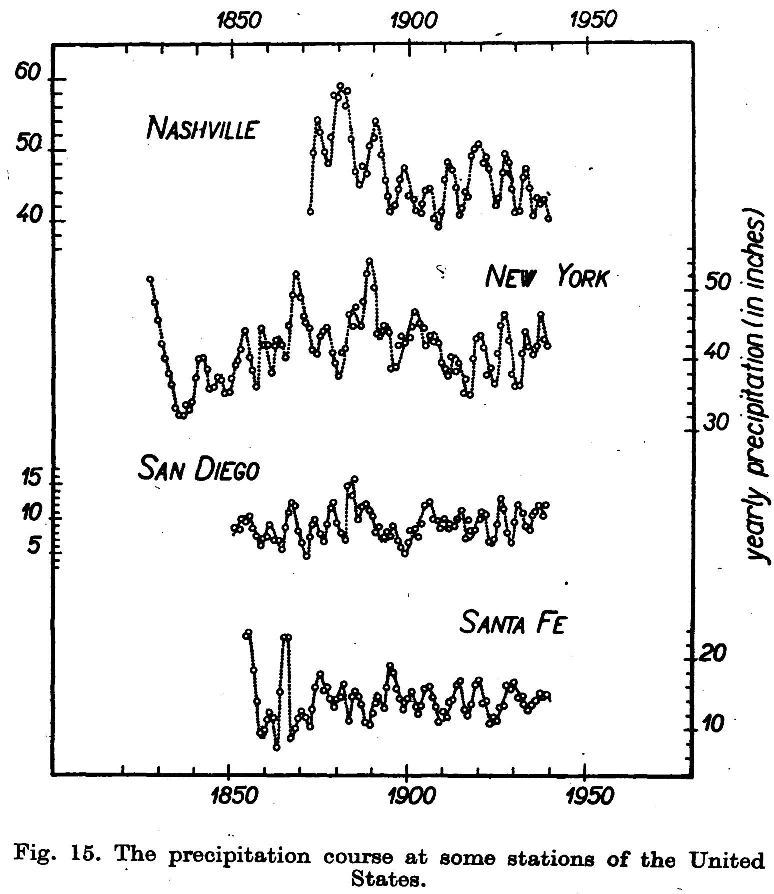

Fig. 15. The precipitation course at some stations of the United States.

7. FUNDAMENTAL FEATURES OF THE PRECIPITATION ACTIVITY IN THE UNITED STATES

Analogous investigations have been carried out for ten stations of the United States. The most important precipitation features of this area are as follows:

Table

Precipitation activity at 10

station	λ	φ	int t	P	$P_{\odot}$
	°	°	y	y	y
Charleston	+ 79.9	+32.8	1738—1765 1832—1940	6.34 ± 0.25	11.0
Chicago	+ 87.6	+41.9	1871—1940	6.61 ± 0.40	11.0
Nashville	+ 86.8	+36.2	1871—1940	6.37 ± 0.29	11.0
New York	+ 74.0	+40.7	1826—1940	6.06 ± 0.20	11.0
Phoenix	+112.0	+33.6	1896—1940	6.00 ± 0.43	10.9
Salt Lake City	+111.9	+40.8	1875—1940	6.47 ± 0.32	10.9
San Diego	+117.2	+32.7	1850—1940	6.35 ± 0.23	11.2
San Francisco	+122.4	+37.8	1850—1940	5.82 ± 0.19	11.2
Santa Fe	+106.0	+35.7	1853—1940	5.89 ± 0.24	11.2
Washington	+ 77.1	+38.9	1852—1940	6.29 ± 0.31	11.2

(1) The smoothed-out yearly averages of precipitation vary with the periods between 5.8 and 6.6 years, which is in good agreement with what has been found at the European stations in the preceding sections; the list of data for the ten stations is on the same lines as before included in Table 17. The precipitation curves at some American stations are in Figure 15.

(2) The average resulting precipitation period is 6.15 ± 0.06 years, so that $k = 0.555 \pm 0.005$, when $P_{\odot} = 11.09$ years; statistics of the lengths of the precipitation periods is included in Figure 16.

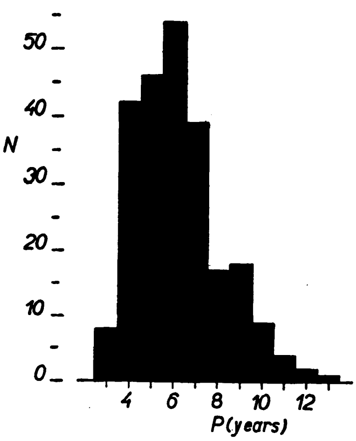

Fig. 16. The distribution of the lengths of precipitation periods at American stations.

(3) There exists a relation between the correlation coefficient, $\psi^*(o, e)$, and the reduced period $k = \frac{P}{P_{\odot}}$; it results from Figure 17. The stations are denoted as follows: *Ca* - Charleston, *Ch* - Chicago, *Ci* - Salt Lake City, *D* - San Diego, *Fe* - Santa Fe, *Fr* - San Francisco, *N* - Nashville, *P* - Phoenix, *Y* - New York, *W* - Washington.

(4) The amplitude drops with the increasing geographical latitude (Figure 18). This effect has the same character as at European stations. No turning point appears on the curve because of low geographical latitudes of the American stations.

(5) An asynchrony in the positions of the average values of the extremes for various stations as well as the dispersion in the phase

shifts of maxima relative to preceding minima are conspicuously expressed.

(6) A study of the average precipitation activity carried out by means of function ΔA indicates no systematic differences between both types of solar cycles (see Table 18).

The integrated precipitation course $\Delta A = \Delta A(t)$ cannot be analyzed according to formula (30) because only data regarding ten stations are at our disposal.

17.

stations of the United States

k	T_0 (min)	T_0 (max)	p. e.	φ_0	a
	y	y	y		%
0.576 ± 0.023	1902.32	1905.42	±1.54	0.49 ± 0.06	11.3
0.601 ± 0.036	1902.30	1905.60	±1.70	0.50 ± 0.09	7.8
0.579 ± 0.026	1899.75	1902.77	±1.26	0.47 ± 0.07	7.6
0.551 ± 0.018	1904.62	1907.62	±1.13	0.50 ± 0.05	7.5
0.551 ± 0.039	1902.79	1907.00	±1.43	0.70 ± 0.11	20.2
0.594 ± 0.029	1902.90	1906.25	±1.32	0.52 ± 0.07	10.6
0.567 ± 0.021	1902.31	1905.81	±1.17	0.55 ± 0.05	23.9
0.520 ± 0.017	1899.43	1902.45	±0.99	0.52 ± 0.05	13.2
0.526 ± 0.021	1901.41	1905.01	±1.25	0.61 ± 0.06	18.8
0.562 ± 0.028	1900.11	1903.41	±1.52	0.52 ± 0.07	10.3

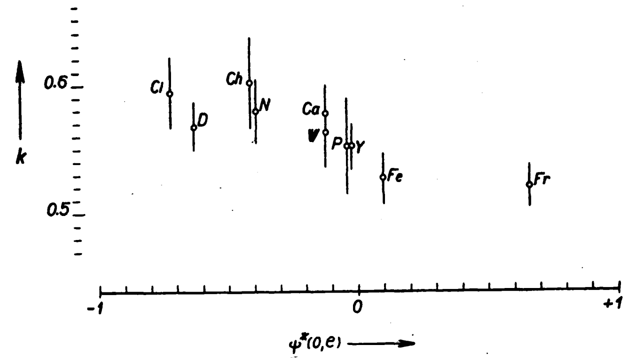

Fig. 17. Dependence of correlation coefficient ψ^* on the reduced precipitation period at American stations.

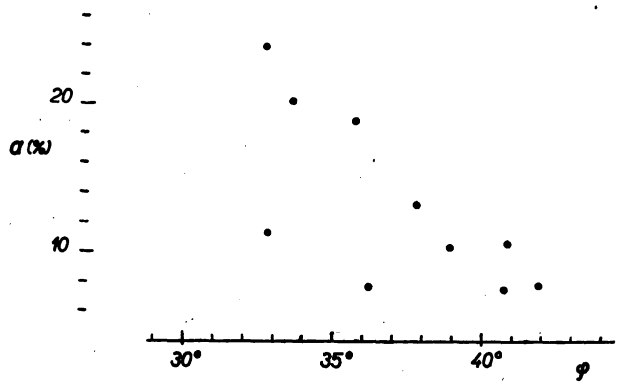

Fig. 18. Dependence of the precipitation amplitude on the geographical latitude at American stations.

Table 18.

Precipitation activity in individual solar cycles at American stations

solar cycle	$\overline{\Delta A}$
8	+0.76
9	0.00
10	—0.85
11	+0.78
12	0.00
13	—1.37
14	+0.71
15	+0.73
16	0.00
17	(—0.48)

8. CONCLUSIONS

The analysis of the precipitation activity in West and Central Europe and in the United States has led to the conclusions as follows:

(I) By expanding the dependence of the precipitation amount on the solar-cycle phase in the Fourier series it has been indicated that even if the precipitation period is a year longer than half a solar cycle, the above mentioned dependence is expressed by a conspicuous double-wave. Hence, there is no evidence for the assertion that the existence of the double-wave yields the precipitation period of 5.5 years.

(II) The investigation of the lengths of the average precipitation periods at 44 West and Central European stations has showed that they are ranged within 5 4 years and 7.5 years and that they are 6.2 years on the average.

(III) The lengths of the individual precipitation periods are ranged in much wider intervals of time, from 2 years (one out of 929 cases) till 15 years (three out of 929 events).

(IV) The moments of the precipitation maxima as well as of the minima at various stations in West and Central Europe take place asynchronically. The shifts show a strong correlation with the geographical longitude: with the increasing distance of the station from the Atlantic Ocean increases the retardation with which the station is attained by the mechanism controlling the development of the weather over West and Central Europe. The average retardation rate is 1.4 km per day.

(V) The material gives a suggestion of the increase of the phase distance of the time of the maximum precipitation from that of the preceding minimum precipitation with the geographical longitude, i. e. with the increasing distance from the Atlantic Ocean. This effect may also be interpretted as an influence of inertia of the mechanism controlling the weather over Europe. However, the respective correlation coefficient is too low for the relation to be considered real. The uncertainty is increased by comparatively high proper errors of the values as computed from the observational material.

(VI) The amplitude of precipitation in West and Central Europe shows a distinct correlation with the geographic latitude. In low latitudes the amplitude drops northwards rapidly, the minimum is reached for the latitude of 56° and then it slightly increases.

(VII) A hypothesis has been worked out concerning the dependence of the correlation coefficient $\psi^*(o, e)$ between the precipitation courses in odd and even cycles of solar activity on the reduced average length of precipitation--activity period, $\frac{P}{P_\odot}$, and a mathematical expression of this relation has been theoretically derived. Good agreement has been achieved when compared the theory with the observed precipitation curves of 44 European stations.

(VIII) On the basis of the values of coefficients ψ^* for the 44 European stations, a new system of lines of equal correlation degree has been constructed all over West and Central Europe, which spreads and partly corrects the former system published by KŘIVSKÝ (1951).

(IX) As a compromise between the requirements of meteorology and cometary statistics a term of the relative precipitation activity, ΔA, has been introduced, defined as the relative difference between the numbers of individual maxima and minima on the curve of precipitation for not less than 10^1 stations. The values of ΔA determined from the 44 European stations yield a double-wave in the course of an eleven-year solar cycle (which confirms the earlier investigations), and as a function of time they give a period of 6.2 $\pm$ 0.1 years with the minimum at 1903.9 $\pm$ 0.3 and the maximum at 1906.8 $\pm$ 0.3.

(X) The average values of the relative precipitation activity show that odd solar cycles are much more active than even cycles, as to the precipitation over West and Central Europe. For odd cycles it is $\overline{\Delta A} > 0$, while for even $\overline{\Delta A} < 0$.

(XI) The length of the period of the relative precipitation activity tends to a 40-year periodicity within the interval 1820 to 1940.

(XII) A long-term precipitation variation in West and Central Europe has also been described by means of the relative precipitation activity. Within the interval of 1870 till 1940, for when the abundant enough material is available, the precipitation curve attains its minimum round 1910, which falls on the time of the minimum of the eighty-year solar period.

(XIII) A concise analysis has been carried out of the precipitation activity at 10 stations of the United States. Analogous causalities have been ascertained to those given under (II), (III), (VI) and (VII): the lengths of the average precipitation periods at the stations are ranged within 5.8 years to 6.6 years, on the average 6.15 years, while the lengths of the individual periods are ranged within 3 years (eight out of 240 cases) to 13 years (one out of 240 events). However, the relation described under (X) has not been confirmed at the American stations. No long-term precipitation variation has been investigated in the U. S. area.

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SRÁŽKOVÁ ČINNOST V ZÁPADNÍ A STŘEDNÍ EVROPĚ A V USA VE VZTAHU K SLUNEČNÍ ČINNOSTI

Souhrn

Na základě srážkových dat ze 44 stanic západní a střední Evropy je zkoumán chod srážek v této oblasti se zřetelem k jejich souvislosti s kometárními objevy. Nejdůležitějším výsledkem je zjištění, že kolísání srážek probíhá v periodě asi 6,2 let, přičemž na jednotlivých stanicích kolísá v mezích 5,4 roku až 7,5 roku. Dále byly zjištěny závislosti parametrů srážkové činnosti na zeměpisné poloze stanic v Evropě, variace v délce periody srážek i dlouhodobé kolísání srážkové činnosti. Velká pozornost je věnována korelaci mezi chodem srážek v lichých a sudých cyklech a její interpretaci pomocí délky srážkové periody. Pro území západní a střední Evropy je dále konstruována síť isočar vyjadřující tuto korelaci. Dále se ukazuje, že liché cykly jsou ve srážkách na území západní a střední Evropy mnohem aktivnější než sudé.

Méně podrobný výzkum téhož charakteru je proveden i pro U.S.A. Ze srážkových dat na 10 amerických stanicích dochází se vcelku k velmi podobným závěrům jako u evropských stanic. Jediný zřetelný nesouhlas nastává ve vzájemném vztahu mezi totální srážkovou činností v obou typech slunečních cyklů.

АТМОСФЕРНЫЕ ОСАДКИ В ЗАПАДНОЙ И ЦЕНТРАЛЬНОЙ ЕВРОПЕ И В СОЕДИНЕННЫХ ШТАТАХ АМЕРИКИ В ЗАВИСИМОСТИ ОТ СОЛНЕЧНОЙ ДЕЯТЕЛЬНОСТИ

Резюме

На основании данныых об атмосферных осадках полученных на 44 станциях западной и центральной Европы изучается ход осадков в этой области в связи с кометными открытиями. Важнейший результат—колебание атмосферных осадков с средним периодом 6,2 лет, при чем на отдельных станциях изменяется в пределах 5,4 лет и 7,5 лет. Далее были установлены зависимости параметров атмосферных осадков от географического положения станций в Европе, изменения в длине периода атмосферных осадков и их вековое колебание. Большая внимательность посвящена корреляции между ходом атмосферных осадков в четных и нечетных циклах солнечной деятельности и ее интерпретации при помощи длины периода осадков. Для западной и центральной Европы далее конструирована сеть изолиний выражающая эту корреляцию. Далее оказывается, что нечетные циклы более активные четных, что касается атмосферных осадков в западной и центральной Европе.

Менее подробное исследование сделано для Соединенных Штатов Америки. Данные с 10 американских станций ведут вообще к подобным заключениям как у европейских станций. Несогласие возникает только в случае взаимного отношения между атмосферными осадками в обеих типах солнечных циклов.