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CONTRA G_δ-CONTINUITY IN SMOOTH FUZZY TOPOLOGICAL SPACES

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Abstract. In this paper the concept of fuzzy contra G_δ-continuity in the sense of A. P. Sostak (1985) is introduced. Some interesting properties and characterizations are investigated. Also, some applications to fuzzy compact spaces are established.

Keywords: fuzzy contra G_δ-continuity, fuzzy strong G_δ-continuity, fuzzy perfect G_δ-continuity, fuzzy G_δ-compact space, fuzzy S-closed space

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1. Introduction and preliminaries

The concept of the fuzzy set was introduced by Zadeh [14] in his classical paper. Fuzzy sets have applications in many fields such as information [10] and control [12]. G. Balasubramanian [1] introduced the concept of the fuzzy G_δ-set. The concept of fuzzy G_δ-continuity was introduced and studied by E. Roja, M. K. Uma and G. Balasubramanian [7]. Dontchev [2] introduced the notion of the contra continuous mapping. Ekici and Kerre [3], Thangaraj [13] introduced the concept of fuzzy contra continuous mappings. The concept of fuzzy contra strongly precontinuity was established by Biljana Krsteska and Erdal Ekici [4]. The purpose of this paper is to introduce the concept of fuzzy contra G_δ-continuity in the sense of A. P. Sostak [11]. Some interesting properties and interrelations between the concepts introduced are established. Also, some properties concerning fuzzy G_δ-compactness, almost fuzzy G_δ-compactness and fuzzy S-closed spaces are studied.

Definition 1.1 [1]. Let (X,T) be a fuzzy topological space and λ a fuzzy set in X. Then λ is called a fuzzy G_δ-set if $\lambda = \bigwedge\limits_{i=1}^{\infty} \lambda_i$ where $\lambda_i \in T$ for $i \in I$.

Definition 1.2 [1]. Let (X,T) be a fuzzy topological space and λ a fuzzy set in X. Then λ is called a fuzzy F_σ-set if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$ where $1-\lambda_i \in T$ for $i \in I$.

Definition 1.3 [7]. Let (X,T) be a fuzzy topological space and λ a fuzzy set in X. Then $\mathrm{int}_\sigma(\lambda) = \bigvee\{\mu\colon \mu \leqslant \lambda, \mu$ is a fuzzy G_δ-set$\}$ is called the fuzzy G_δ-interior of λ and $\mathrm{cl}_\sigma(\lambda) = \bigwedge\{\mu\colon \mu \geqslant \lambda, \mu$ is a fuzzy F_σ-set$\}$ is called the fuzzy G_δ-closure of λ.

Definition 1.4 [8]. Let (X,T) be a fuzzy topological space and λ a fuzzy set in X. Then λ is said to be a fuzzy regular G_δ-set if $\lambda = \mathrm{int}_\sigma(\mathrm{cl}_\sigma(\lambda))$.

Definition 1.5 [8]. Let (X,T) be a fuzzy topological space and λ a fuzzy set in X; λ is said to be a fuzzy regular F_σ-set if $\lambda = \mathrm{cl}_\sigma(\mathrm{int}_\sigma(\lambda))$.

Definition 1.6. [5]. A fuzzy point x_t in X is a fuzzy set taking value $t \in I_0$ at x and zero elsewhere; $x_t \in \lambda$ if and only if $t \leqslant \lambda(x)$. A fuzzy set λ is quasi-coincident with a fuzzy set μ, denoted by $\lambda\ q\ \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise λ is not quasi-coincident with a fuzzy set μ, denoted by $\lambda\ \not{q}\ \mu$ if $\lambda(x) + \mu(x) \leqslant 1$.

Throughout this paper, let X be a non-empty set, $I = [0,1]$ and $I_0 = (0,1]$. For $\langle \in I$, $T(x) = \langle$ for all $x \in X$.

Definition 1.7 [11]. A function $T\colon I^X \to I$ is called a smooth fuzzy topology on X if it satisfies the following conditions:

a) $T\left(\overline{0}\right) = T\left(\overline{1}\right) = 1$,
b) $T(\mu_1 \wedge \mu_2) \geqslant T(\mu_1) \wedge T(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$,
c) $T\Big(\bigvee_{i\in\Gamma} \mu_i\Big) \geqslant \bigwedge_{i\in\Gamma} T(\mu_i)$ for any $\{\mu_i\}_{i\in\Gamma} \in I^X$.

The pair (X,T) is called a smooth fuzzy topological space.

Remark 1.1. Let (X,T) be a smooth fuzzy topological space. Then, for each $r \in I_0$, $T_r = \{\mu \in I^X\colon T(\mu) \geqslant r\}$ is Chang's fuzzy topology on X.

Proposition 1.1 [9]. *Let (X,T) be a smooth fuzzy topological space. For each $r \in I_0$, $\lambda \in I^X$, an operator $C_T\colon I^X \times I_0 \to I^X$ is defined as follows:*

$$C_T(\lambda, r) = \bigwedge\{\mu\colon \mu \geqslant \lambda, T(\overline{1} - \mu) \geqslant r\}.$$

For λ, $\mu \in I^X$ and $r, s \in I_0$ it satisfies the following conditions:
(1) $C_T(\overline{0}, r) = \overline{0}$,
(2) $\lambda \leqslant C_T(\lambda, r)$,
(3) $C_T(\lambda, r) \vee C_T(\mu, r) = C_T(\lambda \vee \mu, r)$,

(4) $C_T(\lambda, r) \leqslant C_T(\lambda, s)$, *if* $r \leqslant s$,
(5) $C_T(C_T(\lambda, r), r) = C_T(\lambda, r)$.

Proposition 1.2 [9]. *Let* (X, T) *be a smooth fuzzy topological space. For each* $r \in I_0$, $\lambda \in I^X$, *an operator* $I_T\colon I^X \times I_0 \to I^X$ *is defined as follows:*

$$I_T(\lambda, r) = \bigvee\{\mu\colon \mu \leqslant \lambda, T(\mu) \geqslant r\}.$$

For λ, $\mu \in I^X$ *and* $r, s \in I_0$ *it satisfies the following conditions:*
(1) $I_T(\overline{1} - \lambda, r) = \overline{1} - C_T(\lambda, r)$,
(2) $I_T(\overline{1}, r) = \overline{1}$,
(3) $\lambda \geqslant I_T(\lambda, r)$,
(4) $I_T(\lambda, r) \wedge I_T(\mu, r) = I_T(\lambda \wedge \mu, r)$,
(5) $I_T(\lambda, r) \geqslant I_T(\lambda, s)$, *if* $r \leqslant s$,
(6) $I_T(I_T(\lambda, r), r) = I_T(\lambda, r)$.

Definition 1.8 [6]. Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let $f\colon (X, T) \to (Y, S)$ be a mapping. Then

(1) f is called fuzzy continuous iff $S(\mu) \leqslant T(f^{-1}(\mu))$ for each $\mu \in I^Y$;
(2) f is called fuzzy open iff $T(\lambda) \leqslant S(f(\lambda))$ for each $\lambda \in I^X$;
(3) f is called fuzzy closed iff $T(\overline{1} - \lambda) \leqslant S(\overline{1} - f(\lambda))$ for each $\lambda \in I^X$.

2. Fuzzy contra G_δ-continuity

Definition 2.1. Let (X, T) be a smooth fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$, λ is called an r-fuzzy G_δ-set iff $\lambda = \bigwedge\limits_{i \in \Gamma} \lambda_i$ where $\{\lambda_i\}_{i \in \Gamma} \in I^X$ is such that $T(\lambda_i) \geqslant r$.

Definition 2.2. Let (X, T) be a smooth fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$, λ is called an r-fuzzy F_σ-set iff $\lambda = \bigvee\limits_{i \in \Gamma} \lambda_i$ where $\{\lambda_i\}_{i \in \Gamma} \in I^X$ is such that $T(\overline{1} - \lambda_i) \geqslant r$.

Definition 2.3. Let (X, T) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, the r-fuzzy σ closure of λ, denoted by $C_{T(\sigma)}(\lambda, r)$, is defined by

$$C_{T(\sigma)}(\lambda, r) = \bigwedge\{\mu\colon \mu \geqslant \lambda, \mu \text{ is an } r\text{-fuzzy } F_\sigma\text{-set}\}.$$

Definition 2.4. Let (X, T) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, the r-fuzzy σ interior of λ, denoted by $I_{T(\sigma)}(\lambda, r)$, is defined by

$$I_{T(\sigma)}(\lambda, r) = \bigvee\{\mu\colon \mu \leqslant \lambda, \mu \text{ is an } r\text{-fuzzy } G_\delta\text{-set}\}.$$

R e m a r k 2.1. Let (X, T) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$,

(1) λ is an r-fuzzy F_σ-set iff $\lambda = C_{T(\sigma)}(\lambda, r)$,
(2) λ is an r-fuzzy G_δ-set iff $\lambda = I_{T(\sigma)}(\lambda, r)$.

Definition 2.5. Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let $f\colon (X, T) \to (Y, S)$ be a mapping. Then

(1) f is called fuzzy G_δ-continuous if $f^{-1}(\mu)$ is an r-fuzzy G_δ-set for each $S(\mu) \geqslant r$, $\mu \in I^Y$ and $r \in I_0$;
(2) f is called fuzzy irresolute G_δ-continuous if $f^{-1}(\mu)$ is an r-fuzzy G_δ-set for each r-fuzzy G_δ-set $\mu \in I^Y$ and $r \in I_0$;
(3) f is called fuzzy irresolute G_δ if $f(\lambda)$ is an r-fuzzy G_δ-set for each r-fuzzy G_δ-set $\lambda \in I^X$ and $r \in I_0$;
(4) f is called fuzzy contra irresolute G_δ-continuous if $^{f^{-1}}(\mu)$ is an r-fuzzy G_δ-set for each r-fuzzy F_σ-set $\mu \in I^Y$ and $r \in I_0$.

Definition 2.6. Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let $f\colon (X, T) \to (Y, S)$ be a mapping. Then f is called fuzzy contra continuous iff $T(f^{-1}(\mu)) \geqslant S(\overline{1} - \mu)$, $\mu \in I^Y$.

Definition 2.7. Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. $f\colon (X, T) \to (Y, S)$ is called fuzzy contra G_δ-continuous iff $f^{-1}(\mu)$ is an r-fuzzy F_σ-set for each $S(\mu) \geqslant r$, $\mu \in I^Y$ and $r \in I_0$.

By using the concept of the neighbourhood and Q-neighbourhood structures [11], the Q^* neighbourhood structure is defined as follows:

Definition 2.8. Let (X, T) be a smooth fuzzy topological space. Its Q^* neighbourhood structure is a mapping $Q^*\colon X \times I^X \to I$ (X denotes the totality of all fuzzy points in X), defined by

$$Q^*(x_0^t, \lambda) = \sup\{\mu\colon\ \mu \text{ is an } r\text{-fuzzy } G_\delta\text{-set},\ \mu \leqslant \lambda,\ x_0^t \in \mu\} \text{ and}$$
$$\lambda = \inf_{x_0^t q \lambda} Q^*(x_0^t, \lambda) \text{ is } r\text{-fuzzy } G_\delta.$$

Proposition 2.1. *Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let $f\colon (X, T) \to (Y, S)$ be a mapping. Then the following statements are equivalent:*
(1) *f is fuzzy contra G_δ-continuous.*
(2) *For each fuzzy point x_0^t in X, $\mu \in I^Y$, $S(\overline{1} - \mu) \geqslant r$ and $r \in I_0$ with $f(x_0^t) \in \mu$, there exists an r-fuzzy G_δ-set $\lambda \in I^X$ with $x_0^t \in \lambda$ such that $\lambda \leqslant f^{-1}(\mu)$.*

(3) *For each fuzzy point* x_0^t *in* X, $\mu \in I^Y$, $S(\overline{1}-\mu) \geqslant r$ *and* $r \in I_0$ *with* $f(x_0^t) \in \mu$, *there exists an* r*-fuzzy* G_δ*-set* $\lambda \in I^X$ *with* $x_0^t \in \lambda$ *such that* $f(\lambda) \leqslant \mu$.

Proof. (1) $\Rightarrow$ (2) Let f be a fuzzy contra G_δ-continuous function. Let x_0^t be a fuzzy point in X, $\mu \in I^Y$ and $S(\overline{1}-\mu) \geqslant r$ with $f(x_0^t) \in \mu$. Then $x_0^t \in f^{-1}(\mu) = I_{T(\sigma)}(f^{-1}(\mu), r)$. Let $\lambda = I_{T(\sigma)}(f^{-1}(\mu), r)$. Then λ is an r-fuzzy G_δ-set and $\lambda = I_{T(\sigma)}(f^{-1}(\mu), r) \leqslant f^{-1}(\mu)$. Then

$$\lambda \leqslant f^{-1}(\mu). \tag{2.1}$$

(2) $\Rightarrow$ (3) By (2.1), $\lambda \leqslant f^{-1}(\mu)$. That is, $f(\lambda) \leqslant f(f^{-1}(\mu)) \leqslant \mu$. Hence the result.

(3) $\Rightarrow$ (1) Let $\lambda \in I^Y$ and $S(\lambda) \geqslant r$. Suppose that $f(x_0^t) \leqslant \overline{1}-\lambda$ for each fuzzy point x_0^t in X. By (3), there exists an r-fuzzy G_δ-set $\mu \in I^X$ with $x_0^t \in \mu$ and $f(\mu) \leqslant \overline{1}-\lambda$. Hence $x_0^t \in \mu \leqslant f^{-1}(f(\mu)) \leqslant f^{-1}(\overline{1}-\lambda)$. By definition 2.8, $f^{-1}(\overline{1}-\lambda)$ is an r-fuzzy G_δ-set. But $f^{-1}(\overline{1}-\lambda) = \overline{1} - f^{-1}(\lambda)$. Hence $f^{-1}(\lambda)$ is an r-fuzzy F_σ-set. Therefore f is fuzzy contra G_δ-continuous.

Remark 2.2. A fuzzy contra G_δ-continuous function need not be a fuzzy G_δ-continuous function. This is illustrated in the following example.

Example 2.1. Define smooth fuzzy topologies $T, S\colon I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \text{ or } \overline{1}, \\ 0.5, & \lambda = \overline{0.4}, \\ 0, & \text{otherwise}, \end{cases}$$

$$S(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \text{ or } \overline{1}, \\ 0.5, & \lambda = \overline{0.3}, \\ 0.5, & \lambda = \overline{0.6}, \\ 0 & \text{otherwise}. \end{cases}$$

We can obtain the following:

$$C_{T(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.6}, & \overline{0} < \lambda \leqslant \overline{0.6},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}, \end{cases}$$

$$C_{S(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.4}, & \overline{0} < \lambda \leqslant \overline{0.4},\ 0 < r \leqslant 0.5, \\ \overline{0.7}, & \overline{0.4} < \lambda \leqslant \overline{0.7},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}. \end{cases}$$

The mapping $f = \mathrm{id}_x\colon (X,T) \to (X,S)$ is fuzzy contra G_δ-continuous but not fuzzy G_δ-continuous because for $S(\lambda = \overline{0.6}) = 0.5$, $f^{-1}(\lambda = \overline{0.6})$ is 0.5-fuzzy F_σ but not 0.5-fuzzy G_δ.

Definition 2.9. Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ be a mapping. Then f is said to be fuzzy strongly G_δ-continuous iff $f^{-1}(\mu)$ is both r-fuzzy G_δ and r-fuzzy F_σ for every $\mu \in I^Y$ and $r \in I_0$.

Definition 2.10. Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ be a mapping. Then f is said to be fuzzy perfectly G_δ-continuous iff $f^{-1}(\mu)$ is both r-fuzzy G_δ and r-fuzzy F_σ for each $S(\mu) \geqslant r$, $\mu \in I^Y$ and $r \in I_0$.

Proposition 2.2. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ be a mapping. Then the following statements are equivalent:*

(1) *f is fuzzy perfectly G_δ-continuous.*

(2) *f is fuzzy G_δ-continuous and fuzzy contra G_δ-continuous.*

Proof. (1) $\Rightarrow$ (2) Let $S(\mu) \geqslant r$ for all $\mu \in I^Y$ and $r \in I_0$. Since f is fuzzy perfectly G_δ-continuous, $f^{-1}(\mu)$ is both r-fuzzy G_δ and r-fuzzy F_σ. Hence f is both fuzzy G_δ-continuous and fuzzy contra G_δ-continuous.

(2) $\Rightarrow$ (1) Let $S(\mu) \geqslant r$ for all $\mu \in I^Y$ and $r \in I_0$. Since f is fuzzy G_δ-continuous and fuzzy contra G_δ-continuous, $f^{-1}(\mu)$ is r-fuzzy G_δ and r-fuzzy F_σ. Since $f^{-1}(\mu)$ is both r-fuzzy G_δ and r-fuzzy F_σ, f is fuzzy perfectly G_δ-continuous. $\square$

Remark 2.3. From the above definitions, it can be concluded that the following diagram of implications is true.

$$\begin{array}{ccc} & & \text{fuzzy contra } G_\delta\text{-continuous} \\ & & \Uparrow \\ \text{fuzzy strongly } G_\delta\text{-continuous} & \Longrightarrow & \text{fuzzy perfectly } G_\delta\text{-continuous} \\ & & \Downarrow \\ & & \text{fuzzy } G_\delta\text{-continuous} \end{array}$$

The following examples show that the converse statements need not be true.

E x a m p l e 2.2. Define smooth fuzzy topologies $T, S\colon I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \ \text{ or } \ \overline{1}, \\ 0.5, & \lambda = \overline{0.4}, \\ 0, & \text{otherwise}, \end{cases}$$

$$S(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \text{ or } \overline{1}, \\ 0.5, & \lambda = \overline{0.3}, \\ 0.5, & \lambda = \overline{0.6}, \\ 0, & \text{otherwise}. \end{cases}$$

We can obtain

$$C_{T(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.6}, & \overline{0} < \lambda \leqslant \overline{0.6},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}, \end{cases}$$

$$C_{S(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.4}, & \overline{0} < \lambda \leqslant \overline{0.4},\ 0 < r \leqslant 0.5, \\ \overline{0.7}, & \overline{0.4} < \lambda \leqslant \overline{0.7},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}. \end{cases}$$

The mapping $f = \mathrm{id}_x\colon (X, T) \to (X, S)$ is fuzzy contra G_δ-continuous but not fuzzy perfectly G_δ-continuous because for $S(\lambda = \overline{0.6}) = 0.5$, $f^{-1}(\lambda = \overline{0.6})$ is 0.5-fuzzy F_σ but not 0.5-fuzzy G_δ.

E x a m p l e 2.3. Define smooth fuzzy topologies $T, S\colon I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1, & \lambda = \overline{0}, \overline{1}, \\ 0.5, & \lambda = \overline{0.3}, \\ 0.5, & \lambda = \overline{0.4}, \\ 0, & \text{otherwise}, \end{cases}$$

$$S(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \text{ or } \overline{1}, \\ 0.5 & \lambda = \overline{0.3}, \\ 0, & \text{otherwise}. \end{cases}$$

We can obtain

$$C_{T(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.6}, & \overline{0} < \lambda \leqslant \overline{0.6},\ 0 < r \leqslant 0.5, \\ \overline{0.7}, & \overline{0.6} < \lambda \leqslant \overline{0.7},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}, \end{cases}$$

$$C_{S(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.7}, & \overline{0} < \lambda \leqslant \overline{0.7},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}. \end{cases}$$

The mapping $f = \mathrm{id}_x\colon (X, T) \to (X, S)$ is fuzzy G_δ-continuous but not fuzzy perfectly G_δ-continuous, because for $S(\lambda = \overline{0.3}) = 0.5$, $f^{-1}(\lambda = \overline{0.3})$ is 0.5-fuzzy G_δ but not 0.5-fuzzy F_σ.

Example 2.4. Define smooth fuzzy topologies $T, S\colon I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \text{ or } \overline{1}, \\ 0.5, & \lambda = \overline{0.4}, \\ 0.6, & \lambda = \overline{0.6}, \\ 0, & \text{otherwise}, \end{cases}$$

$$S(\lambda) = \begin{cases} 1, & \lambda = \overline{0} \text{ or } \overline{1}, \\ 0.5, & \lambda = \overline{0.3}, \\ 0.5, & \lambda = \overline{0.4}, \\ 0.6, & \lambda = \overline{0.5}, \\ 0, & \text{otherwise}. \end{cases}$$

We can obtain

$$C_{T(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.4}, & \overline{0} < \lambda \leqslant \overline{0.4},\ 0 < r \leqslant 0.5, \\ \overline{0.6}, & \overline{0.4} < \lambda \leqslant \overline{0.6},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}, \end{cases}$$

$$C_{S(\sigma)}(\lambda, r) = \begin{cases} \overline{0}, & \lambda = \overline{0},\ r \in I_0, \\ \overline{0.5}, & \overline{0} < \lambda \leqslant \overline{0.5},\ 0 < r \leqslant 0.5, \\ \overline{0.6}, & \overline{0.5} < \lambda \leqslant \overline{0.6},\ 0 < r \leqslant 0.5, \\ \overline{0.7}, & \overline{0.6} < \lambda \leqslant \overline{0.7},\ 0 < r \leqslant 0.5, \\ \overline{1}, & \text{otherwise}. \end{cases}$$

The mapping $f = \mathrm{id}_x\colon (X,T) \to (X,S)$ is fuzzy perfectly G_δ-continuous but not fuzzy strongly G_δ-continuous since for $S(\lambda = \overline{0.4}) = 0.5$, $f^{-1}(\lambda = \overline{0.4})$ is both 0.5-fuzzy G_δ and 0.5-fuzzy F_σ. Therefore the mapping is fuzzy perfectly G_δ-continuous. Now, for $S(\lambda = \overline{0.3}) = 0.5$, $f^{-1}(\lambda = \overline{0.3})$ is neither 0.5-fuzzy G_δ nor 0.5-fuzzy F_σ. Therefore the mapping is not fuzzy strongly G_δ-continuous.

Proposition 2.3. *Let (X,T), (Y,S) and (Z,R) be smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ and $g\colon (Y,S) \to (Z,R)$ be two mappings. Then the following statements hold.*

(1) *$g \circ f$ is fuzzy contra G_δ-continuous if g is fuzzy continuous and f is fuzzy contra G_δ-continuous.*

(2) *$g \circ f$ is fuzzy contra G_δ-continuous if g is fuzzy contra G_δ-continuous and f is fuzzy irresolute G_δ-continuous.*

(3) *If f is fuzzy contra G_δ-continuous and g is fuzzy contra continuous then $g \circ f$ is fuzzy G_δ-continuous.*

(4) *If f is a fuzzy irresolute G_δ-surjective mapping and $g \circ f$ is a fuzzy contra G_δ-continuous mapping then g is a fuzzy contra G_δ-continuous mapping.*

(5) *If $g \circ f$ is a fuzzy contra G_δ-continuous mapping and g is a fuzzy open injective mapping then f is a fuzzy contra G_δ-continuous mapping.*

Proof. (1) Let $R(\lambda) \geqslant r$ for all $\lambda \in I^Z$ and $r \in I_0$. Since g is fuzzy continuous, $S(g^{-1}(\lambda)) \geqslant R(\lambda) \geqslant r$. Since f is fuzzy contra G_δ-continuous, $f^{-1}(g^{-1}(\lambda))$ is an r-fuzzy F_σ-set. The relation $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$, yields that $g \circ f$ is fuzzy contra G_δ-continuous.

(2) Let $R(\lambda) \geqslant r$ for all $\lambda \in I^Z$ and $r \in I_0$. Since g is fuzzy contra G_δ-continuous, $g^{-1}(\lambda)$ is an r-fuzzy F_σ-set. Since f is fuzzy irresolute G_δ-continuous, $f^{-1}(g^{-1}(\lambda))$ is an r-fuzzy F_σ-set. The relation $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$, yields that $g \circ f$ is fuzzy contra G_δ-continuous.

(3) Let $R(\overline{1} - \lambda) \geqslant r$ for all $\lambda \in I^Z$ and $r \in I_0$. Since g is fuzzy contra continuous, $S(g^{-1}(\lambda)) \geqslant r$. Since f is fuzzy contra G_δ-continuous, $f^{-1}(g^{-1}(\lambda))$ is an r-fuzzy F_σ-set. But $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$, hence it follows that $g \circ f$ is fuzzy G_δ-continuous.

(4) Let $R(\overline{1} - \lambda) \geqslant r$ for all $\lambda \in I^Z$ and $r \in I_0$. Since $g \circ f$ is fuzzy contra G_δ-continuous, $(g \circ f)^{-1}(\lambda)$ is an r-fuzzy G_δ-set. Since f is a fuzzy irresolute G_δ-surjective mapping, $f(g \circ f)^{-1}(\lambda)$ is an r-fuzzy G_δ-set. But $g^{-1}(\lambda) = f(g \circ f)^{-1}(\lambda)$, hence it follows that g is fuzzy contra G_δ-continuous.

(5) Let $S(\lambda) \geqslant r$ for all $\lambda \in I^Y$ and $r \in I_0$. Since g is fuzzy open, $R(g(\lambda)) \geqslant r$. Since $g \circ f$ is fuzzy contra G_δ-continuous, $(g \circ f)^{-1}(g(\lambda))$ is an r-fuzzy F_σ-set. But $f^{-1}(\lambda) = (g \circ f)^{-1}(g(\lambda))$, hence it follows that f is fuzzy contra G_δ-continuous. □

R e m a r k 2.4. Composition of two fuzzy contra G_δ-continuous functions need not be fuzzy G_δ-continuous. This is illustrated in the following example:

E x a m p l e 2.5. Define smooth fuzzy topologies $T, S, R\colon I^X \to I$ as follows:

$$T(\lambda)=\begin{cases}1, & \lambda=\overline{0} \text{ or } \overline{1},\\ 0.5, & \lambda=\overline{0.4},\\ 0, & \text{otherwise},\end{cases}$$

$$S(\lambda)=\begin{cases}1, & \lambda=\overline{0} \text{ or } \overline{1},\\ 0.5, & \lambda=\overline{0.4},\\ 0.5, & \lambda=\overline{0.6},\\ 0 & \text{otherwise},\end{cases}$$

$$R(\lambda)=\begin{cases}1, & \lambda=\overline{0} \text{ or } \overline{1},\\ 0.5, & \lambda=\overline{0.6},\\ 0, & \text{otherwise}.\end{cases}$$

We can obtain

$$C_{T(\sigma)}(\lambda,r)=\begin{cases}\overline{0}, & \lambda=\overline{0},\ r\in I_0,\\ \overline{0.6}, & \overline{0}<\lambda\leqslant\overline{0.6},\ 0<r\leqslant 0.5,\\ \overline{1}, & \text{otherwise},\end{cases}$$

$$C_{S(\sigma)}(\lambda,r)=\begin{cases}\overline{0}, & \lambda=\overline{0},\ r\in I_0,\\ \overline{0.4}, & \overline{0}<\lambda\leqslant\overline{0.4},\ 0<r\leqslant 0.5,\\ \overline{0.6}, & \overline{0.4}<\lambda\leqslant\overline{0.6},\ 0<r\leqslant 0.5,\\ \overline{1}, & \text{otherwise},\end{cases}$$

$$C_{R(\sigma)}(\lambda,r)=\begin{cases}\overline{0}, & \lambda=\overline{0},\ r\in I_0,\\ \overline{0.4}, & \overline{0}<\lambda\leqslant\overline{0.4},\ 0<r\leqslant 0.5,\\ \overline{1}, & \text{otherwise}.\end{cases}$$

The mapping $f=\mathrm{id}_x\colon (X,T)\to(X,S)$ is fuzzy contra G_δ-continuous because for $S(\lambda=\overline{0.6})=0.5$, $f^{-1}(\lambda=\overline{0.6})$ is 0.5 fuzzy F_σ. The mapping $g=\mathrm{id}_x\colon (X,S)\to(X,R)$ is fuzzy contra G_δ-continuous because for $R(\lambda=\overline{0.6})=0.5$, $g^{-1}(\lambda=\overline{0.6})$ is 0.5 fuzzy F_σ. The mapping $g\circ f\colon (X,T)\to(X,R)$ is not fuzzy G_δ-continuous because for $R(\lambda=\overline{0.6})=0.5$, $(g\circ f)^{-1}(\lambda=\overline{0.6})$ is r-fuzzy F_σ.

Proposition 2.4. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T)\to(Y,S)$ be a mapping. Suppose that one of the following conditions holds:*

(1) $f^{-1}(C_{S(\sigma)}(\mu,r))\leqslant I_{T(\sigma)}(C_{T(\sigma)}(f^{-1}(\mu),r),r)$ *for each* $\mu\in I^Y$ *and* $r\in I_0$.
(2) $C_{T(\sigma)}(I_{T(\sigma)}(f^{-1}(\mu),r),r)\leqslant f^{-1}(I_{S(\sigma)}(\mu,r))$ *for each* $\mu\in I^Y$ *and* $r\in I_0$.
(3) $f(C_{T(\sigma)}(I_{T(\sigma)}(\lambda,r),r))\leqslant I_{S(\sigma)}(f(\lambda),r)$ *for each* $\lambda\in I^X$ *and* $r\in I_0$.
(4) $f(C_{T(\sigma)}(\lambda,r))\leqslant I_{S(\sigma)}(f(\lambda),r)$ *for each* $\lambda\in I^X$ *and* $r\in I_0$.

Then f is fuzzy contra G_δ-continuous.

Proof. (1) $\Rightarrow$ (2) This can be proved using the complement.

(2) $\Rightarrow$ (3) Let $\lambda\in I^X$. Suppose that $f(\lambda)=\mu$, $\mu\in I^Y$, then $\lambda\leqslant f^{-1}(\mu)$. By (2), $C_{T(\sigma)}(I_{T(\sigma)}(\lambda,r),r)\leqslant C_{T(\sigma)}(I_{T(\sigma)}(f^{-1}(\mu),r),r)\leqslant f^{-1}(I_{S(\sigma)}(\mu,r))$. Therefore $f(C_{T(\sigma)}(I_{T(\sigma)}(\lambda,r),r))\leqslant I_{S(\sigma)}(\mu,r)=I_{S(\sigma)}(f(\lambda),r)$. Hence

$$f(C_{T(\sigma)}(I_{T(\sigma)}(\lambda,r),r))\leqslant I_{S(\sigma)}(f(\lambda),r).$$

(3) $\Rightarrow$ (4) Let $\lambda\in I^X$ be any r-fuzzy G_δ-set. Then $\lambda=I_{T(\sigma)}(\lambda,r)$. Now $f(C_{T(\sigma)}(\lambda,r))=f(C_{T(\sigma)}(I_{T(\sigma)}(\lambda,r),r))$. Further, by (3), $f(C_{T(\sigma)}(I_{T(\sigma)}(\lambda,r),r))\leqslant I_{S(\sigma)}(f(\lambda),r)$. Hence $f(C_{T(\sigma)}(\lambda,r))\leqslant I_{S(\sigma)}(f(\lambda),r)$.

Suppose that (4) holds. Let $S(\mu)\geqslant r$ for $\mu\in I^Y$ and $r\in I_0$. According to the assumption, $f(C_{T(\sigma)}(f^{-1}(\mu),r))\leqslant I_{S(\sigma)}(f(f^{-1}(\mu)),r)$ for $f^{-1}(\mu)\in I^X$. That is, $f(C_{T(\sigma)}(f^{-1}(\mu),r))\leqslant I_{S(\sigma)}(\mu,r)\leqslant\mu$. So $C_{T(\sigma)}(f^{-1}(\mu),r)\leqslant f^{-1}(\mu)$. But $f^{-1}(\mu)\leqslant C_{T(\sigma)}(f^{-1}(\mu),r)$. Therefore $f^{-1}(\mu)=C_{T(\sigma)}(f^{-1}(\mu),r)$. Thus $f^{-1}(\mu)$ is an r-fuzzy F_σ-set. Hence f is fuzzy contra G_δ-continuous.

Proposition 2.5. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T)\to(Y,S)$ be a mapping. Suppose that one of the following conditions holds:*

(1) $f(C_{T(\sigma)}(\lambda,r))\leqslant I_{S(\sigma)}(f(\lambda),r)$ *for each* $\lambda\in I^X$ *and* $r\in I_0$.
(2) $C_{T(\sigma)}(f^{-1}(\mu),r)\leqslant f^{-1}(I_{S(\sigma)}(\mu,r))$ *for each* $\mu\in I^Y$ *and* $r\in I_0$.
(3) $f^{-1}(C_{S(\sigma)}(\mu,r))\leqslant I_{T(\sigma)}(f^{-1}(\mu),r)$ *for each* $\mu\in I^Y$ *and* $r\in I_0$.

Then f is fuzzy contra G_δ-continuous.

Proof. (1) $\Rightarrow$ (2) Let $f(\lambda)=\mu$ for $\mu\in I^Y$. Then $\lambda\leqslant f^{-1}(\mu)$. By (1), $f(C_{T(\sigma)}(\lambda,r))\leqslant f(C_{T(\sigma)}(f^{-1}(\mu),r))\leqslant I_{S(\sigma)}(f(f^{-1}(\mu)),r)\leqslant I_{S(\sigma)}(\mu,r)$. Therefore $C_{T(\sigma)}(f^{-1}(\mu),r)\leqslant f^{-1}(I_{S(\sigma)}(\mu,r))$.

(2) $\Rightarrow$ (3) This can be proved using the complement.

Suppose that (3) holds. Let $S(\overline{1}-\mu)\geqslant r$ for $\mu\in I^Y$ and $r\in I_0$. Then $C_{S(\sigma)}(\mu,r)=\mu$. By (3), $f^{-1}(\mu)=f^{-1}(C_{S(\sigma)}(\mu,r))\leqslant I_{T(\sigma)}(f^{-1}(\mu),r)$. But $I_{T(\sigma)}(f^{-1}(\mu),r)\leqslant f^{-1}(\mu)$. Thus $f^{-1}(\mu)=I_{T(\sigma)}(f^{-1}(\mu),r)$. Therefore $f^{-1}(\mu)$ is an r-fuzzy G_δ-set. Hence f is a fuzzy contra G_δ-continuous mapping. □

Proposition 2.6. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T)\to(Y,S)$ be a bijective mapping. The mapping f is fuzzy contra G_δ-continuous if $C_{S(\sigma)}(f(\lambda),r)\leqslant f(I_{T(\sigma)}(\lambda,r))$ for each $\lambda\in I^X$ and $r\in I_0$.*

Proof. Suppose that $S(\overline{1}-\mu)\geqslant r$ for each $\mu\in I^Y$ and $r\in I_0$. Then $C_{S(\sigma)}(\mu,r)=\mu$. For each $\lambda\in I^X$ and $r\in I_0$, put $f(\lambda)=\mu$. Since f is surjective, from the assumption it follows that $f(I_{T(\sigma)}(f^{-1}(\mu)),r)\geqslant C_{S(\sigma)}(f(f^{-1}(\mu)),r)=C_{S(\sigma)}(\mu,r)=\mu$. Therefore $f^{-1}(f(I_{T(\sigma)}(f^{-1}(\mu)),r))\geqslant f^{-1}(\mu)$. Since f is a injective mapping, $I_{T(\sigma)}(f^{-1}(\mu),r)=f^{-1}(f(I_{T(\sigma)}(f^{-1}(\mu)),r)))\geqslant f^{-1}(\mu)=\lambda$. But $I_{T(\sigma)}(f^{-1}(\mu),r)\leqslant f^{-1}(\mu)$. Thus $f^{-1}(\mu)=I_{T(\sigma)}(f^{-1}(\mu),r)$. Therefore $f^{-1}(\mu)$ is an r-fuzzy G_δ-set. Hence f is a fuzzy contra G_δ-continuous mapping.

3. Application to fuzzy compact spaces

Definition 3.1. A smooth fuzzy topological space (X,T) is called fuzzy compact iff every T-cover $\{\eta_j\colon T(\eta_j)\geqslant r,\ j\in J\}$ of each $\mu\in I^X$ with $T(\overline{1}-\mu)\geqslant r$ and $r\in I_0$ has a finite subcollection such that for each $x_t\in\overline{1}-\mu$ there exists $j_0\in J$ such that $x_t\in\eta_{j_0}$.

Definition 3.2. Let (X,T) be a smooth fuzzy topological space and $\mu\in I^X$, $r\in I_0$. Then the family $\{\eta_j\colon \eta_j$ is an r-fuzzy G_δ-set, $j\in J\}$ is called a fuzzy G_δ-cover of μ iff for each $x_t\in\mu$ there exists $j_0\in J$ such that $x_t\in\eta_{j_0}$.

Definition 3.3. Let (X,T) be a smooth fuzzy topological space and $\mu\in I^X$, $r\in I_0$. Then the family $\{\eta_j\colon \eta_j$ is an r-fuzzy F_σ-set, $j\in J\}$ is called a fuzzy F_σ-cover of μ iff for each $x_t\in\mu$ there exists $j_0\in J$ such that $x_t\in\eta_{j_0}$.

Definition 3.4. A smooth fuzzy topological space (X,T) is called fuzzy G_δ-compact iff every fuzzy G_δ-cover $\{\eta_j\colon \eta_j$ is an r-fuzzy G_δ-set, $j\in J\}$ of each $\lambda\in I^X$ with $T(\overline{1}-\lambda)\geqslant r$ and $r\in I_0$ has a finite subcollection such that for each $x_t\in\overline{1}-\lambda$, there exists $j_0\in J$ such that $x_t\in\eta_{j_0}$.

Definition 3.5. A smooth fuzzy topological space (X,T) is called fuzzy almost G_δ-compact iff every fuzzy G_δ-cover $\{\eta_j\colon \eta_j$ is an r-fuzzy G_δ-set, $j\in J\}$ of each $\lambda\in I^X$ with $T(\overline{1}-\lambda)\geqslant r$ and $r\in I_0$ has a finite subcollection such that for each $x_t\in\overline{1}-\lambda$ there exists $j_0\in J$ such that $x_t\in C_{T(\sigma)}(\eta_{j_0},r)$.

Proposition 3.1. *The image of a fuzzy almost G_δ-compact space under a fuzzy contra G_δ-continuous, fuzzy G_δ-continuous and onto mapping is fuzzy compact.*

Proof. Let (X,T) be a fuzzy almost G_δ-compact space and (Y,S) a smooth fuzzy topological space. Let $f\colon (X,T)\to(Y,S)$ be a fuzzy contra G_δ-continuous fuzzy G_δ-continuous and onto mapping. Let $\mu \in I^Y$ with $S(\overline{1} - \mu) \geqslant r$, $r \in I_0$ and $\{\eta_j \colon S(\eta_j) \geqslant r,\ j \in J\}$ form an S-cover of μ. For $\lambda \in I^X$ put $f(\lambda) = \mu$. Then $\mu = f(\lambda) = \bigvee_{j\in J} \eta_j$. Now, $\lambda = f^{-1}(f(\lambda)) = f^{-1}\Big(\bigvee_{j\in J} \eta_j\Big) = \bigvee_{j\in J} f^{-1}(\eta_j)$. Since f is fuzzy contra G_δ-continuous and fuzzy G_δ-continuous, $f^{-1}(\eta_j)$ is r-fuzzy F_σ and r-fuzzy G_δ. Let $y_t \in \overline{1} - \mu$ and put $y_t = f(x_t)$. Then $x_t \in \overline{1} - \lambda$. Since (X,T) is fuzzy almost G_δ-compact, every fuzzy G_δ-cover $\{f^{-1}(\eta_j) \colon f^{-1}(\eta_j)$ is an r-fuzzy G_δ-set, $j \in J\}$ has a finite subcollection such that for $x_t \in \overline{1} - \lambda$ there exists $j_0 \in J$ such that $x_t \in C_{T(\sigma)}(f^{-1}(\eta_{j_0}), r) = f^{-1}(\eta_{j_0})$. That is, $f(x_t) \in \eta_{j_0}$. Hence $y_t \in \eta_{j_0}$. Therefore (Y,S) is fuzzy compact.

Proposition 3.2. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ be a fuzzy strongly G_δ-continuous function. If (X,T) is fuzzy almost G_δ-compact then (Y,S) is fuzzy compact.*

Proof. Since f is fuzzy strongly G_δ-continuous, f is both fuzzy G_δ-continuous and fuzzy contra G_δ-continuous. Hence by Proposition 3.1, (Y,S) is fuzzy compact. □

Proposition 3.3. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces, let $f\colon (X,T) \to (Y,S)$ be a fuzzy contra irresolute G_δ-continuous onto mapping. If (X,T) is fuzzy G_δ-compact, then (Y,S) is fuzzy almost G_δ-compact.*

Proof. Let $\mu \in I^Y$ be such that $S(\overline{1} - \mu) \geqslant r$, $r \in I_0$ and let $\{\eta_j \colon \eta_j$ is an r-fuzzy G_δ-set, $j \in J\}$ be a fuzzy G_δ-cover of μ. For $\lambda \in I^X$ put $f(\lambda) = \mu$. Then $\mu = \bigvee_{j\in J} \eta_j$. It follows that $\mu = \bigvee_{j\in J} C_{s(\sigma)}(\eta_j, r)$. Then $\mu = f(\lambda) = \bigvee_{j\in J} C_{s(\sigma)}(\eta_j, r)$. Now, $\lambda = f^{-1}(f(\lambda)) = f^{-1}\Big(\bigvee_{j\in J} C_{s(\sigma)}(\eta_j, r)\Big) = \bigvee_{j\in J} f^{-1}(C_{s(\sigma)}(\eta_j, r))$. Since f is fuzzy contra irresolute G_δ-continuous, $f^{-1}(C_{S(\sigma)}(\eta_j, r))$ is r-fuzzy G_δ. Let $y_t \in \overline{1} - \mu$ and put $y_t = f(x_t)$. Then $x_t \in \overline{1} - \lambda$. Since (X,T) is fuzzy G_δ-compact, every fuzzy G_δ-cover $\{f^{-1}(C_{s(\sigma)}(\eta_j, r)) \colon f^{-1}(C_{s(\sigma)}(\eta_j, r))$ is an r-fuzzy G_δ-set, $j \in J\}$ has a finite subcollection such that for $x_t \in \overline{1} - \lambda$ there exists $j_0 \in J$ such that $x_t \in f^{-1}(C_{S(\sigma)}(\eta_{j_0}, r))$. That is, $f(x_t) \in C_{S(\sigma)}(\eta_{j_0}, r)$. Hence $y_t \in C_{S(\sigma)}(\eta_{j_0}, r)$. Therefore (Y,S) is fuzzy almost G_δ-compact. □

Definition 3.6. Let (X,T) be a smooth fuzzy topological space. For $\lambda \in I^X$, $r \in I_0$, λ is said to be an r-fuzzy regular G_δ-set iff $\lambda = I_{T(\sigma)}(C_{T(\sigma)}(\lambda, r), r)$.

Definition 3.7. Let (X,T) be a smooth fuzzy topological space. For $\lambda \in I^X$, $r \in I_0$, λ is said to be an r-fuzzy regular F_σ-set iff $\lambda = C_{T(\sigma)}(I_{T(\sigma)}(\lambda, r), r)$.

R e m a r k 3.1. (1) Every r-fuzzy regular G_δ-set is r-fuzzy G_δ.
(2) Every r-fuzzy regular F_σ-set r-fuzzy F_σ.

Definition 3.8. Let (X,T) be a smooth fuzzy topological space and $\mu \in I^X$, $r \in I_0$. Then the family $\{\eta_j\colon \eta_j$ is an r-fuzzy regular F_σ-set, $j \in J\}$ is called a fuzzy regular F_σ-cover of μ iff for each $x_t \in \mu$ there exists $j_0 \in J$ such that $x_t \in \eta_{j_0}$.

Definition 3.9. A smooth fuzzy topological space (X,T) is called fuzzy strongly S-closed iff every fuzzy F_σ-cover of $\lambda \in I^X$ with $T(\overline{1}-\lambda) \geqslant r$ and $r \in I_0$ has a finite subcover.

Definition 3.10. A smooth fuzzy topological space (X,T) is called fuzzy S-closed iff every fuzzy regular F_σ-cover of $\lambda \in I^X$ with $T(\overline{1}-\lambda) \geqslant r$ and $r \in I_0$ has a finite subcover.

Proposition 3.4. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ be a fuzzy contra G_δ-continuous onto function. If (X,T) is fuzzy strongly S-closed, then (Y,S) is fuzzy compact.*

P r o o f. Let $\mu \in I^Y$ with $S(\overline{1}-\mu) \geqslant r$, $r \in I_0$ and $\{\eta_j\colon S(\eta_j) \geqslant r,\ j \in J\}$ form an S-cover of μ. For $\lambda \in I^X$ put $f(\lambda) = \mu$. Then $\mu = f(\lambda) = \bigvee_{j\in J} \eta_j$. Now, $\lambda = f^{-1}(f(\lambda)) = f^{-1}\Big(\bigvee_{j\in J} \eta_j\Big) = \bigvee_{j\in J} f^{-1}(\eta_j)$. Since f is fuzzy contra G_δ-continuous, $f^{-1}(\eta_j)$ is an r-fuzzy F_σ set. Let $y_t \in \overline{1}-\mu$ and put $y_t = f(x_t)$. Then $x_t \in \overline{1}-\lambda$. Since (X,T) is fuzzy strongly S-closed, every fuzzy F_σ-cover $\{f^{-1}(\eta_j)\colon f^{-1}(\eta_j)$ is an r-fuzzy F_σ-set, $j \in J\}$ has a finite subcollection such that for $x_t \in \overline{1}-\lambda$ there exists $j_0 \in J$ such that $x_t \in f^{-1}(\eta_{j_0})$. That is, $f(x_t) \in \eta_{j_0}$. Hence $y_t \in \eta_{j_0}$. Therefore (Y,S) is fuzzy compact. □

Proposition 3.5. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces.*

Let $f\colon (X,T) \to (Y,S)$ be a fuzzy contra irresolute G_δ-continuous onto function. If (X,T) is fuzzy strongly G_δ-compact, then (Y,S) is fuzzy strongly S-closed.

P r o o f. Let $\mu \in I^Y$ be such that $S(\overline{1}-\mu) \geqslant r$, $r \in I_0$, and let $\{\eta_j\colon \eta_j$ is an r-fuzzy F_σ-set, $j \in J\}$ be a fuzzy F_σ-cover of μ. For $\lambda \in I^X$ put $f(\lambda) = \mu$. Then $\mu = f(\lambda) = \bigvee_{j\in J} \eta_j$. Now, $\lambda = f^{-1}(f(\lambda)) = f^{-1}\Big(\bigvee_{j\in J} \eta_j\Big) = \bigvee_{j\in J} f^{-1}(\eta_j)$. Since f is fuzzy contra irresolute G_δ-continuous, $f^{-1}(\eta_j)$ is an r-fuzzy G_δ-set. Let $y_t \in \overline{1}-\mu$ and put $y_t = f(x_t)$. Then $x_t \in \overline{1}-\lambda$. Since (X,T) is fuzzy strongly G_δ-compact, $\{f^{-1}(\eta_j)\colon f^{-1}(\eta_j)$ is an r-fuzzy G_δ-set, $j \in J\}$ has a finite subcollection such that for $x_t \in \overline{1}-\lambda$ there exists $j_0 \in J$ such that $x_t \in f^{-1}(\eta_{j_0})$. That is, $f(x_t) \in \eta_{j_0}$. Hence $y_t \in \eta_{j_0}$. Therefore (Y,S) is fuzzy strongly S-closed. □

Proposition 3.6. *Every fuzzy strongly S-closed space (X,T) is fuzzy S-closed.*

P r o o f. Let (X,T) be a fuzzy strongly S-closed space. For $\lambda \in I^X$ and $r \in I_0$ put $\lambda = \bigvee_{j\in J} \eta_j$, where η_j is an r-fuzzy regular F_σ-set. Since every r-fuzzy regular F_σ-set is r-fuzzy F_σ and (X,T) is fuzzy strongly S-closed, there exists a finite subcollection $\{\eta_j\colon \eta_j$ is an r-fuzzy F_σ-set, $j \in J\}$ such that $\lambda = \bigvee_{j=1}^{n} \eta_j$. Hence (X,T) is fuzzy S-closed. □

Definition 3.11. Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. $f\colon (X,T) \to (Y,S)$ is called fuzzy almost G_δ-continuous iff $f^{-1}(\mu)$ is an r-fuzzy G_δ-set for each r-fuzzy regular G_δ-set $\mu \in I^Y$.

Proposition 3.7. *Let (X,T) and (Y,S) be any two smooth fuzzy topological spaces. Let $f\colon (X,T) \to (Y,S)$ be a fuzzy almost G_δ-continuous and onto function. If (X,T) is fuzzy strongly S-closed, (Y,S) is fuzzy S-closed.*

P r o o f. Let $\mu \in I^Y$ be such that $S(\overline{1}-\mu) \geqslant r$, $r \in I_0$ and let $\{\eta_j\colon \eta_j$ is an r-fuzzy regular F_σ-set, $j \in J\}$ be a fuzzy regular F_σ-cover of μ. For $\lambda \in I^X$ put $f(\lambda) = \mu$. Then $\mu = f(\lambda) = \bigvee_{j\in J} \eta_j$. Now, $\lambda = f^{-1}(f(\lambda)) = f^{-1}\Big(\bigvee_{j\in J} \eta_j\Big) = \bigvee_{j\in J} f^{-1}(\eta_j)$. Since f is fuzzy almost G_δ-continuous, $f^{-1}(\eta_j)$ is an r-fuzzy F_σ-set. Let $y_t \in \overline{1} - \mu$ and put $y_t = f(x_t)$. Then $x_t \in \overline{1} - \lambda$. Since (X,T) is fuzzy strongly S-closed, $\{f^{-1}(\eta_j)\colon f^{-1}(\eta_j)$ is an r-fuzzy F_σ-set, $j \in J\}$ has a finite subcollection such that for $x_t \in \overline{1} - \lambda$ there exists $j_0 \in J$ such that $x_t \in f^{-1}(\eta_{j_0})$. That is, $f(x_t) \in \eta_{j_0}$. Hence $y_t \in \eta_{j_0}$. Therefore (Y,S) is fuzzy S-closed. □

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References

[1] *Balasubramanian G.*: Maximal fuzzy topologies. Kybernetika *31* (1995), 459–464.

[2] *Dontchev J.*: Contra-continuous functions and strongly s-closed spaces. Inter. J. Math. Sci. *19* (1996), 303–310.

[3] *Ekici E., Kerre E.*: On fuzzy contra continuities. Advances in Fuzzy Mathematics *1* (2006), 35–44.

[4] *Krsteska B., Ekici E.*: Fuzzy contra strong precontinuity. Indian J. Math. *50* (2008), 149–161.

[5] *Pu P. M., Liu Y. M.*: Fuzzy topology. I: Neighbourhood structure of a fuzzy point and Moor-Smith convergence. J. Math. Anal. Appl. *76* (1980), 571–599.

[6] *Ramadan A. A, Abbas S. E., Yong Chan Kim*: Fuzzy irresolute mappings in smooth fuzzy topological spaces. The Journal of Fuzzy Mathematics *9* (2001), 865–877.

[7] *Roja E, Uma M. K., Balasubramanian G.*: Some generalization of fuzzy G_δ-continuous mappings. Mathematical Forum *17* (2004–2005), 1–16.

[8] *Roja E, Uma M. K., Balasubramanian G.*: G_δ-connectedness in fuzzy topological spaces. East Asian Math. J. *20* (2004), 87–95.
[9] *Samanta S. K., Chattopadhyay K. C.*: Fuzzy topology. Fuzzy Sets Systems *54* (1993), 207–212.
[10] *Smets P.*: The degree of belief in a fuzzy event. Inform. Sci. *25* (1981), 1–19.
[11] *Sostak A. P.*: On a fuzzy topological structure. Rend. Circ. Matem. Palermo (Ser. II) *11* (1985), 89–103.
[12] *Sugeno M.*: An introductory survey of fuzzy control. Inform. Sci. *36* (1985), 59–83.
[13] *Thangaraj G.*: Contributions to the study on some aspects of fuzzy topological structures. Ph.D. Thesis, University of Madras, 2003.
[14] *Zadeh L. A.*: Fuzzy sets. Inf. Control *8* (1965), 338–353.

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