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Kybernetika, Vol. 47 (2011), No. 4, 612--629

Persistent URL: http://dml.cz/dmlcz/141662

KYBERNETIKA — VOLUME 47 (2011), NUMBER 4, PAGES 612–629

ROBUST CONTROL OF CHAOS IN MODIFIED FITZHUGH–NAGUMO NEURON MODEL UNDER EXTERNAL ELECTRICAL STIMULATION BASED ON INTERNAL MODEL PRINCIPLE

Yuan Jiang and Jiyang Dai

This paper treats the question of robust control of chaos in modified FitzHugh–Nagumo neuron model under external electrical stimulation based on internal model principle. We first present the solution of the global robust output regulation problem for output feedback system with nonlinear exosystem. Then we show that the robust control problem for the modified FitzHugh–Nagumo neuron model can be formulated as the global robust output regulation problem and the solvability conditions for the output regulation problem for the modified FitzHugh–Nagumo neuron model are all satisfied. Then we apply the obtained output regulation results to the control problem for modified FitzHugh–Nagumo neuron model. Finally, an output feedback control law is designed for the modified FitzHugh–Nagumo neuron model to achieve global stability of the closed-loop system in the presence of uncertain parameters and external stimulus. An example is shown that the proposed algorithm can completely reject the external electrical stimulation generated from a Van der Pol circuit.

Keywords: control theory, Lyapunov methods, internal model principle, modified FitzHugh–Nagumo model, Van der Pol circuit

Classification: 93E12, 62A10

1. INTRODUCTION

Recently, the study of biology at the system level has received considerable attention, giving rise to the field of "systems biology" [16, 23]. As we know, many problems in control can be framed as regulation problems, where the goal is to design a controller that can drive the output of a system that has been perturbed by possibly unmeasurable disturbances to a constant set point [14]. Output regulation is also crucial for the functioning of all biological systems, where it is known as homeostasis, from the simplest bacteria to humans. Not surprisingly, feedback control is essential in achieving homeostasis. For example, the production of tryptophan which is an essential amino acid is regulated in bacteria by a series of three feedback loops [35].

Chaos which is a universal phenomenon in nonlinear systems exists in a variety of neural systems ranging from the simple to complex [24]. Since determining the dynamical behavior of an ensemble of coupled neurons is an important problem in computational neuroscience. So the primary step for understanding this complex problem is to understand the dynamical behavior of individual neurons. Commonly used models for the study of individual neurons which display spiking/bursting behavior include the modified FitzHugh–Nagumo model. So our studies are based on the modified FitzHugh–Nagumo model.

The modified FitzHugh–Nagumo model was originally produced by Rinzel in 1987 for exhibiting the qualitative behavior observed in neurons, viz quiescence, excitability and periodicity [31]. The dynamic equations for the controlled modified FitzHugh–Nagumo neuron model can be derived as follows:

$$\begin{aligned} \dot{z}_1 &= z_1 - \tfrac{1}{3}z_1^3 - z_2 + z_3 + L(t) + I(u) \\ \dot{z}_2 &= \epsilon_1(z_1 + a_1 - a_2 z_2) \\ \dot{z}_3 &= \epsilon_2(-z_1 + a_3 - a_4 z_3). \end{aligned} \tag{1}$$

The function $L(t)$ represents the external stimulus. From biological point of view, the variable z_1 represents the potential difference between the dendritic spine head and its surrounding medium, z_2 is recovery variable and z_3 represents the slowly moving current in the dendrite. In this model, z_1 and z_2 together make up a fast subsystem relative to z_3. $I(u)$ is the control input, and $a_1, \dots, a_4$, ϵ_1 and ϵ_2 are positive constants, and the external electrical stimulation $L(t)$ is generated by the following Van der Pol circuit

$$\begin{aligned} \dot{w}_1 &= w_2 - \varsigma(\tfrac{1}{3}w_1^3 - w_1) \\ \dot{w}_2 &= -w_1 \end{aligned} \tag{2}$$

where $1.6 \le \varsigma \le 2.4$. The parameter ς can be treated as a tuning parameter for adjusting the period of current/voltage cycle. The eigenvalues of the Jacobian matrix at the origin of (2) are $\frac{1}{2}(\varsigma \pm \sqrt{\varsigma^2 - 4})$. When $\varsigma \ge 2$, the eigenvalues are positive; when $0 < \varsigma \le 2$, the eigenvalues are complex conjugates with positive real parts. Thus, the origin is an unstable equilibrium point and there exists a limit cycle.

Given a class of external electrical stimulation $L(t) = w_1$, we are interested in the problem of designing an output feedback controller such that, in the presence of uncertain parameters and external stimulus, the trajectory of the closed-loop system starting from any initial state of the modified FitzHugh–Nagumo neuron model and the controller exists and is globally bounded for all $t \ge 0$.

It will be shown in Section 3 that the above problem can be fallen into robust output regulation problem for output feedback systems described in Section 2. Therefore, we will first describe the robust output regulation problem and present the solution to this problem in Section 2. In Section 3, we will show that the control problem for the modified FitzHugh–Nagumo neuron model can be dealt with by the results in Section 2. Furthermore, we will design an output feedback controller to solve the above problem with computer simulation in Section 3. Finally, a conclusion is given in Section 4.

Output regulation problem was first studied for the class of linear time-invariant systems and was completely solved by the collective efforts of many researchers (see,

e. g. [6, 14, 22]). A celebrated outcome of their research was what is called the internal model principle. The internal model principle was initially developed for linear systems, but its applications have also been extended to some nonlinear control problems (see, e. g. [7, 8, 20]). In particular, Huang et al. [18, 19] are the pioneering papers that give the solvability conditions for robust nonlinear output regulation problem in terms of internal model principle. According to the internal model principle, the outputs of a linear dynamic system, namely exosystem, are treated as deterministic external disturbances. The influence on the system response, which is caused by exosystem's disturbances, can be suitably reduplicated on the feedback path of the closed-loop system.

As we know, various control problems of the FitzHugh–Nagumo neuron model have been considered extensively. Mishra et al. [28] studied the dynamics of modified FitzHugh–Nagumo neuron model and investigated bifurcation in the dynamics of two modified FitzHugh–Nagumo neurons coupled to each other through an electrical coupling. Controlling synchronization between pair of modified FitzHugh–Nagumo neuron models under external electrical stimulation by a nonlinear controlling mechanism is proposed in [29]. Synchronization of FitzHugh–Nagumo neural systems under external electrical stimulation via the nonlinear control is investigated in [36]. In [9], a robust adaptive neural network (NN) controller is proposed to realize the synchronization of two gap junction coupled chaotic FitzHugh–Nagumo neurons under external electrical stimulation. Recently, output feedback control of FitzHugh–Nagumo model was studied in [34] by employing the internal model approach. In this paper, we will address the problem of robust control of chaos in modified FitzHugh–Nagumo neuron model under external electrical stimulation based on internal model principle. In particular, we will demonstrate how the adaptive control method developed recently for nonlinearly parameterized systems in [25] can be used to solve the problem of global robust output regulation for nonlinear systems in the output feedback form with unknown parameters and exogenous signals belonging to a compact set whose bound is unknown. Our result is different from all the previous works in the following senses. First, our algorithm allows for the nonlinear systems in the output feedback form with unknown parameters and exogenous signals belonging to a compact set whose bound is unknown. Second, it is possible for us to achieve global robust stabilization for the modified FitzHugh–Nagumo neuron model under external electrical stimulation generated by a Van der Pol circuit. Third, our algorithm provides a nonlinear internal model for a class of external stimulus and a nonlinear regulator with complete servocompensation of the class of external stimulus.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this section, we will formulate the control problem and present some preliminaries. Consider the following uncertain nonlinear systems which can be transformed into the output feedback form:

$$\begin{aligned}\dot{x} &= \bar{F}(a)x + \bar{G}(y,w,a)y + \bar{D}_1(w,a)\\ \dot{y} &= \bar{H}(a)x + \bar{K}(y,w,a)y + h(a)u + \bar{D}_2(w,a)\\ e &= y - q(w,a)\end{aligned} \tag{3}$$

where $(x,y) \in R^n$, $y \in R$, and $u \in R$ are the system states, output and input, respectively. The error signal $e \in R$ is the only measurable variable that can be used in feedback design. The unknown constant parameter vector a belongs to a compact set $\wp \subset R^q$ whose bound is unknown. It is assumed that all the functions in system (3) are sufficiently smooth and $\bar{D}_1(0,a)$, $\bar{D}_2(0,a)$, and $q(0,a)=0$ for all $a \in \wp$. The exogenous signal $w \in R^m$ represents either the disturbance signal or the reference input or both, which generated from a nonlinear exosystem

$$\dot{w} = s(w) \tag{4}$$

To introduce our problem, let us first make some assumptions in the following.

Assumption 1. There exists a positive definite function $V(w)$ such that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\partial V}{\partial w}s(w) \leq 0, \quad when \;\; |w(t)| \geq W_0$$

where W_0 is an unknown positive constant.

Remark 1. Assumption 1 is to say that the flows of vector field $s(w)$ are bounded. Based on Assumption 1, the periodic solutions of the exosystems can include many functions, such as harmonic functions and limit cycles of nonlinear dynamic systems.

Assumption 2. For all $a \in \wp$, $\bar{F}(a)$ is Hurwitz, $h(a)$ is continuous in a and $h(a) > 0$.

The global robust output regulation problem that we are going to solve is to find a finite dimensional system

$$\begin{aligned}\dot{\mu} &= \upsilon(\mu, e(t)), \mu \in R^s,\\ u &= u(\mu, e(t))\end{aligned} \tag{5}$$

such that the closed-loop system (3)-(5) is globally bounded for any initial condition $x(0) \in R^{n-1}$, $w(0) \in \Omega \subset R^m$ and any $a \in \wp$. Moreover, $\lim_{t\to\infty} e(t) = 0$.

The global robust output regulation problem for output feedback systems has been studied under various assumptions (see, e. g. [4, 33, 34]). We note that this formulation is more general than those in that both of these papers require that w and a belong to some known compact subsets. Here, accounting for the unknown bound of $w(t)$ and a, we will integrate both adaptive control and robust control techniques from [27] and [21].

According to the results in reference [13], the output regulation problem of systems $(3)-(4)$ can be solved only the following assumption is satisfied.

Assumption 3. There exists $\varpi(w,a)$ with $\varpi(0,0)=0$ such that

$$\frac{\partial \varpi}{\partial w}s(w)=\bar{F}(a)\varpi+\bar{G}(q(w,a),w,a)q(w,a)+\bar{D}_1(w,a) \tag{6}$$

Thus, the solution of the regulator equations associated with equations (3) and (4) is given by $(\varpi(w,a),q(w,a))$ and $\iota(w,a)$, where

$$\begin{aligned}\iota(w,a)=h^{-1}(a)[&\frac{\partial q(w,a)}{\partial w}s(w)-\bar{H}(a)\varpi(w,a)\\ &-\bar{K}(q(w,a),w,a)q(w,a)-\bar{D}_2(w,a)]\end{aligned} \tag{7}$$

In order to solve the robust output regulation problem, we need an assumption on the structure of the nonlinear exosystem (4) in the following. In most of the papers on the robust output regulation of nonlinear systems, for example, [2, 5, 10, 12, 15, 17, 32, 33], the exosystem is linear and the feed-forward term is a polynomial of the state of the exosystem. More recently, some progresses are reported on output regulation with nonlinear exosystems (see, e. g. [3, 5, 11, 26, 30, 34, 37, 38]).

Assumption 4. For the exosystem (4), there exists an immersion system

$$\begin{aligned}\dot{\eta}&=F\eta+G\gamma(J\eta)\\ \iota(w,a)&=H\eta\end{aligned} \tag{8}$$

where $\eta\in R^r$, the known matrices F,G,H and J have appropriate dimensions, and the pair (F,H) is observable, and there exists a positive definite matrix $P_{\hat{\eta}}$ satisfying $P_{\hat{\eta}}G+J^T=0$, and the nonlinear function $\gamma(J\eta)=\begin{bmatrix}\gamma_1(\sum_{i=1}^r J_{1i}\eta_i)\\ \vdots\\ \gamma_m(\sum_{i=1}^r J_{mi}\eta_i)\end{bmatrix}$ satisfies that $(v_1-v_2)^T(\gamma(v_1)-\gamma(v_2))\geq 0$.

Remark 2. Assumption 4 bases on the assumption made on the exosystems in [3], and we can know this is a condition for which the circle criterion in [1] can be applied. For the Van der Pol circuit, let $\eta=w$ and choose the matrix parameters as follows: $F=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$, $G=\begin{bmatrix}-2 & 0\\ 0 & 0\end{bmatrix}$, $J=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$, $H=\begin{bmatrix}1 & 0\end{bmatrix}$, $\gamma_1(s)=\frac{1}{3}s^3$, $\gamma_2(s)=0$, $P_{\hat{\eta}}=\mathrm{diag}(1/2,1/4)$. It can be seen that Assumption 4 is satisfied.

Since the feed-forward term $\iota(w,a)$ in the immersion system model η is unknown, so we design the following internal model as

$$\dot{\hat{\eta}}=(F-KH)(\hat{\eta}-Kh^{-1}(a)e)+G\gamma(J(\hat{\eta}-Kh^{-1}(a)e))+Ku \tag{9}$$

where $K\in R^r$ such that $F_0=F-KH$ is Hurwitz and there exist positive definite matrices $P_{\hat{\eta}}$ and $Q_{\hat{\eta}}$ satisfying

$$P_{\hat{\eta}}F_0+F_0^TP_{\hat{\eta}}=-Q_{\hat{\eta}} \tag{10}$$

Remark 3. It is noted that there exist positive definite matrices $P_{\hat{\eta}}$ and $Q_{\hat{\eta}}$ satisfying

$$\begin{aligned} &P_{\hat{\eta}}F_0 + F_0^T P_{\hat{\eta}} = -Q_{\hat{\eta}} \\ &P_{\hat{\eta}}G + J^T = 0. \end{aligned} \tag{11}$$

In particular, if G and J^T are two column vectors, and the pair (F_0, G) is controllable, and the pair (J, F_0) is observable, and the triple (F_0, G, J) satisfies the strictly positive real condition $Re[-J(jwI - F_0)^{-1}G] > 0, \forall w \in R$, then there exists a solution of (11) from the well known Meyer–Kalman–Yacubovic Theorem.

Attaching (9) to (3) leads to what is called the augmented system in reference [15]. Furthermore, performing on the augmented system the following coordinate and input transformation

$$\tilde{x} = x - \varpi(w, a),$$

$$e = y - q(w, a),$$

$$\tilde{\eta} = \eta - \hat{\eta} + Kh^{-1}(a)e.$$

Finally, we have the model for the control design

$$\begin{aligned} \dot{\tilde{x}} &= \bar{F}(a)\tilde{x} + \tilde{G}(e, w, a)e, \\ \dot{\tilde{\eta}} &= F_0\tilde{\eta} + G\gamma(J\eta) - G\gamma(J(\eta - \tilde{\eta})) + Kh^{-1}(a)\bar{H}(a)\tilde{x} + Kh^{-1}(a)\tilde{K}(e, w, a)e, \\ \dot{e} &= \bar{H}(a)\tilde{x} + \tilde{K}(e, w, a)e + h(a)(u - \iota(w, a)) \end{aligned} \tag{12}$$

where

$$\tilde{G}(e, w, a)e = \bar{G}(q(w, a) + e, w, a)(q(w, a) + e) - \bar{G}(q(w, a), w, a)q(w, a),$$

$$\tilde{K}(e, w, a)e = \bar{K}(q(w, a) + e, w, a)(q(w, a) + e) - \bar{K}(q(w, a), w, a)q(w, a).$$

System (12) is called an augmented system associated with the given system and the nonlinear exosystem. It can be shown that $\tilde{G}(0, w, a) = 0$ and $\tilde{K}(0, w, a) = 0$ for all w and a. Therefore, the origin is the equilibrium of system (12). Moreover, if a feedback control law solves the global robust stabilization problem of the lower triangular system (12), then the global robust output regulation problem of the given system and the exosystem is also solvable.

Recall that both $w(t)$ and a are in compact sets whose bounds are unknown. By Lemma 2.1 in [25],

$$\|P(a)\tilde{G}(e, w, a)e\|^2 \leq \bar{\theta}_1(w, a)\alpha_1(e)e^2 \leq \theta_1\alpha_1(e)e^2,$$

$$\frac{1}{h^2(a)}\|\tilde{K}(e, w, a)e\|^2 \leq \bar{\theta}_2(w, a)\alpha_2(e)e^2 \leq \theta_2\alpha_2(e)e^2,$$

$$\frac{1}{h^2(a)}\|\bar{H}(a)\|^2 \leq \theta_3, \tag{13}$$

where $\alpha_1(e) \geq 1$, $\alpha_2(e) \geq 1$ are smooth known functions and $\theta_i \geq 1, i = 1, \ldots, 3$, are unknown constants.

Let $V_{\tilde{x}} = \tilde{x}^T P(a)\tilde{x}$, where

$$P(a)\bar{F}(a) + \bar{F}^T(a)P(a) = -I,$$

then using $2bc \leq db^2 + d^{-1}c^2$ there exists an unknown constant θ_4 such that

$$\begin{aligned}
\dot{V}_{\tilde{x}} &= -\|\tilde{x}\|^2 + 2\tilde{x}^T P(a)\tilde{G}(e,w,a)e \\
&\leq -\frac{3}{4}\|\tilde{x}\|^2 + 4\|P(a)\tilde{G}(e,w,a)e\|^2 \\
&\leq -\frac{3}{4}\|\tilde{x}\|^2 + 4\theta_1\alpha_1(e)e^2 \\
&\leq -\frac{3}{4}\|\tilde{x}\|^2 + \theta_4\alpha_1(e)e^2
\end{aligned} \tag{14}$$

noting that

$$2\tilde{x}^T P(a)\tilde{G}(e,w,a)e \leq \frac{1}{4}\|\tilde{x}\|^2 + 4\|P(a)\tilde{G}(e,w,a)e\|^2$$

Let $V_{\tilde{\eta}} = \tilde{\eta}^T P_{\hat{\eta}}\tilde{\eta}$, where $P_{\hat{\eta}}F_0 + F_0^T P_{\hat{\eta}} = -Q_{\hat{\eta}}$, then there exist unknown positive real constants θ_5 and θ_6 such that

$$\begin{aligned}
\dot{V}_{\tilde{\eta}} &= -\tilde{\eta}^T Q_{\hat{\eta}}\tilde{\eta} + 2\tilde{\eta}^T P_{\hat{\eta}}[Kh^{-1}(a)\bar{H}(a)\tilde{x} + Kh^{-1}(a)\tilde{K}(e,w,a)e] \\
&\quad + 2\tilde{\eta}^T P_{\hat{\eta}}G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \\
&\leq -\frac{3}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + \frac{8\|P_{\hat{\eta}}K\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\|h^{-1}(a)\bar{H}(a)\|^2\|\tilde{x}\|^2 \\
&\quad + \frac{8\|P_{\hat{\eta}}K\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\|h^{-1}(a)\tilde{K}(e,w,a)e\|^2 \\
&\leq -\frac{3}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + \frac{8\|P_{\hat{\eta}}K\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\theta_3\|\tilde{x}\|^2 + \frac{8\|P_{\hat{\eta}}K\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\theta_2\alpha_2(e)e^2 \\
&\leq -\frac{3}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_6\alpha_2(e)e^2
\end{aligned} \tag{15}$$

noting that

$$2\tilde{\eta}^T P_{\hat{\eta}}Kh^{-1}(a)\bar{H}(a)\tilde{x} \leq \frac{1}{8}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + \frac{8\|P_{\hat{\eta}}K\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\|h^{-1}(a)\bar{H}(a)\|^2\|\tilde{x}\|^2,$$

$$2\tilde{\eta}^T P_{\hat{\eta}}Kh^{-1}(a)\tilde{K}(e,w,a)e \leq \frac{1}{8}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + \frac{8\|P_{\hat{\eta}}K\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\|h^{-1}(a)\tilde{K}(e,w,a)e\|^2.$$

It follows from Assumption 4 that $P_{\hat{\eta}}G = -J^T$, and thus we have

$$
\begin{aligned}
&2\tilde{\eta}^T P_{\hat{\eta}} G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \\
&= -2(J\eta - J(\eta - \tilde{\eta})))^T(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \leq 0.
\end{aligned} \tag{16}
$$

From previous discussions

$$
\begin{aligned}
\dot{e} &= \bar{H}(a)\tilde{x} + \tilde{K}(e, w, a)e + h(a)(u - \iota(w, a)) \\
&= \bar{H}(a)\tilde{x} + \tilde{K}(e, w, a)e + h(a)(u - H\eta) \\
&= \bar{H}(a)\tilde{x} + \tilde{K}(e, w, a)e + h(a)(u - H\tilde{\eta} - H\hat{\eta} + HKh^{-1}(a)e).
\end{aligned} \tag{17}
$$

We now design control u as

$$
\begin{aligned}
u &= -r\rho(e)e - (3 + \frac{\|H\|^2}{\lambda_{\min}(Q_{\hat{\eta}})})e - \hat{\Theta}\tilde{l}(e) + H\hat{\eta}, \\
\dot{r} &= \rho(e)e^2
\end{aligned} \tag{18}
$$

where r is a time-varying design parameter, $\rho(e) \geq 1$ and $\tilde{l}(e)$ are smooth functions to be determined, $\hat{\Theta}$ is adaptive coefficient. Then we have the resultant error dynamics

$$
\begin{aligned}
\dot{e} = \bar{H}(a)\tilde{x} + \tilde{K}(e, w, a)e + h(a)\Big[&-r\rho(e)e - \Big(3 + \frac{\|H\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}\Big)e - \hat{\Theta}\tilde{l}(e) \\
&- H\tilde{\eta} + HKh^{-1}(a)e\Big].
\end{aligned} \tag{19}
$$

Then for $V_e = \frac{1}{2h(a)}e^2$ there exist unknown positive real constants θ_7, θ_8 and θ_9 such that

$$
\dot{V}_e \leq -r\rho(e)e^2 + \theta_7\|\tilde{x}\|^2 + \theta_8\alpha_2(e)e^2 + \theta_9 e^2 - e\hat{\Theta}\tilde{l}(e) + \frac{1}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 \tag{20}
$$

noting that

$$
\begin{aligned}
&\frac{1}{h(a)}e\bar{H}(a)\tilde{x} \leq e^2 + \frac{1}{4}\|h^{-1}(a)\bar{H}(a)\|^2\|\tilde{x}\|^2, \\
&\frac{1}{h(a)}e\tilde{K}(e, w, a)e \leq e^2 + \frac{1}{4}\|h^{-1}(a)\tilde{K}(e, w, a)\|^2 e^2, \\
&-eH\tilde{\eta} \leq \frac{\|H\|^2}{\lambda_{\min}(Q_{\hat{\eta}})}e^2 + \frac{1}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2, \\
&HKh^{-1}(a)e^2 \leq e^2 + \frac{1}{4}\|HKh^{-1}(a)\|^2 e^2.
\end{aligned}
$$

Let

$$
V = \beta V_{\tilde{x}} + V_{\tilde{\eta}} + V_e + \frac{1}{2}\gamma_{\Theta}^{-1}(\hat{\Theta} - \Theta)^2 + \frac{1}{2}r^2 \tag{21}
$$

where $\beta \geq \frac{4}{3}(\theta_5 + \theta_7 + 1)$ is chosen, and γ_Θ is a positive real constant. Then

$$\begin{aligned}\dot{V} \leq & -\frac{3}{4}\beta\|\tilde{x}\|^2 + \beta\theta_4\alpha_1(e)e^2 - \frac{3}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_6\alpha_2(e)e^2 \\ & - r\rho(e)e^2 + \theta_7\|\tilde{x}\|^2 + \theta_8\alpha_2(e)e^2 + \theta_9 e^2 - e\hat{\Theta}\tilde{l}(e) + \frac{1}{4}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 \\ & - \gamma_\Theta^{-1}\tilde{\Theta}\dot{\hat{\Theta}} + \dot{r}r \end{aligned} \tag{22}$$

Let $\Theta = \beta\theta_4 + \theta_6 + \theta_8 + \theta_9$, which is an unknown constant and

$$\dot{\hat{\Theta}} = e\gamma_\Theta\tilde{l}(e) \tag{23}$$

where $\tilde{l}(e)$ is a function of e given by

$$\tilde{l}(e) = \alpha_1(e)e + 2\alpha_2(e)e + e \tag{24}$$

Note that $\tilde{l}(e)$ is a continuous function of e. This property is guaranteed by the fact that $\alpha_1(e)$ and $\alpha_2(e)$ are smooth functions of e and the fact $\alpha_1(0) = 0$ and $\alpha_2(0) = 0$. Then

$$\begin{aligned}\dot{V} \leq & (\dot{r} - \rho(e)e^2)r - \frac{1}{2}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 - \frac{3}{4}\beta\|\tilde{x}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_7\|\tilde{x}\|^2 \\ & + \Theta[\alpha_1(e)e^2 + \alpha_2(e)e^2 + \alpha_2(e)e^2 + e^2] - e\hat{\Theta}\tilde{l}(e) - \gamma_\Theta^{-1}\tilde{\Theta}\dot{\hat{\Theta}} \\ \leq & (\dot{r} - \rho(e)e^2)r - \frac{1}{2}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 - \frac{3}{4}\beta\|\tilde{x}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_7\|\tilde{x}\|^2 \\ & + e\Theta\tilde{l}(e) - e\hat{\Theta}\tilde{l}(e) - \gamma_\Theta^{-1}\tilde{\Theta}\dot{\hat{\Theta}} \\ \leq & (\dot{r} - \rho(e)e^2)r - \frac{1}{2}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 - \frac{3}{4}\beta\|\tilde{x}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_7\|\tilde{x}\|^2 \\ & + e\Theta\tilde{l}(e) - e\hat{\Theta}\tilde{l}(e) - e\tilde{\Theta}\tilde{l}(e) \\ \leq & (\dot{r} - \rho(e)e^2)r - \frac{1}{2}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 - \frac{3}{4}\beta\|\tilde{x}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_7\|\tilde{x}\|^2 \end{aligned} \tag{25}$$

It follows from (18), and we have

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 - \frac{3}{4}\beta\|\tilde{x}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_7\|\tilde{x}\|^2 \tag{26}$$

It follows from

$$\beta \geq \frac{4}{3}(\theta_5 + \theta_7 + 1),$$

and we have

$$-\frac{3}{4}\beta\|\tilde{x}\|^2 + \theta_5\|\tilde{x}\|^2 + \theta_7\|\tilde{x}\|^2 \leq -\|\tilde{x}\|^2 \tag{27}$$

Substituting (27) into (26), we obtain

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 - \|\tilde{x}\|^2 \tag{28}$$

Therefore we can conclude that $\|\tilde{x}\| \in L_2 \cap L_\infty$, $\|\tilde{\eta}\| \in L_2 \cap L_\infty$ and $e \in L_2 \cap L_\infty$, and the variable $\hat{\Theta}$ is bounded. The boundedness of $\hat{\eta}$ follows from the boundedness of $\tilde{\eta}$ and w. Therefore we can conclude that all the variables in the closed-loop control system are bounded. Furthermore, together with the derivatives of $\tilde{x}$, $\tilde{\eta}$ and e are bounded, by invoking Barbalat's lemma, we have $\lim_{t\to\infty}\tilde{x} = 0$, $\lim_{t\to\infty}\tilde{\eta} = 0$ and $\lim_{t\to\infty} e = 0$. The result of this section is summarized in the following.

Theorem 2.1. Suppose that there exist positive definite matrices $P_{\hat{\eta}}$ and $Q_{\hat{\eta}}$, and a nonzero constant vector $K \in R^r$, such that $F_0 = F - KH$ is Hurwitz. For the uncertain nonlinear system (3) , satisfying Assumption $1-4$, then the control input (18), the adaptive law (23) and the nonlinear internal model (9) solve the global robust output regulation problem with nonlinear exosystem (4).

3. SOLVABILITY OF THE PROBLEM FOR MODIFIED FITZHUGH–NAGUMO NEURON MODEL

In order to formulate the control problem of the modified FitzHugh–Nagumo neuron model described in Section 1 into the robust output regulation problem described in Section 2. Let us first note that the system (1) with $I(u) = 0$ has been studied the bursting mechanism in exciting systems in [31]. Nevertheless, performing the following simple coordinate transformation

$$\begin{aligned} x_1 &= z_2 - \tfrac{a_1}{a_2} \\ x_2 &= z_3 - \tfrac{a_3}{a_4} \\ e &= y = z_1 \\ u &= I(u) - \tfrac{a_1}{a_2} + \tfrac{a_3}{a_4} \end{aligned} \tag{29}$$

gives

$$\begin{aligned} \dot{x}_1 &= -\epsilon_1 a_2 x_1 + \epsilon_1 y \\ \dot{x}_2 &= -\epsilon_2 a_4 x_2 - \epsilon_2 y \\ \dot{y} &= -x_1 + x_2 + (1 - \tfrac{1}{3}y^2)y + L(t) + u \\ e &= y. \end{aligned} \tag{30}$$

Next, to take into account the variation of the system parameters, let $\Omega = (a_2 > 0, a_4 > 0, \epsilon_1 > 0, \epsilon_2 > 0)$ and $\bar{\Omega} = (\bar{a}_2, \bar{a}_4, \bar{\epsilon}_1, \bar{\epsilon}_2)$ denote the actual and nominal value of the system parameters, respectively. Then $\Omega = \bar{\Omega} + a$ for some $a \in R^4$ where a represents the parameter variation from its nominal value. The external electrical stimulation $L(t) = w_1$ is generated by the following Van der Pol circuit

$$\begin{aligned} \dot{w}_1 &= w_2 - \varsigma\left(\frac{1}{3}w_1^3 - w_1\right) \\ \dot{w}_2 &= -w_1 \end{aligned}$$

where $1.6 \leq \varsigma \leq 2.4$.

Our objective is to design an output feedback controller such that, in the presence of large parameter variations and external stimulus $L(t) = w_1$, the trajectory of the closed-loop system starting from any initial state of the modified FitzHugh–Nagumo neuron model and the controller exists and is globally bounded for all $t \geq 0$, and all the plant states (y, x_1, x_2) converge to zero, that is, (z_1, z_2, z_3) converge to $(0, \frac{a_1}{a_2}, \frac{a_3}{a_4})$ as $t \to \infty$.

It is easy to see that $V(w) = \frac{1}{2}w_1^2 + \frac{1}{2}w_2^2$ satisfies

$$\frac{\mathrm{d}V}{\mathrm{d}w} = -\frac{1}{2}w_1^2 - \frac{1}{6}\varsigma w_1^4 \leq 0, \quad 1.6 \leq \varsigma \leq 2.4 \tag{31}$$

which satisfies Assumption 1.

Now, let $x = (x_1, x_2)$, and thus we have

$$\bar{F}(a) = \begin{bmatrix} -\varepsilon_1 a_2 & 0 \\ 0 & -\varepsilon_2 a_4 \end{bmatrix}, \bar{G}(y, w, a) = \begin{bmatrix} \varepsilon_1 & -\varepsilon_2 \end{bmatrix}^T, \bar{H}(a) = \begin{bmatrix} -1 & 1 \end{bmatrix},$$

$\bar{K}(y, w, a) = 1 - \frac{1}{3}y^2$, $h(a) = 1$, $\bar{D}_1(w, a) = \bar{D}_2(w, a) = 0$, and $q(w, a) = 0$.

It can be seen that (30) is in the form of (3) with $(x_1, x_2) = x$ viewing $y = z_1$ as the output, and the control problem described in Section 1 is the robust output regulation problem in Section 2. To apply the results of Section 2 to solve the control problem of the modified FitzHugh–Nagumo neuron model, we need to verify that the modified FitzHugh–Nagumo neuron model satisfies Assumption 2-4. It can seen that $\bar{F}(a)$ is a Hurwitz matrix and $h(a) > 0$, which satisfies Assumption 2. The solution of the regulator equations can be easily calculated as $\varpi_1(w, a) = 0$, $\varpi_2(w, a) = 0$, $y(w, a) = 0$ and $\iota(w, a) = -w_1$. Thus Assumption 3 is also satisfied. For the Van der Pol circuit, let $\eta = w$ and choose the matrix parameters as in Remark 2, then Assumption 4 is satisfied. We have verified that the modified FitzHugh–Nagumo neuron model satisfies all the assumptions needed to solve the global robust output regulation problem in Section 2, so we can apply the approach in Section 2 to design a dynamic output feedback controller for modified FitzHugh–Nagumo neuron model.

Let $K = [4, 1]^T$, then we have

$$F_0 = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}, Q_{\hat{\eta}} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, P_{\hat{\eta}} = \mathrm{diag}(1/2, 1/4).$$

Based on the control algorithm proposed in Section 2, we can obtain the internal model, the control input, and the adaptive law are designed as follows

$$\begin{aligned} \dot{\hat{\eta}}_1 &= -2(\hat{\eta}_1 - 4e) + (\hat{\eta}_2 - e) - \frac{2}{3}(\hat{\eta}_1 - 4e)^3 + 4u, \\ \dot{\hat{\eta}}_2 &= -2(\hat{\eta}_1 - 4e) + u, \\ u &= -re[(1 + e^2)^2 + 1] + \hat{\eta}_1 - 4e, \\ \dot{r} &= e^2[(1 + e^2)^2 + 1] \end{aligned} \tag{32}$$

In the simulation, assume the nominal value of the parameters in the modified FitzHugh–Nagumo neuron model are $\bar{a}_2 = 0.02, \bar{a}_4 = 0.03, \bar{\epsilon}_1 = 0.3, \bar{\epsilon}_2 = 0.1$.

Specifically, the parameter variation a is assumed to be $a = (0.08, 0.07, 1, 1)$. Let the initial condition be $x(0) = [0.5, 1]^T$, $y(0) = -0.5$, $\hat{\eta}(0) = [0, 0]^T$, $w(0) = [1, -1]^T$ and $r(0) = 0$. The response of the open-loop system with initial condition $x(0) = [0.5, 1]^T$, $y(0) = -0.5$ and $w(0) = [1, -1]^T$ is shown in Figure 1. Figure 2 shows the phase portrait of the Van der Pol circuit. Figure 3 shows the profile of the system states of the open-loop system. Figure 4 shows the profile of the system states of the closed-loop system. It can be observed that they converge to zero. As shown in the Figures 5 and 6, the disturbances are successfully reproduced by the designed internal model. Figures 7 and 8 show the profiles of the adaptive control gain $r(t)$ and controller u, respectively.

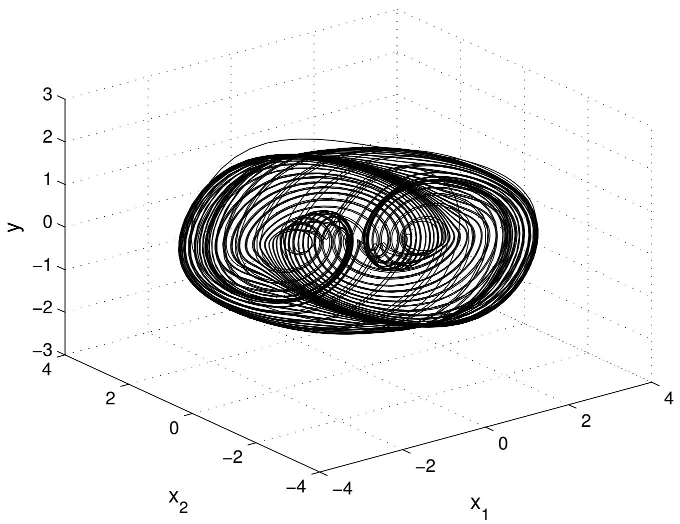

Fig. 1. Phase portrait of modified FitzHugh–Nagumo neuron model under external electrical stimulation $L(t) = w_1$.

To make the problem more interesting, we could let the parameter ς to be uncertain. To characterize the uncertainty, let $1.6 \leq \varsigma \leq 2.4$. Due to the space limit, we only show the simulation results under $\varsigma = 2$.

4. CONCLUSION

In this paper, an algorithm is proposed to design a dynamic output feedback controller for the modified FitzHugh–Nagumo neuron model in the presence of external electrical stimulation. We first present the solution of the global robust output regulation problem for an output feedback system with nonlinear exosystem. Then we apply the obtained output regulation results to a control problem for the modified FitzHugh–Nagumo neuron model. Moreover, a dynamic output feedback control law is designed such that the external electrical stimulation can be asymptotically

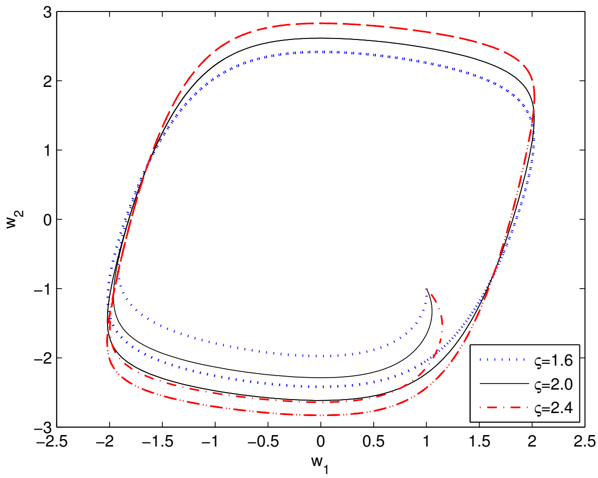

Fig. 2. Phase portrait of the Van der Pol circuit.

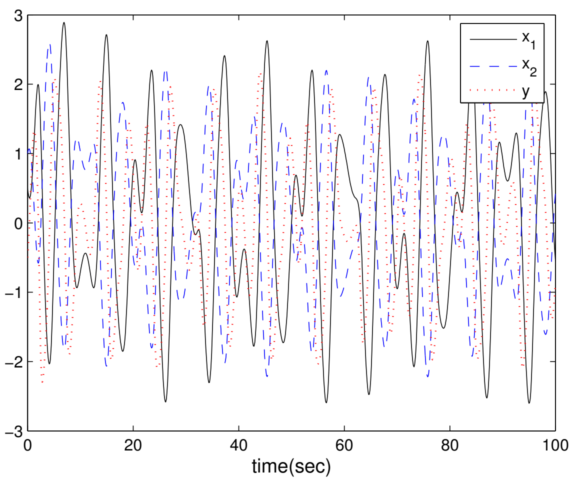

Fig. 3. Time response of the system states of the open-loop system.

rejected while maintaining the global stability of the closed-loop system. Simulation results illustrate the effectiveness of our algorithm. The proposed algorithm can be used in many applications, e. g. active vibration control, and the avoidance of nonharmonic distortion in nonlinear circuits. In the future research, we will consider to design an output feedback controller for the modified FitzHugh–

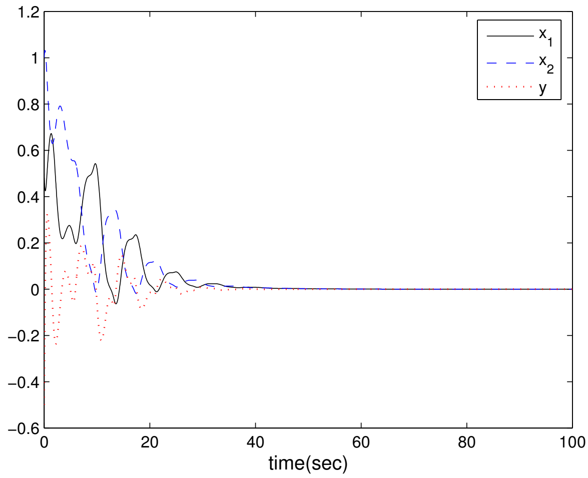

Fig. 4. Time response of the system states of the closed-loop system.

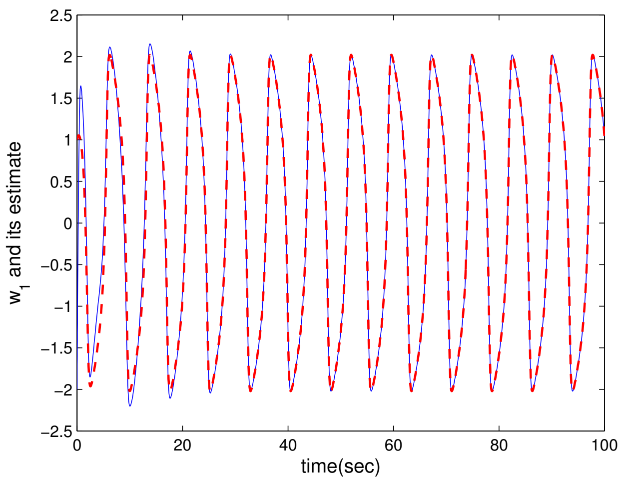

Fig. 5. The external electrical stimulation $F(t) = w_1$ and its estimate (dash-dotted line: w_1; solid line: $\hat{\eta}_1 - 4e$).

Nagumo neuron model by subjecting it under the following electrical stimulation $L(t) = (A/W)\cos(Wt)$. Here, A represents the magnitude of the stimulus and W refers to the frequency of given stimulus.

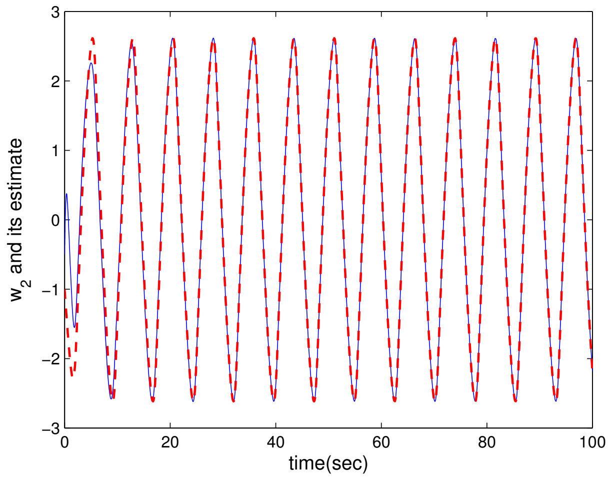

Fig. 6. w_2 and its estimate (dash-dotted line: w_2; solid line: $\hat{\eta}_2 - e$)

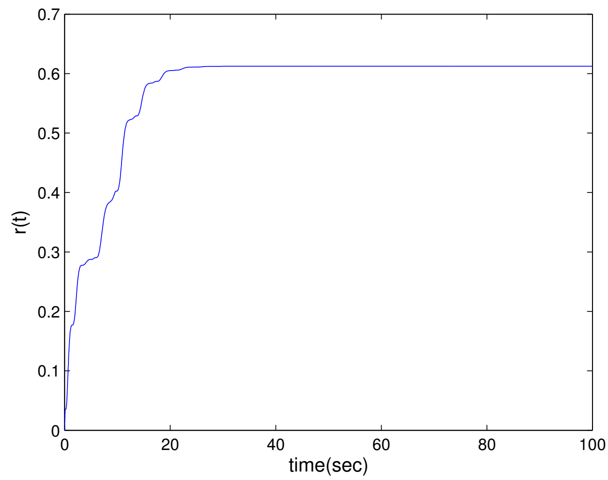

Fig. 7. Time response of the state $r(t)$.

ACKNOWLEDGEMENTS

This work was partially supported by the National Natural Science Foundation of China (Nos.60864001, 60874009), the Natural Science Foundation of Jiangxi Province (No.2009GZS0085) and the Project Sponsored by the Scientific Research Foundation for the Doctors of Nanchang Hangkong University (No.EA201104184). The authors would like to thank all the

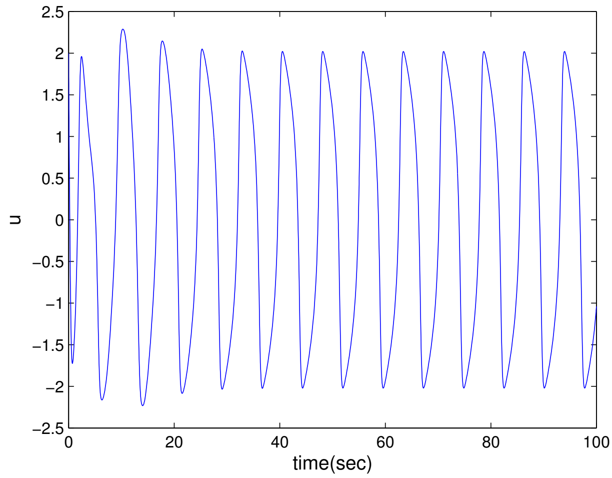

Fig. 8. Input u.

constructive comments from the Editors and anonymous reviewers.

(Received June 25, 2010)

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