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THE SECULAR DECREASE OF THE ABSOLUTE MAGNITUDE OF SHORT-PERIODIC COMETS

SEKULÁRNÍ POKLES ABSOLUTNÍ VELIKOSTI KRÁTKOPERIODICKÝCH KOMET

СЕКУЛЯРНОЕ ПАДЕНИЕ АБСОЛЮТНОЙ ЗВЕЗДНОЙ ВЕЛИЧИНЫ КОРОТКО-ПЕРИОДИЧЕСКИХ КОМЕТ

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1. INTRODUCTION

The comets with orbital periods shorter than 10 years, i. e. with semi-major axis generally smaller than is mean heliocentric distance of Jupiter, will be called "short-periodic comets". The absolute magnitude is the value, defined by the relation

$$m_0 = m - 5 \log \Delta - 10 \log r, \tag{1}$$

m is the magnitude at heliocentric distance r and at geocentric distance Δ. The equation (1) is fulfilled for the moment t. For this moment the absolute magnitude is constant. If we are considering time intervals about 10—100 years or more, it is necessary to take into account its secular variations. We must search the origin of these variations in the physical processes, proceeding in the coma of a comet. They depend on the number of expelled gaseous molecules, able to radiate, and on the quantity of the dust, reflecting solar radiation. The problem of the secular variations of the absolute magnitude was not theoretically solved.

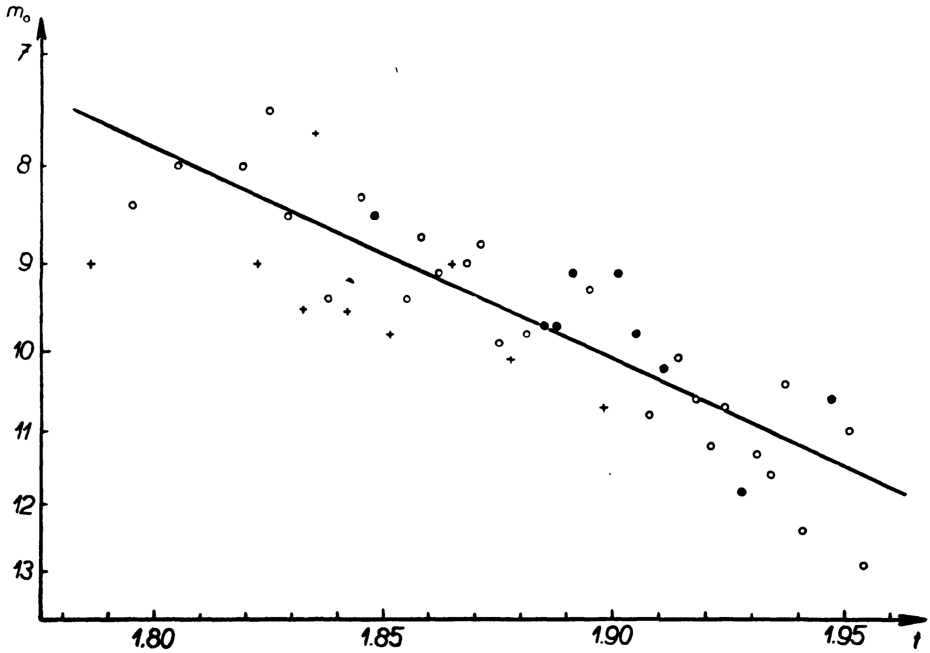
2. MATERIAL AND ITS GRAPHICAL PRESENTATION

The time-course of the absolute magnitudes and their variations for the short-periodic comets on the base of the material from the Catalogue of the Absolute Magnitudes of Comets, published by S. K. VSESSVIATSKY[1] is studied in this paper. Nine comets with the greatest number of observed returns were chosen. These are: Encke 1954 IX (44), Grigg—Skjellerup 1952 IV (8), Tempel 2 1951 VIII (10), Pons—Winnecke 1951 VI (15), Biela 1852 III (6), d'Arrest 1950 II (8), Fayet 1955 II (14), Brooks 2 1953 V (8), Wolf 1 1950 VI (9).

The accuracy of an estimate of the absolute magnitude is classiffied in VSESSVIATSKY's paper using this scale:

1 — very inaccurate, error 1^m or more,
2 — accuracy about $0^m.5$,
3 — very accurate, error $0^m.1$ — $0^m.2$.

In figures 1—5 the dependence of the absolute magnitude (in logarithmical scale) on time (in thousands of years) is plotted for some short-periodic comets. Crosses refer to estimations 1, open circles to estimations 2 and filled circles to estimations 3.



1. The time-course of the absolute magnitude of the comet Encke.

3. THE ANALYTICAL FORM OF THE FUNCTION $m_0 = f(t)$ AND THE TIME-COURSE OF THE DECREASE OF THE ABSOLUTE MAGNITUDE DURING 100 YEARS $\Delta_{100}\, m_0$

If we plotted the dependence of the absolute magnitude in a linear scale on time, we would find that for a given comet the decrease of the luminosity is proportional to the absolute luminosity of the comet (in the magnitude scale). We can thus write:

$$\mathrm{d}m_0(t) = C_2 \,.\, m_0(t) \,.\, \mathrm{d}t, \qquad (2)$$

i. e.

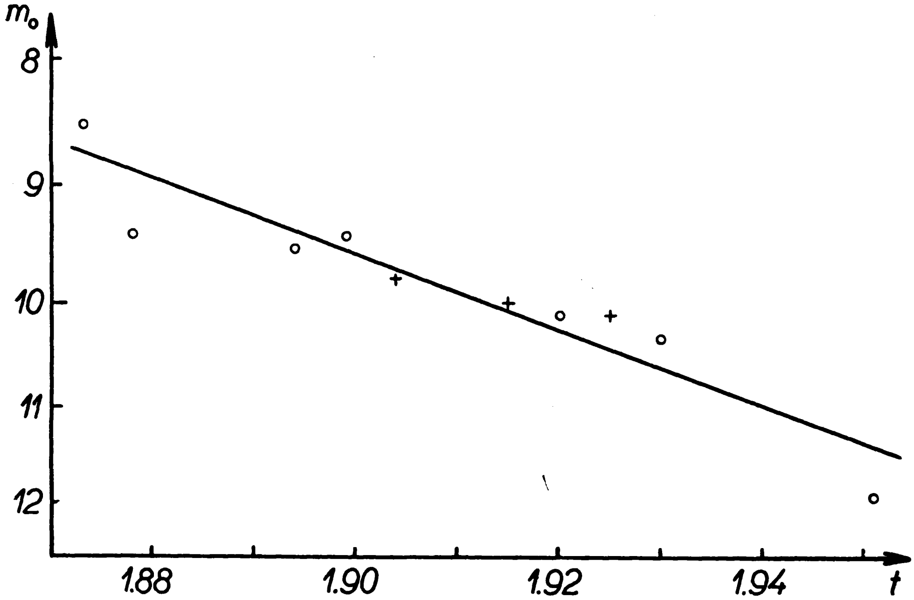
$$m_0(t) = C_1 \, . \, e^{C_2 t}, \tag{3}$$

the values C_1, C_2 are constants charactetistic for the given comet. Regarding the numerical practice we can write the equation (3) in the form:

$$m_0(t) = \alpha \, . \, 10^{\beta t}, \tag{4}$$

β is the disintegration coefficient characterizing the rate of the decrease of the magnitude (the rate of "the disintegration" of the comet), t is expressed in thousands of years. The relation between constants C_1, C_2 and constants α, β is evidently

$$C_1 = \alpha,$$
$$C_2 \log e = \beta.$$



2. The time-course of the absolute magnitude of the comet Tempel 2.

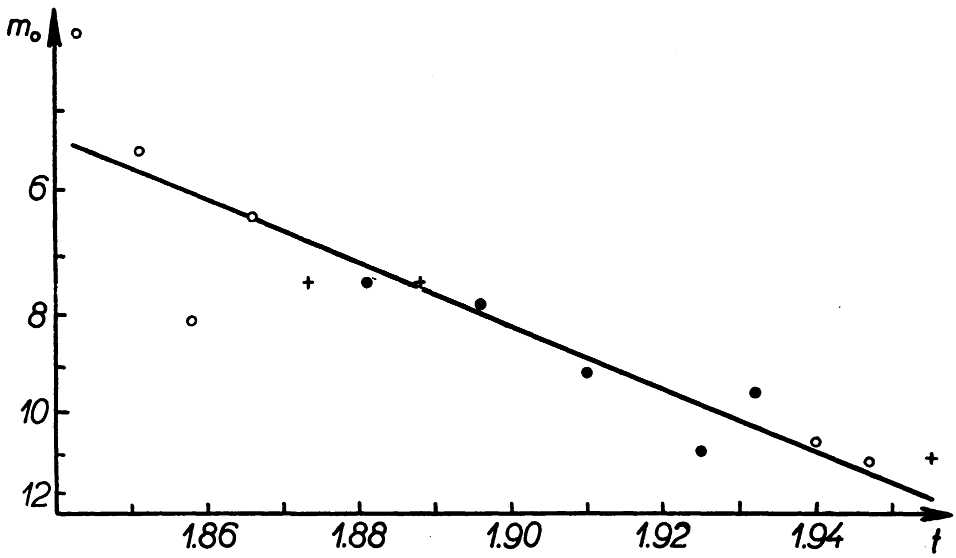
The change of the absolute magnitude during 100 years at the moment t is given by

$$\Delta_{100} m_0(t) = \frac{\alpha \cdot \beta}{\log e} \cdot 10^{\beta t - 1}, \tag{5}$$

where e is the base of Napier's logarithmus. The relation between $\Delta_{100} m_0(t)$ and the absolute magnitude can be written by the relation

$$\Delta_{100} m_0(t) = 0{,}23 \, \beta \, m_0(t). \tag{5'}$$

The disintegration coefficient is specific for each special comet, and it depends on several factors. The comet of a short orbital period must lose a greater quantity of gas during the defined time interval than the comet of a longer orbital period, because each comet loses the greatest quantity of gas while passing near perihelion. Thus the disintegration coefifcient depends on the orbital period of a comet.

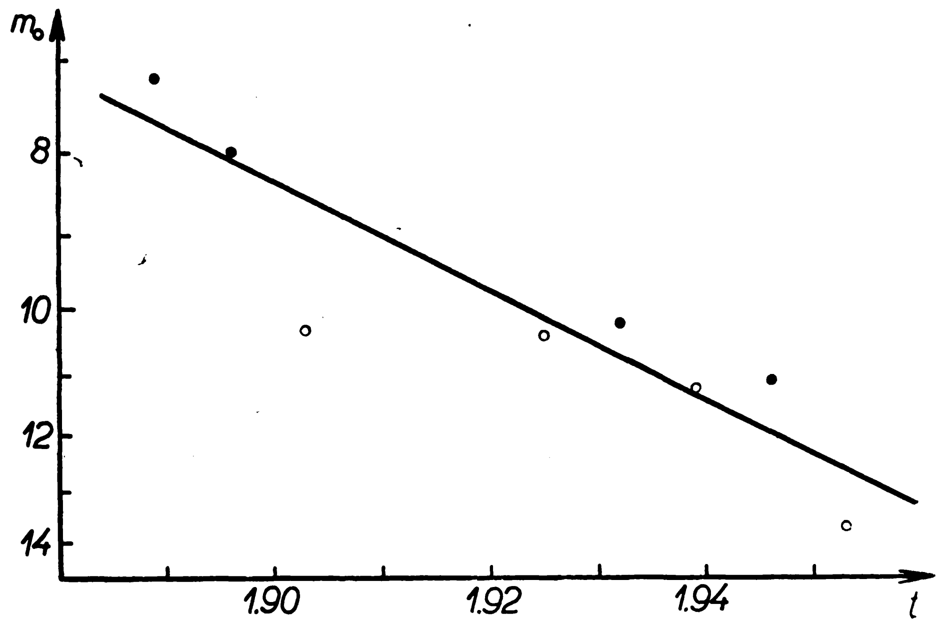


3. The time-course of the absolute magnitude of the comet Fayet.

Table 1.

Name	log α	β	y P_0	A.U. L	% δP	m_0 (1,9)	$\Delta_{100}m_0$	$\Delta_p m_0$	r (t, log m_0)	w
Encke	— 1,031	1,073	3,297	330,8	0,487	10,2	2,5	0,08	0,862	8,8
Grigg-Skjellerup	— 3,895	2,594	4,944	318,6	1,188	10,8	6,5	0,32	0,738	1,3
Tempel 2	— 1,798	1,463	5,224	333,8	0,978	9,6	3,2	0,17	0,934	1,7
Pons-Winnecke	— 1,478	1,308	5,891	307,1	2,795	10,2	3,1	0,18	0,755	3,0
Biela	— 0,071	0 529	6,683	279,0	0,865	8,6	1,0	0,07	0,816	0,9
d'Arrest	— 1,158	1,135	6,597	301,0	1,537	10,0	2,6	0,17	0,833	1,7
Fayet	— 4,991	3,108	7,445	294,2	0,799	8,2	5,9	0,44	0,908	3,0
Brooks 2	— 5,494	3,377	7,009	307,9	0,188	8,4	6,5	0,46	0,926	2,0
Wolf 1	— 8,641	5,021	7,483	308,2	10,044	7,9	9,2	0,69	0,958	2,1

If we further assume that each comet slowly disintegrates along the whole orbit, the disintegration coefficient becomes function of the circular orbit during a defined time interval. As we consider the short-periodic comets, the difference between the actual lenght and the mean one of the orbit is very



4. The time-course of the absolute magnitude of the comet Brooks 2.

small for a time interval long enough. The mean elliptical orbit, made during 100 years, is approximatively

$$L = 100\,\pi a^{-1/2} \left[\frac{3}{2} \left(1 + \frac{b}{a} \right) - \left(\frac{b}{a} \right)^{1/2} \right], \qquad (6)$$

a is the semi-major axis, b the semi-minor axis (in A. U.).

Comets are distrubed by great planets. Accidental perturbations act upon the changes of the orbital period of the comet. In order to characterise this type of perturbations we can use the variation of the period, defined by the relation

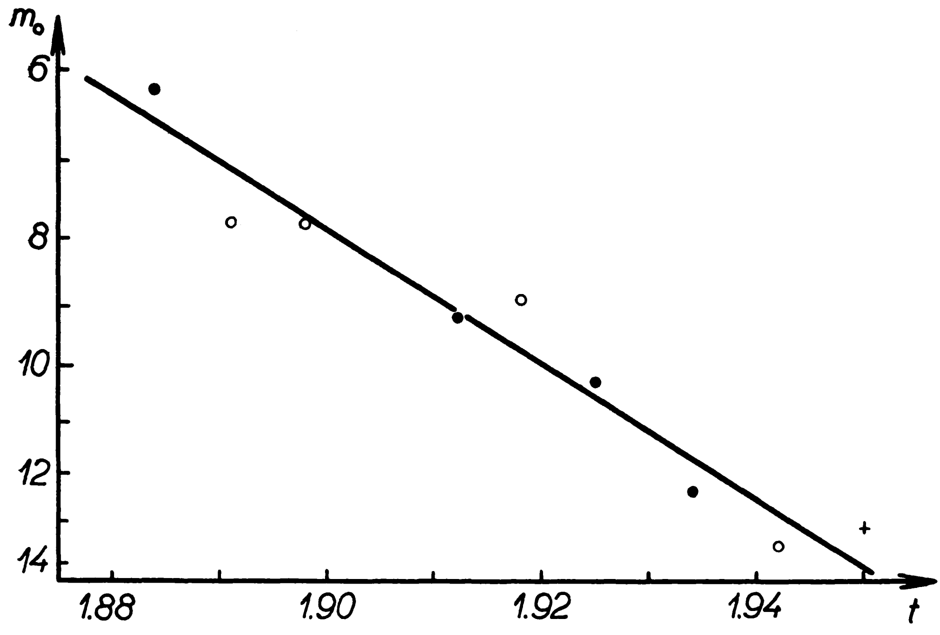
$$\delta P = \frac{100}{nP_0} \sum_{i=1}^{n} |P_i - P_0| \, (\%), \qquad (7)$$

n is the numer of returns, for which the period P_i was determined, P_0 is the mean period of P_i. It must be noted that the effect of disintegration is small in this case.

First of all the disintegration coefficient depends on the inside structure of a comet. Some comets are stable (Encke), some unstable (Biela, Holmes).

Table 2.

Comet	Q	K	w
Encke	0,5236	— 0,5307	8,8
Grigg-Skjellerup	0,5802	— 0,5172	1,3
Tempel 2	0,5207	— 0,5241	1,7
Pons-Winnecke	0,5312	— 0,5253	3,0
Biela	0,4793	— 0,5467	0,9
d'Arrest	0,5236	— 0,5279	1,7
Fayet	0,4793	— 0,5267	3,0
Brooks 2	0,5172	— 0,5204	2,0
Wolf 1	0,5083	— 0,5222	2,1



5. The time-course of the absolute magnitude of the comet Wolf 1.

In the equation (4) the disintegration coefficient is considered to be constant. That is fulfilled only approximately. We don't consider short-periodical anomalies during a comet's radiation. These are, however, sometimes very considerable (explosions). We are interested only in the secular systematic variations. Relatively to what has been said, even during a longer time interval (10—100 years) the disintegration coefficient needn't be constant. That happens when the orbital period and the form of the orbit change suddenly. The perihelion distance acts the disintegration coefficient too, because the evaporation of the gas as well as a radiation pressure become greater near the Sun.

From the equations (4), (5) it follows that the course of the absolute magnitude as well as the course of the decrease of the absolute magnitude are determined by the coefficients α, β. These coefficients we can determine by the least squares method.

Table 1 gives the numerical results. The succesive columns are as follows:

name — name of the comet,
$\log \alpha$ — Briggs's logarithmus of the coefficient α, determined by the least squares method,
β — disintegration coefficient, determined by the least squares method,
P_o — mean orbital period by equation (7); it is a fuction of time,
L — average distance travelled by the comet in 100 years, determined from the equation (6),
δP — variation of the period, determined from (7),
m_o (1,9) — absolute magnitude, determined from (4) for t = 1,9,
$\Delta_{100}\, m_o$ (1,9) — decrease of the absolute magnitude during 100 years, determined from (5),
$\Delta_p\, m_o$ (1,9) — decrease of the absolute magnitude during the period,
r (t, $\log m_o$) — correlation factor of the relation between time and $\log m_o$,
w — weight: the arithmetical sum of individual weights of the observations of the comet (taken according to Vsessviatsky's scale), divided by ten. Accuracy of the data in Tables 2 and 3 was estimated in the same way.

4. THE RELATION $\beta = f(\alpha)$

There appears to be a definite correlation between the values of $\log\alpha$ and β. If we plot the disintegration coefficient against $\log \alpha$ (Figure 6), we see that the relation is very closely linear, and we can consequently write:

$$\beta = Q + K \log \alpha \qquad (8)$$

where Q, K are constants. The least squares method gives:

$$Q = 0{,}5349,$$
$$K = -0{,}5198.$$

The correlation coefficient is $r = -0{,}99994$.

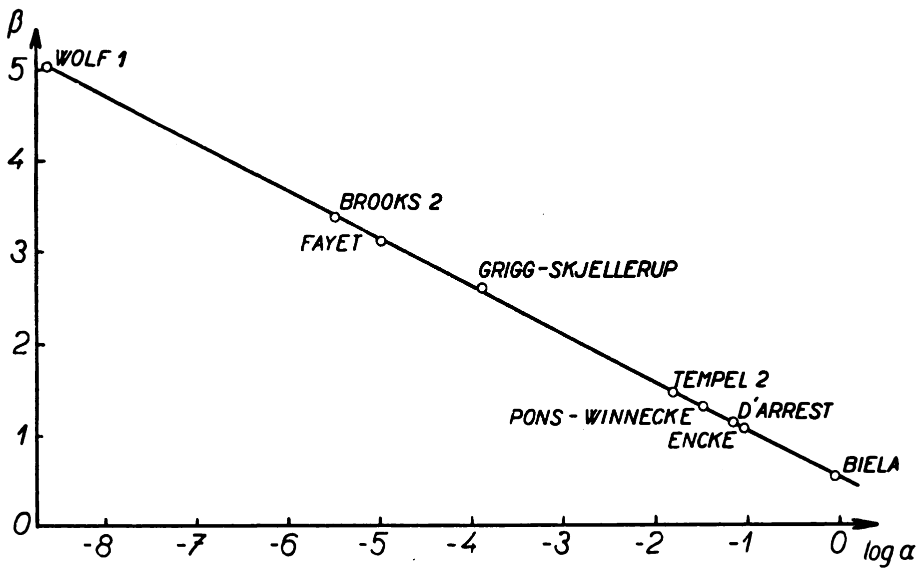
The relation (8) can be derived from theoretical considerations put the equation (4) into logarithmic form. Writing this equation for succesive returns, we get n equations of the following form

$$\log m_0(t_i) = \log \alpha + \beta t_i, \qquad i = 1, \ldots, n,$$

and, adding, we have

$$\beta = \frac{[\log m_0]}{[t]} - \frac{n}{[t]} \log \alpha , \tag{9}$$

where the brackets denote the sum. For a given comet, β and $\log \alpha$ are constants. If we form the quotiens $[\log m_0) : [t]$ and $- n : [t]$ for various comets,



6. Relation $\beta = f(\alpha)$ for short-periodic comets.

we realize that their values are about the same. For various comets, the disintegration coefficients are different. We can consider the equation (9) as the equation of a straightline and a comparison of coefficients in the equations (8), (9) gives:

$$Q = \frac{[\log m_0]}{[t]} , \tag{10}$$

$$K = - \frac{n}{[t]} .$$

Table 2 shows the values of the factors K and Q for nine short-periodic comets. It is evident that the differences among the individual factors K and the differences among the individual factors Q as well are small. They converge to the values K and Q determined by the least squares method.

In the equation (10), the quantities Q and K are functions of time; but they can be considered as constants over the time-interval of 100 years.

Table 3.

Comet	log α	β	w
Tempel 1 1873 I	— 1,44	1,28	0,2
Brorsen 1873 VI	— 2,17	1,66	0,7
De Vico-Swift 1894 IV	— 1,08	1,09	0,5
Giacobini-Zinner 1946 V	— 7,67	4,50	1,0
Forbes 1948 VIII	— 4,98	3,12	0,6
Reinmuth 1 1950 IV	— 1,29	1,20	0,7
Daniel 1950 V	— 7,03	4,18	0,7
Finlay 1953 VII	— 1,08	1,10	1,5
Honda-Mrkos-Pajdušáková 1954 III	— 3,40	2,31	0,4
Reinmuth 2 1954 V	— 1,34	1,23	0,4
Wirtanen 1954 XI	— 8,60	5,01	0,4

5. THE DETERMINATION OF THE FACTORS log α AND β FROM THE KNOWLEDGE OF n VALUES $m_0(t_i)$ CORRESPONDING TO MOMENTS t_i

If we want to compute the approximate values β and log α for little-observed comets, the least squares method is inadequate for it can lead to false results. Thus I have proceeded in anither way starting from the differences between the mean values of Q and K (defined by the equation (8)) and their actual values for a given comet (as defined by the equation (10)).

Let us have n values of the absolute magnitude for a comet:

$$m_0(t_1),\ m_0(t_2), \ldots,\ m_0(t_n)$$

corresponding to epochs

$$t_1,\ t_3, \ldots,\ t_n.$$

According to (4) we can write:

$$\log m_0(t_i) = \log \alpha + \beta t_i, \qquad i = 1, \ldots, n,$$

i. e.

$$[\log m_0] = n \log \alpha + \beta[t]. \tag{11}$$

Let us insert for β the right-hand side of (8), and further denote:

$$\frac{1}{n}\,[\log m_0] = M_0\,,$$

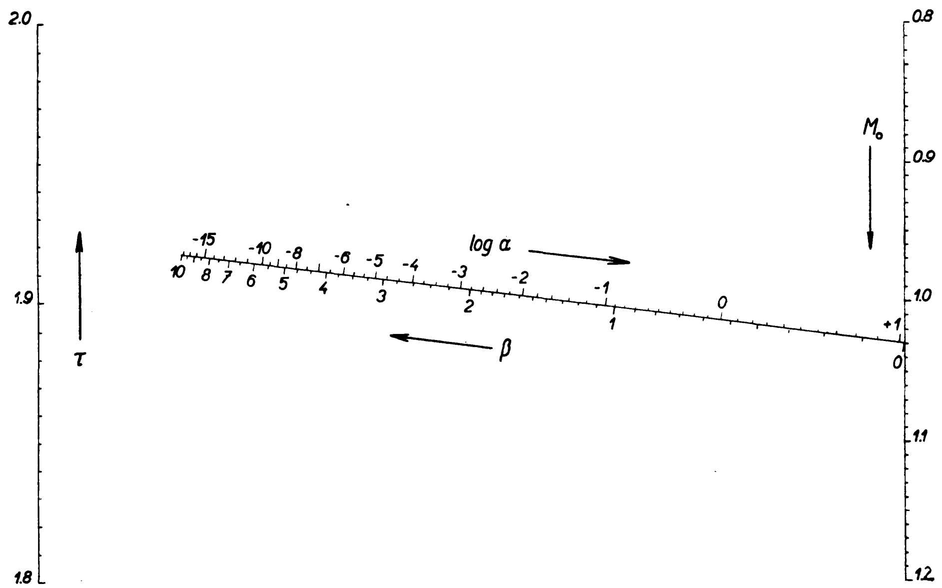
$$\frac{1}{n}\,[t] = \tau,$$

then we have

$$\log \alpha = \frac{M_0 - Q\tau}{1 + K\tau}. \qquad (12)$$

The disintegration factor is:

$$\beta = \frac{KM_0 + Q}{1 + K\tau}. \qquad (13)$$



7. The nomogram for determining the coefficients log α and β.

Regarding the physical meaning of the disintegration coefficient, we must de mand the following conditions for the quantities M_0, τ:

a) the equations are not defined for $\tau \to -\frac{1}{K}$; thus

$$\tau \notin \left\langle -\frac{1}{K} - \varepsilon; -\frac{1}{K} + \varepsilon \right\rangle, \ \varepsilon \sim 5 \cdot 10^{-3},$$

b) for $\tau > -\frac{1}{K}$ it must be $M_0 > -\frac{Q}{K}$,

c) for $\tau < -\frac{1}{K}$ it must be $M_0 < -\frac{Q}{K}$.

It will be shown that, for most of the short-periodic comets, these conditions are fulfilled. Table 3 contains the quantities log α, β, determined by the above-showed method for some other comets.

6. THE NOMOGRAM FOR DETERMINING THE COEFFICIENTS log α AND β

The nomogram of the relations log $\alpha = f_1(M_o, \tau)$, $\beta = f_2(M_o, \tau)$ is presented in Figure 7. The scales for M_o and τ are parallels. The form of the scale equation for log α and β is:

$$y\,(\log\,\alpha,\,\beta) = \frac{A + BQ}{DK}x\,(\log\,\alpha,\,\beta) - \frac{A}{K}\,, \tag{14}$$

A, B, D are modulli of the nomogram.

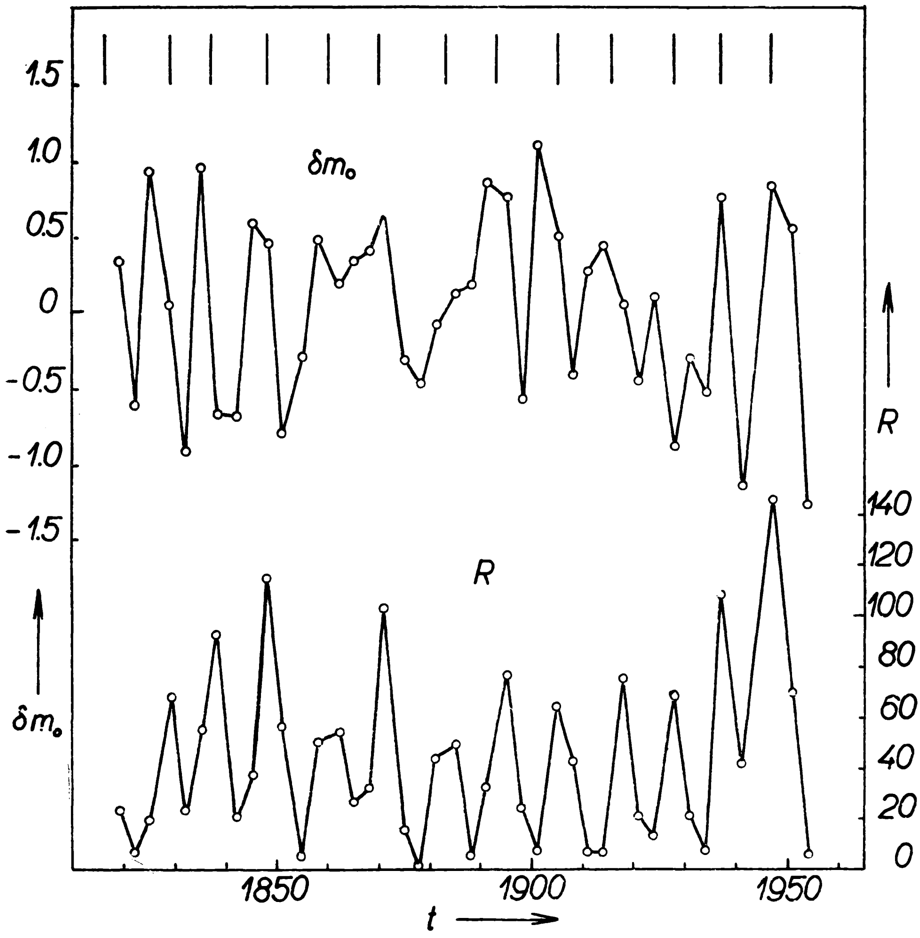
7. THE DEPENDENCE OF THE LUMINOSITY OF A COMET ON THE SOLAR ACTIVITY

If we turn our attention to the correlation coefficients $r(t, \log m_o)$ in Table 1, it is seen that smaller correlation coefficients correspond to the comets having smaller perihelion distances. It is clear that the spread of values is real at least to certain degree, for the differences oscillate in regular intervals around a certain value. If we plot the differences

$$\delta m_o = m_o\ \text{(computed)} - m_o\ \text{(observed)}$$

[m_o(computed) from the equation (4)] against time (upper part of Figure 8 shows this dependence for the comet Encke), we conclude that these differences oscillate around a zero-position, defined by (4). So we get a broken line which more or less converges to an actual curve, showing the course of differences δm_o. The discrepancy is due partly to the incertainty in the estimates of m_o, partly to the fact that we can observe the comet only each P^{th} year, if P is the orbital period. Although a time-course of δm_o is partly distorted by these effects, the dependence δm_o on the solar relative numbers R is quite evident. The time-course of the relative numbers R is plotted in figure 8 below. The maxima of the solar activity are denoted by strikes in the upper part of Figure 8. The correlation between these quantities is visible from the calculation of the mean period of δm_o and of R, too. The mean period of the curve $\delta m_o(t)$ during years 1819—1954 is $11{,}1^y$, the mean period of the curve R during the same years is $10{,}9^y$. The mean phase-shift of the maxima of these curves is about 2^y. But the maximum of the curve $\delta m_o(t)$ for comet Encke cannot be determined with greater accuracy. It is very difficult to find a similar correlation for other comets, because their orbital periods are more than 5^y. Therefore, we can determine no more than two values of the quantity δm_o during a solar cycle. Even, if we could observe comets along their whole orbital ellipse, we should get false results, because the number of shining molecules decreases with heliocentric distance, and around aphelion we should measure the brightness of the dust coma only.

Because the intensity of the radiation of the gas in the coma depends on the intensity of the solar radiation (especially in the ultra-violet part), it is possible to estimate the changes of the intensity of the solar ultra-violet radiation during the 11-years cycle from the magnitude of the oscilations of $m_0(t)$. It results that the difference between the mean minimum and the mean maximum of the function $\delta m_0(t)$ is about $0^m.8$, i. e. the radiation of the comet Encke at maximum is 2.1 times greater than at minimum.



8. The time-course of differences δm_0 of the comet Encke and the time-course of the solar relative numbers *R*.

It appears that comet Encke is a very suitable object even for these studies, for owing to its small perihelion distance, the fluctuations due to the changes in the solar radiation are best perceptible.

I am obliged to Professor Dr J. M. Mohr and Dr Plavec for discussions of problems connected with the secular decrease of the absolute magnitude.

REFERENCES

1 S. K. Vsessviatsky, AŽ 33, 516 (1956).

SOUHRN

Na základě uvedeného materiálu je ukázáno, že absolutní jasnost krátkoperiodických komet klesá exponenciálně s časem. Absolutní velikost každé komety je určena dvěma konstantami α, β, jež jsou spolu svázány těsným vztahem. Dále je ukázáno, že jasnost komety Enckeovy závisí na sluneční činnosti.

РЕЗЮМЕ

На основании приведенного материала оказывается, что абсолютная звездная величина в зависимости от времени падает показательно. Абсолютная величина каждой кометы определяется двумя постоянными α, β, которые связаны между собой тесным соотношением. Далее оказывается, что светимость кометы Энке зависит от солнечной активности.