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Dedicated to Lubomír Kubáček on the occasion of his 80th birthday

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Abstract

Mathematical programming under multiple objectives has emerged as a powerful tool to assist in the process of searching for decisions which best satisfy a multitude of conflicting objectives. In multiobjective linear programming problems it is usually impossible to optimize all objectives in a given system. Trade-offs are properties of inadequately designed system a thus can be eliminated through designing better one. Multiobjective De Novo linear programming is problem for designing optimal system by reshaping the feasible set. The paper presents approaches for solving the MODNLP problem, extensions of the problem, examples, and applications.

Key words: De Novo programming, multiple objectives, linear programming, trade-offs

2010 Mathematics Subject Classification: 90C29

1 Introduction

Traditional concepts of optimality focus on valuation of already given systems. New concept of designing optimal systems is applied (Zeleny [7]). Multiobjective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. In MOLP problems it is usually impossible to optimize all objectives together in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing this for another objective. Trade-offs are properties of inadequately designed system and

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thus can be eliminated through designing better one. The purpose is not to measure and evaluate tradeoffs, but to minimize or even eliminate them. An optimal system should be tradeoff-free. As a methodology of optimal system design can be employed De Novo programming for reshaping feasible sets in linear systems. De Novo concept was introduced by Milan Zelený (see [5]). Basic concepts of the De Novo optimization are summarized. The paper presents approaches for solving the Multi-objective De Novo linear programming (MODNLP) problem, possible extensions, methodological and real applications, and an illustrative example. The approach is based on reformulation of MOLP problem by given prices of resources and the given budget. Searching for meta-optimum with a minimal budget is used. The instrument of optimum-path ratio is used for achieving the best performance for a given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of systems boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources.

2 Multi-objective linear programming problem

Multi-objective linear programming (MOLP) problem can be described as follows

$$\begin{aligned} \text{Max } z &= Cx \\ \text{s.t. } Ax &\leq b \\ x &\geq 0 \end{aligned} \tag{1}$$

where C is a (k, n)-matrix of objective coefficients, A is a (m, n)-matrix of structural coefficients, b is an m-vector of known resource restrictions, x is an n-vector of decision variables. In MOLP problems it is usually impossible to optimize all objectives in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing this for another objective. For multi-objective programming problems the concept of non-dominated solutions is used. (see for example Fiala [2]). A compromise solution is selected from the set of non-dominated solutions. Two subjects Decision Maker and Analyst are introduced.

Classification of methods for solution of MOLP problems according to information mode:

• Methods with a priori information.

Decision Maker provides global preference information (weights, utility, goal values, . . .). Analyst solves a single objective problem.

• Methods with progressive information—interactive methods.

Decision Maker provides local preference information. Analyst solves local problems and provides current solutions.

• Methods with a posteriori information.

Analyst provides a non-dominated set. Decision Maker provides global preference information on the non-dominated set. Analyst solves a single objective problem.

There are proposed many methods from these categories. Most of the methods are based on trade-offs. The next part is devoted to the trade-off free approach.

3 Multi-objective De Novo linear programming problem

Multi-objective De Novo linear programming (MODNLP) is problem for designing optimal system by reshaping the feasible set. By given prices of resources and the given budget the MOLP problem (1) is reformulated in the MODNLP problem (2)

$$\begin{aligned} \text{Max } z &= Cx \\ \text{s.t. } Ax - b &\le 0 \\ pb &\le B \\ x &\ge 0 \end{aligned} \tag{2}$$

where b is an m-vector of unknown resource restrictions, p is an m-vector of resource prices, and B is the given total available budget.

From (2) follows

$$pAx \le pb \le B.$$

Defining n-vector of unit cost $v = pA$ we can rewrite problem (2) as

$$\begin{aligned} \text{Max } z &= Cx \\ \text{s.t. } vx &\le B \\ x &\ge 0 \end{aligned} \tag{3}$$

Solving single objective problems

$$\begin{aligned} \text{Max } z^i &= C^i x \quad i = 1, 2, \ldots, k \\ \text{s.t. } vx &\le B \\ x &\ge 0 \end{aligned} \tag{4}$$

z^* is k-vector of objective values for the ideal system with respect to B.

The problems (4) are continuous "knapsack" problems, the solutions are

$$x_i^j = \begin{cases} 0 & j \ne j_i \\ B/v_{j_i} & j = j_i \end{cases}$$

where $j_i \in \{j \in (1, \ldots, n) \mid \max_j (c_j^i / v_j)\}$.

The meta-optimum problem can be formulated as follows

$$\begin{aligned} \text{Min } f &= vx \\ \text{s.t. } Cx &\ge z^* \\ x &\ge 0 \end{aligned} \tag{5}$$

Solving problem (5) provides solution:

$$\begin{aligned} &x^* \\ B^* &= vx^* \\ b^* &= Ax^* \end{aligned}$$

The value B^* identifies the minimum budget to achieve z^* through solution x^* and b^*.

4 Optimum-path ratios

The given budget level $B \leq B^*$. The optimumpath ratio for achieving the best performance for a given budget B is defined as

$$r_1 = \frac{B}{B^*}.$$

The optimum-path ratio provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems. Optimal system design for the budget B:

$$x = r_1 x^*, \quad b = r_1 b^*, \quad z = r_1 z^*$$

If the number of criteria k is less than that of variables n, we can individually solve the problem and obtain synthetic solutions. Shi [4] defined the synthetic optimal solution as follows $x^{**} = (x^1_{j_1}, \ldots, x^k_{j_k}, 0, \ldots, 0) \in R^n$, where $x^q_{j_q}$ is the optimal solution of (4). For the synthetic optimal solution a budget is used. There is possible define six types of optimum-path ratios (Shi [4]):

$$r_1 = \frac{B}{B^*}, \quad r_2 = \frac{B}{B^{**}}, \quad r_2 = \frac{B^*}{B^{**}},$$

$$r_4 = \frac{\sum_i \lambda_i B_i^j}{B}, \quad r_5 = \frac{\sum_i \lambda_i B_i^j}{B^*}, \quad r_6 = \frac{\sum_i \lambda_i B_i^j}{B^{**}}.$$

Optimum-path ratios are different. There is possible to establish different optimal system design as options for decision maker.

5 Extensions

There are extension possibilities of De Novo programming (DNP):

- Fuzzy DNP.
- Interval DNP.
- Complex types of objective functions.
- Continuous innovations.

Fuzzy De Novo Programming (FDNP) uses instruments as fuzzy parameters, fuzzy goals, fuzzy relations, and fuzzy approaches (Li and Lee [3]).

Inexact De Novo programming (IDNP) incorporates the interval programming and de Novo programming, allowing uncertainties represented as intervals within the optimization framework. The IDNP approach has the advantages in constructing optimal system design via an ideal system by introducing the flexibility toward the available resources in the system constraints (Zhang et al. [8]).

Complex types of objective functions are defined. The multi-objective form of Max (cx - pb) appears to be the right function to be maximized in a globally competitive economy (Zeleny [6]).

Searching for a better portfolio of resources leads to continuous reconfiguration and "reshaping" of systems boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources. The technological innovation matrix $T = (t_{ij})$ is introduced. The elements in the structural matrix A should be reduced by technological progress. T should be continuously explored. The problem (2) is reformulated in to innovation MODNLP problem (6)

$$\begin{aligned} \text{Max } z &= Cx \\ \text{s.t. } TAx - b &\leq 0 \\ pb &\leq B \\ x &\geq 0 \end{aligned} \tag{6}$$

The multi-objective optimization can be then seen as a dynamic process in three time horizons:

1. short – term equilibrium:
   - trade-off,
   - operational thinking.
2. mid – term equilibrium:
   - trade-off free,
   - tactical thinking.
3. long – term equilibrium:
   - beyond trade-off free,
   - strategic thinking.

6 Applications

The tradeoffs-free decision making has a significant number of methodological applications. All such applications have the tradeoffs-free alternative in common:

- Compromise programming minimize distance from the ideal point.
- Risk management—portfolio selection—tradeoffs between investment returns and investment risk.
- Game theory – win-win solutions.
- Added value – value for the producer and value for the customer—both must benefit.

There are real applications of De Novo approach. For example production plan for a real production system is defined taking into account financial constraints and given objective functions (Babic and Pavic [1]). The paper

(Zhang et al. [8]) presents an Inexact DNP approach for the design of optimal water-resources-management systems under uncertainty. Optimal supplies of good-quality water are obtained in considering different revenue targets of municipalindustrialagricultural competition under a given budget.

7 Illustrative example

The MOLP problem is formulated:

$$\begin{gathered}\text{Max } \ z_1 = x_1 + x_2\\ \text{Max } \ z_2 = x_1 + 4x_2\\ 3x_1 + 4x_2 \leq 60,\\ x_1 + 3x_2 \leq 30,\\ x_1 \geq 0, \ x_2 \geq 0.\end{gathered}$$

The MODNLP problem is formulated:
Input: $p = (0.5, 0.4) \quad B = 42,$
unit costs $v = pA = (1.9, 3.2).$

$$\begin{aligned} \text{Max } z^i = C^i x \quad i = 1, 2, \ldots, k \\ \text{s.t. } vx \leq B \\ x \geq 0 \end{aligned} \qquad z_1^* = 22.11, \ z_2^* = 52.50,$$

$$\begin{aligned} \text{Min } f = vx \\ \text{s.t. } Cx \geq z^* \\ x \geq 0 \end{aligned} \qquad \begin{aligned} &x_1^* = 11.98, \ x_2^* = 10.13 \\ &B^* = vx^* = 55.17 \\ &b^* = Ax^*, \ b_1^* = 76.48, \ b_2^* = 42.39 \end{aligned}$$

$$r_1 = \frac{B}{B^*} = 0.761$$

Optimal system design for B: $x = r_1 x^*$, $b = r_1 b^*$, $z = r_1 z^*$,
$x_1 = 9.12$, $x_2 = 7.71$, $b_1 = 58.23$, $b_2 = 32.25$, $z_1 = 16.83$, $z_2 = 39.96$.

The innovation MODNLP problem is formulated:
Input: $p = (0.5, 0.4) \quad B = 42,$
technological innovation matrix $T = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix},$
unit costs $v = pTA = (1.48, 2.44),$
$z_1^* = 28.38, \ \ z_2^* = 68.85,$
$x_1^* = 14.89, \ \ x_2^* = 13.49,$
$B^* = vx^* = 54.95,$
$r_1 = 0.764,$
$x_1 = 11.38, \ \ x_2 = 10.31,$
$z_1 = 21.69, \ \ z_2 = 52.62.$

The solutions in different time horizons are represented in Fig. 1.

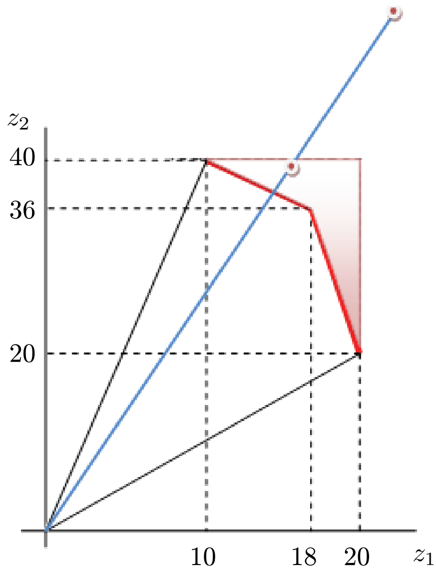

Fig. 1. Solutions for the illustrative example

8 Conclusions

Traditional concepts of optimality focus on valuation of already given system. New concepts of optimality are oriented on designing optimal systems. The purpose is not to measure and evaluate tradeoffs, but to minimize or even eliminate them. An optimal system should be tradeoff-free. De Novo programming is used as a methodology of optimal system design for reshaping feasible sets in linear systems. MOLP problem is reformulated by given prices of resources and the given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of systems boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources. These changes can lead to beyond tradeoff-free solutions. Multiobjective optimization can be taken as a dynamic process. De Novo programming approach is open for further extensions and applications.

References

[1] Babic, Z., Pavic, I.: *Multicriterial production planning by De Novo programming approach.* International Journal of Production Economics **43** (1996), 59–66.

[2] Fiala, P.: Modely a metody rozhodování. *Economia*, Praha, 2008.

[3] Li, R. J., Lee, E. S.: *Fuzzy Approaches to Multicriteria De Novo Programs.* Journal of Mathematical Analysis and Applications **153** (1990), 13–20.

[4] Shi, Y.: *Studies on Optimum-Path Ratios in De Novo Programming Problems.* Computers and Mathematics with Applications **29** (1995), 43–50.

[5] Zelený, M.: *De Novo Programming.* Ekonomicko-matematický obzor **26** (1990), 406–413.

[6] Zeleny, M.: *Multiobjective Optimization, Systems Design and De Novo Programming.* In: Zopounidis C., Pardalos P. M. (eds.): Handbook of Multicriteria Analysis, *Springer*, Berlin, 2010.

[7] Zeleny, M.: *Optimal Given System vs. Designing Optimal System: The De Novo Programming Approach.* International Journal of General System **17** (1990), 295–307.

[8] Zhang, Y. M., Huang, G. H., Zhang, X. D.: *Inexact de Novo programming for water resources systems planning.* European Journal of Operational Research **199** (2009), 531–541.