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Combination of 3D epoch-wise and permanent geodetic networks observed by GNSS*

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Dedicated to Lubomír Kubáček on the occasion of his 80th birthday

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Abstract

The local, regional and global geodetic networks are recently almost exclusively observed by satellite radionavigation methods, such as the U.S. Global Positioning System (GPS), and the Russian navigation system GLONASS. The unprecedented accuracy of geodetic satellite positioning allows determination of the geocentric site coordinates at millimetre level. The paper points to complex adjustment model applied for combination of 3D coordinates observed in permanent and epoch-wise satellite networks. The discussed methods are demonstrated on local and regional GPS networks in Slovakia and in Central Europe.

Key words: 3D Global Navigation Satellite System Network, adjustment model for combination of 3D coordinates

2010 Mathematics Subject Classification: 62P30

1 Introduction

The application of Global Navigation Satellite Systems (GNSS) in geodesy is a revolutionary change in the geodetic positioning associated with new challenges for analysis of observations and their interpretations. The sub-centimetre accuracy of satellite positioning enables to determine besides the precise coordinates

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also their temporal evolution, and in this way to monitor the dynamics of lithosphere. As all the satellite observations are fully automatic, we have to deal with large amount of measured data requiring sophisticated models and algorithms for their effective processing and analysis.

GNSS are satellite-based radionavigation systems providing the precise three-dimensional position, spatial orientation and time information to users. Recently are fully operable the U.S. Global Positioning System and the Russian navigation system GLONASS. The configuration of GNSS consists of 21 and more compatible satellites at medium orbits (about 20 000 km above Earth) fulfilling the requirement that at least four satellites are simultaneously observable above the horizon anywhere on the Earth 24 hours a day. GNSS are one-way ranging systems with signals transmitted from satellites to the users equipped with special receiving instruments. The fundamental observable is the signal travel time between the satellite antenna and the receiver antenna which is used for determination of instantaneous distance between the satellite and the receiver. As the receivers clocks are not synchronized with the satellite clocks, the synchronization error is the reason for denoting the measured distances as the pseudoranges. Simultaneous ranging to four or more satellites enable determination of 3D coordinates of the observing site.

We will introduce complex adjustment model for combination of 3D coordinates observed in permanent and epoch-wise satellite networks. The primary coordinate adjustment has to be performed by specialized GNSS software as e.g the Bernese GPS software, version 5.0 [2]. The following combination of various network solution is aimed to estimation of additional parameters including the site coordinates for reference epoch, site velocities reflecting the dynamics of the Earth's crust, transformation parameters among reference frames and other unknowns, like apparent station shift due to observing equipment changes, etc. The combination procedures require relevant stochastic modelling of observation noise. The applied methods are demonstrated on velocity estimates in local and regional GPS networks.

2 Application of GNSS in geodesy

The geodetic GNSS observations require the extension of the fundamental positioning principle realized by code measurements of pseudoranges using also the measurements of satellite carrier phases. The necessity of elimination of systematic effects and solving for ambiguities due to unknown initial number of cycles of the carrier enable to use phase observations preferably in the differential mode. At least two simultaneously operating GNSS receivers and four simultaneously visible satellites are required to the relative coordinate determination. The new site position is determined relatively to the point with apriori known position (one receiver is observing at the point with known coordinates). The potential accuracy of relative positioning is at millimetre level and is suitable for a large spectrum of applications in geodesy, e.g. geodetic control survey, geodynamics, height determination, cadastral surveying and geographic information systems, monitoring and engineering and photogrammetry [7].

3 Observation techniques applied in geodesy for precise monitoring of stability of the Earth's crust: epoch-wise and permanent networks

The epoch-wise GNSS networks are based on coordinate determination of new geodetic control points and the densification of existing geodetic networks performed in separate observing campaigns taking several hours up to several days. The coordinates in permanent geodetic networks are obtained from continuous observations at specially built and equipped sites providing the data of highest quality, suitable for long-term monitoring of local, regional or global geodynamics. The models applied for combination of GNSS networks are based on ideas originally developed by Professor Lubomír Kubáček for analysis of terrestrial geodetic networks observed by classical geodetic tools yielding horizontal and vertical angles, distances and height differences from levelling observations [3], [4], [5].

The spatial (3D) coordinates related to geocentric reference frame determined by GNSS based geodetic techniques are taken as time dependent, because the observations repeated in various epochs lead to different results.

The main reasons are:

- Global plate tectonics (2–10 cm/year, Eurasia $\sim$ 3 cm/year),
- Local geodynamics (in Central Europe up to 1 cm/year),
- Seasonal variations—amplitudes $\sim$ 1 cm,
- Systematic biases due to GNSS instrumentation and/or GNSS receiving antennae changes < 2 cm,
- GNSS observations and the resulting coordinates are related to various reference frame realizations. There is a necessity to transform the obtained coordinates to unified, homogeneous and stable geocentric reference frame.

4 Combination of individual GNSS network solutions in space-time frame

Parameters to be determined from the combination of GNSS networks observed at varying epochs t_i are as follows [4]:

- $\mathbf{y}$ – site coordinates,
- $\mathbf{v}_y$ – site velocities (linear changes of coordinates),
- $\mathbf{s}$ – amplitudes of periodic seasonal variations,
- $\mathbf{Q}$ – transformation parameters relating epoch observations to unified reference frame,

- $\mathbf{u}$ – biases related to site stabilization, GNSS receiver/antenna manipulation or replacement.

The basic model for network combination relates the input data—the observed coordinates and velocities of reference sites to the estimated parameters

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{v}_{\mathrm{ref}} \end{pmatrix} = \begin{pmatrix} \mathbf{x}_{t_1} \\ \mathbf{x}_{t_2} \\ \vdots \\ \mathbf{x}_{t_m} \\ \mathbf{v}_{\mathrm{ref}} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & \mathbf{D}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{S}_1 & \mathbf{U}_1 \\ \mathbf{I} & \mathbf{D}_2 & \mathbf{T}_2 & \cdots & \mathbf{0} & \mathbf{S}_2 & \mathbf{U}_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{I} & \mathbf{D}_m & \mathbf{0} & \cdots & \mathbf{T}_m & \mathbf{S}_m & \mathbf{U}_m \\ \mathbf{0} & \mathbf{E} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{y} \\ \mathbf{v}_y \\ \boldsymbol{\Theta}_2 \\ \vdots \\ \boldsymbol{\Theta}_m \\ \mathbf{s} \\ \mathbf{u} \end{pmatrix} + \begin{pmatrix} \varepsilon_{x_1} \\ \varepsilon_{x_2} \\ \vdots \\ \varepsilon_{x_m} \\ \varepsilon_v \end{pmatrix} \tag{1}$$

where:

- $\mathbf{x}_{ti}$ – observed coordinates related to epoch t_i (note that $\mathbf{x}$ may be result of single epoch campaign or 'weekly' solution from permanent network),
- $\mathbf{v}_{\mathrm{ref}}$ – reference velocities [1],
- $\mathbf{I}$, $\mathbf{D}$, $\mathbf{T}$, $\mathbf{E}$, $\mathbf{S}$, $\mathbf{U}$ – submatrices of design matrix relating the observations to estimated parameters.
- ε_{xi}, ε_v – random errors of observed coordinates and reference velocities.

The global covariance matrix of observations is composed from:

- $\mathbf{S}_{ti}$ – covariance matrix of estimated network coordinates related to epoch t_i,
- $\mathbf{S}_{\mathrm{ref}}$ – covariance matrix of reference velocities, e.g. the ITRF 2005 velocity field [1].

The general assumption applied in global covariance matrix definition is that coordinates in various epochs t_i are uncorrelated

$$\boldsymbol{\Sigma} = \mathrm{var}\begin{pmatrix} \mathbf{x} \\ \mathbf{v}_{\mathrm{ref}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Sigma}_{x_{t1}} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{x_{t2}} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma}_{x_{tm}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Sigma}_{v_{\mathrm{ref}}} \end{pmatrix} \tag{2}$$

The linear unbiased and efficient estimate of parameters in (1) are obtained by the statistical model for indirect measurements [5], [6]. Practical applicability of such models was proved by combination of several hundreds of weekly network solutions comprising up to 100 sites. The resulting coordinates and other relevant data are estimated with mm accuracy.

5 Examples of local and regional GPS networks for monitoring of recent geokinematics

The epoch local monitoring network of Mochovce Nuclear Power Plant: Reference epoch-wise network observed by GPS once per year from 2001. Aim of the network regular observations is monitoring the stability of the central area and surroundings of the power plant. The set of 11 stable geodetic pillars (Fig. 1) covering area 10 × 10 km is regularly re-observed by GPS in 48-hour epoch campaigns

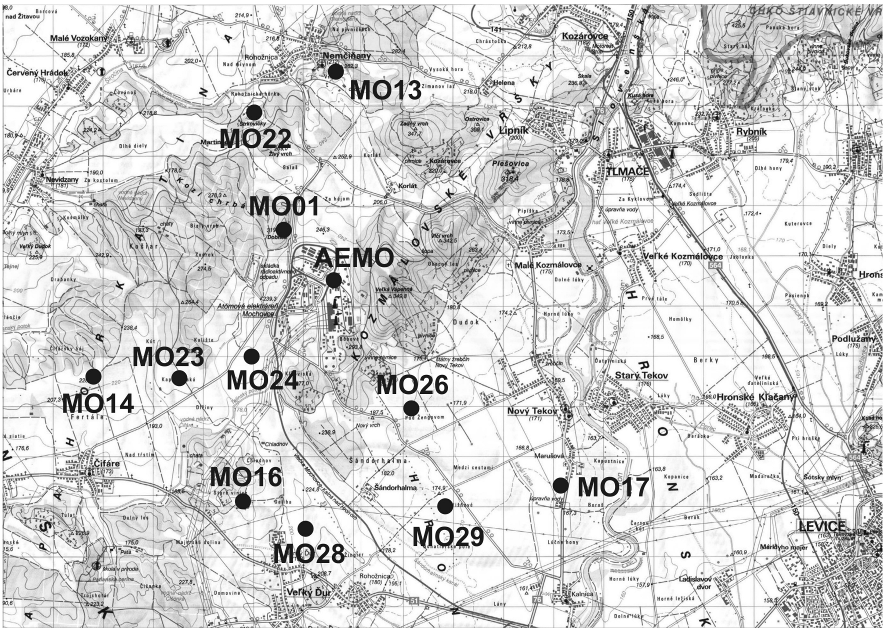

Fig. 1: The GPS monitoring network of Mochovce Nuclear Power Plant

Repeatability of evaluated horizontal position in various years is from 1 to 8 mm for all the monitored pillars. Estimated parameters include coordinates and relative linear movements of the monitored pillars. Results of the long-term stability inspection lead to the conclusion that all pillars are stable, with systematic horizontal movements smaller than 1 mm/year (Fig. 2).

Regional epoch and permanent networks for geokinematical investigations in Central Europe: Central Europe is covered by two, partially overlapping networks: (i) The epoch-wise network monitored from 1994 in one-year or two-year intervals, and (ii) Permanent network with sites continuously observed at least for 3 years.

Horizontal velocities were firstly estimated from separate solutions of epoch and permanent networks. Velocities at identical sites from two independent solutions coincide at mm/year level. Next the combined solution was performed. The general pattern of geokinematics of Central Europe and Balkan territory in clearly visible in Fig. 4.

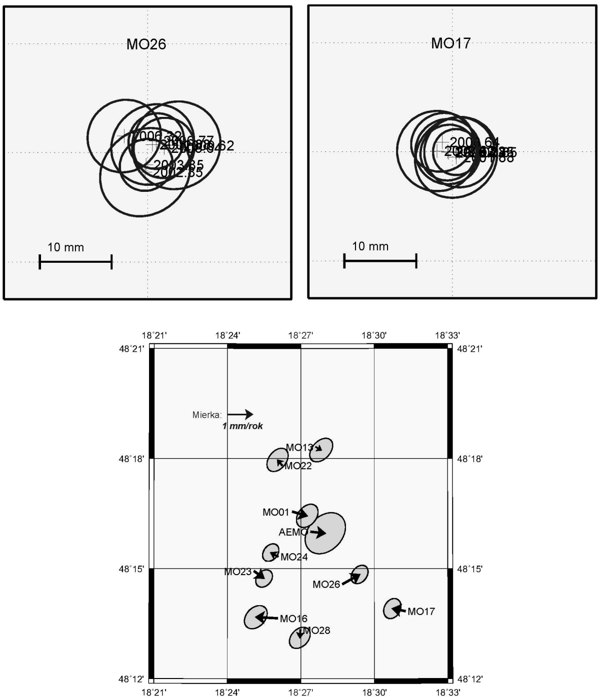

Fig. 2: Horizontal position estimates for individual epoch campaigns at two pillars (MO17 and MO26), and the estimated relative horizontal velocities of the monitored network with their confidence ellipses.

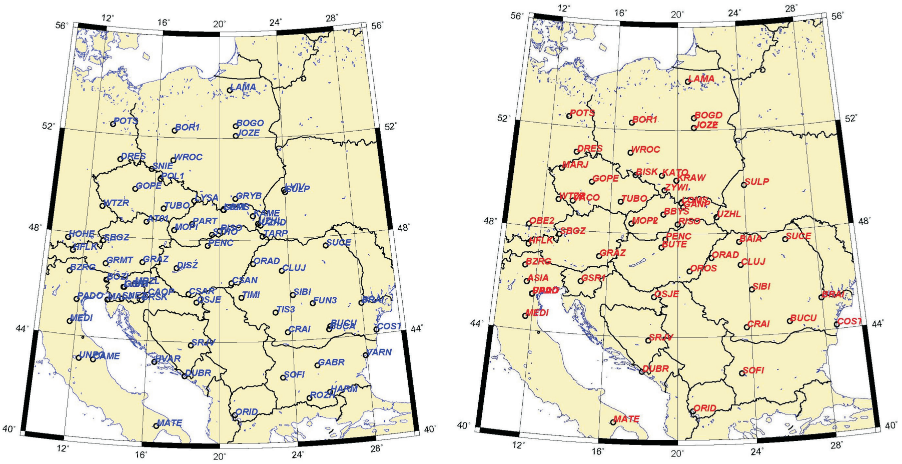

Fig. 3: Epoch-wise (left) and permanent (right) networks used for monitoring of geokinematics of Central Europe.

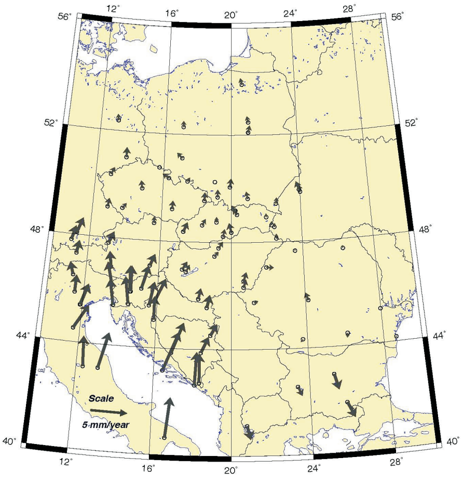

Fig. 4: Estimated horizontal velocity field from combination of epoch-wise and permanent networks.

6 Problems to be solved in future: estimation of accuracy and correlations among individual network solutions

In the combination model (1), (2) it is assumed that the global covariance matrix of observations has form

$$\mathbf{\Sigma} = \mathrm{var}\left(\mathbf{x}\right) = \begin{pmatrix} \mathbf{\Sigma}_{per_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_{epoch_1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Sigma}_{per_n} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{\Sigma}_{epoch_m} \end{pmatrix} \tag{3}$$

Such form of the matrix is based on two assumptions:

i) Accuracy of permanent and epoch network solutions is fully represented by their covariance matrices.

ii) Estimates of coordinates at various epochs are uncorrelated (it concerns both epoch-wise and permanent networks).

More appropriate stochastic modelling should respect the inter-epoch correlations and needs scaling of the covariance matrices of individual network solutions:

$$\mathbf{\Sigma} = \begin{pmatrix} \vartheta_{per_1}\mathbf{\Sigma}_{per_1} & \mathbf{\Sigma}_{p_1e_1} & \cdots & \mathbf{\Sigma}_{p_1p_n} & \mathbf{\Sigma}_{p_1e_m} \\ \mathbf{\Sigma}_{p_1e_1} & \vartheta_{epoch_1}\mathbf{\Sigma}_{epoch_1} & \cdots & \mathbf{\Sigma}_{e_1p_n} & \mathbf{\Sigma}_{e_1e_m} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{\Sigma}_{p_1p_n} & \mathbf{\Sigma}_{e_1p_n} & \cdots & \vartheta_{per_n}\mathbf{\Sigma}_{per_n} & \mathbf{\Sigma}_{p_ne_m} \\ \mathbf{\Sigma}_{p_1e_m} & \mathbf{\Sigma}_{e_1e_m} & \cdots & \mathbf{\Sigma}_{p_ne_m} & \vartheta_{epoch_m}\mathbf{\Sigma}_{epoch_m} \end{pmatrix} \tag{4}$$

Only simplified situations were solved up to now:

i) Variance components ϑ are estimated as representatives only for two types of networks only–e.g. as one representative value for permanent and one for epoch-wise networks.

ii) Observational noise is assumed as frequency dependent. The covariance matrix of observations $\mathbf{x}$ from permanent network is modelled as sum of white noise and coloured noise components $\mathbf{J}_j$ of known structure and with apriori known scale factors σ_j as $\mathbf{\Sigma} = \sum_j \sigma_j^2 \mathbf{J}_j$. Recently only simple 2-component models $j = 1, 2$ are used in practice and the modelling is limited only for independent 1D time series.

7 Conclusions

The geodetic networks observed by satellite GNSS techniques in various epochs are processed by models enabling to simultaneously estimate site coordinates, their time variations and a set of additional parameters necessary to eliminate various kinds of biases. The practical applications demonstrated the achieving millimetre accuracy of site coordinates and velocities in case of local and regional networks. Results obtained from long-term monitoring of epoch and permanent GNSS networks proved the ability of satellite techniques for very precise positioning and geo-kinematic investigations. However, there are still more unresolved problems in the network combinations, like the observational noise modelling, accuracy estimates of epoch and permanent networks and its evolution in time.

References

[1] Altamimi, Z., Collilieux, X., Legrand, J., Garayt, B., Boucher, C.: *A new release of the International Terrestrial Reference Frame based on time series of station positions and Earth Orientation Parameters.* Journal of Geophysical Research **112**, doi: 10.1029/2007JB004949 B09401 (2007).

[2] Dach, R., Hugentobler, U., Fridez, P., Meindl, M. (eds.): Bernese GPS Software Version 5.0. *Astronomical Institute, University of Berne*, 2007.

[3] Dobeš et al: Local Geodetic Networks. *Výskumný ústav geodézie a kartografie*, Bratislava, Slovakia, 1990, (in Slovak).

[4] Hefty, J.: Global Positioning System in Four-dimensional Geodesy. *Slovak University of Technology*, Bratislava, Slovakia, 2004, (in Slovak).

[5] Kubáček, L., Kubáčková, L., Kukuča, J.: Probability and Statistics in Geodesy and Geophysics. *Elsevier*, Amsterdam–Oxford–New York–Tokyo, 1987.

[6] Kubáčková, L.: Methods of Processing Experimental Data. *Veda*, Bratislava, 1990, (in Slovak).

[7] Leick, A.: GPS Satellite Surveying. Third Edition. *Wiley*, Hoboken, New Jersey, 2004.