Friedrich Haslinger

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THE $\overline{\partial}$-NEUMANN OPERATOR AND COMMUTATORS OF THE BERGMAN PROJECTION AND MULTIPLICATION OPERATORS

Friedrich Haslinger, Wien

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Abstract. We prove that compactness of the canonical solution operator to $\overline{\partial}$ restricted to $(0,1)$-forms with holomorphic coefficients is equivalent to compactness of the commutator $[\mathscr{P},\overline{M}]$ defined on the whole $L^2_{(0,1)}(\Omega)$, where $\overline{M}$ is the multiplication by $\overline{z}$ and $\mathscr{P}$ is the orthogonal projection of $L^2_{(0,1)}(\Omega)$ to the subspace of $(0,1)$ forms with holomorphic coefficients. Further we derive a formula for the $\overline{\partial}$-Neumann operator restricted to $(0,1)$ forms with holomorphic coefficients expressed by commutators of the Bergman projection and the multiplications operators by z and $\overline{z}$.

Keywords: $\overline{\partial}$-equation, $\overline{\partial}$-Neumann operator, compactness

MSC 2010: 32W05, 32A36

1. Introduction

We assume that the reader is familiar with the $\overline{\partial}$-Neumann problem. See [15], [4], [8]. Let Ω be a bounded pseudoconvex domain in $\mathbb{C}^n$. We consider the $\overline{\partial}$-complex

$$L^2(\Omega) \xrightarrow{\overline{\partial}} L^2_{(0,1)}(\Omega) \xrightarrow{\overline{\partial}} \dots \xrightarrow{\overline{\partial}} L^2_{(0,n)}(\Omega) \xrightarrow{\overline{\partial}} 0,$$

where $L^2_{(0,q)}(\Omega)$ denotes the space of $(0,q)$-forms on Ω with coefficients in $L^2(\Omega)$. The $\overline{\partial}$-operator on $(0,q)$-forms is given by

$$\overline{\partial}\Bigl(\sum_J{}' a_J \,\mathrm{d}\overline{z}_J\Bigr) = \sum_{j=1}^n \sum_J{}' \frac{\partial a_J}{\partial \overline{z}_j}\,\mathrm{d}\overline{z}_j \wedge \mathrm{d}\overline{z}_J.$$

The derivatives are taken in the sense of distributions, and the domain of $\overline{\partial}$ consists of those $(0,q)$-forms for which the right hand side belongs to $L^2_{(0,q+1)}(\Omega)$. Then $\overline{\partial}$

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is a densely defined closed operator, and therefore has an adjoint operator from $L^2_{(0,q+1)}(\Omega)$ into $L^2_{(0,q)}(\Omega)$ denoted by $\overline{\partial}^*$.

The complex Laplacian

$$\Box = \overline{\partial}\overline{\partial}^* + \overline{\partial}^*\overline{\partial}$$

acts as an unbounded selfadjoint operator on

$$L^2_{(0,q)}(\Omega),\ 1 \leqslant q \leqslant n,$$

it is surjective and therefore has a continuous inverse, the $\overline{\partial}$-Neumann operator N_q. If v is a closed $(0, q+1)$-form, then $\overline{\partial}^*\, N_{q+1}v$ provides the canonical solution to $\overline{\partial}u = v$, which is orthogonal to the kernel of $\overline{\partial}$ and so has minimal norm (see [15], [8], [4]).

A survey of the L^2-Sobolev theory of the $\overline{\partial}$-Neumann problem is given in [1].

The question of compactness of N_q is of interest for various reasons. For example, compactness of N_q implies global regularity in the sense of preservation of Sobolev spaces [16]. Also, the Fredholm theory of Toeplitz operators is an immediate consequence of compactness in the $\overline{\partial}$-Neumann problem [20], [14], [3]. There are additional ramifications for certain C^*-algebras naturally associated to a domain in $\mathbb{C}^n$ [19]. Finally, compactness is a more robust property than global regularity—for example, it localizes, whereas global regularity does not—and it is generally believed to be more tractable than global regularity.

Catlin [2] showed that for sufficiently smooth bounded pseudoconvex domains satisfying what he called property (P), the $\overline{\partial}$-Neumann problem is compact, and that all domains of finite type in the sense of D'Angelo [5] satisfy property (P).

A thorough discussion of compactness in the $\overline{\partial}$-Neumann problem can be found in [7].

Compactness is completely understood on (bounded) locally convexifiable domains. On such domains, the following are equivalent [6], [7]:

(i) N_q is compact,

(ii) the boundary of the domain satisfies (an analogue of) property (P) (for q-forms),

(iii) the boundary contains no q-dimensional analytic variety.

In general, however, the situation is not understood at all.

The study of the $\overline{\partial}$-Neumann problem is essentially equivalent (in a sense that can be made precise) to the study of the canonical solution operator to $\overline{\partial}$. Interestingly, in many situations, the restriction of the canonical solution operator to forms with *holomorphic* coefficients arises naturally [19], [6].

The restriction of the canonical solution operator to forms with holomorphic coefficients has many interesting aspects, which in most cases correspond to certain growth properties of the Bergman kernel. It is also of great interest to clarify to what extent compactness of the restriction already implies compactness of the original solution operator to $\overline{\partial}$. This is the case for convex domains, see [6]. There are many other examples of non-compactness where the obstruction already occurs for forms with holomorphic coefficients (see [18], [17]).

In [9] the canonical solution operator S_1 to $\overline{\partial}$ restricted to $(0,1)$-forms with holomorphic coefficients was investigated. Let $A^2_{(0,1)}(\Omega)$ denote the space of all $(0,1)$-forms with holomorphic coefficients belonging to $L^2(\Omega)$. It is shown that the canonical solution operator $S_1\colon\ A^2_{(0,1)}(\Omega) \longrightarrow L^2(\Omega)$ has the form

$$S_1(g)(z) = \int_\Omega B(z,w)\,\langle g(w), z-w\rangle\,\mathrm{d}\lambda(w),$$

where B denotes the Bergman kernel of Ω and

$$\langle g(w), z-w\rangle = \sum_{j=1}^n g_j(w)(\overline{z}_j - \overline{w}_j),$$

for $z = (z_1,\dots,z_n)$ and $w = (w_1,\dots,w_n)$. (For further results see also [10], [11], [12], [13]).

In this paper we investigate the connection between the $\overline{\partial}$-Neumann operator and commutators of the Bergman projection with multiplication operators. In [3] it is shown that compactness of the $\overline{\partial}$-Neumann operator N on $L^2_{(0,1)}(\Omega)$ implies compactness of the commutator $[P,M]$, where P is the Bergman projection and M is pseudodifferential operator of order 0. Here we show that compactness of the $\overline{\partial}$-Neumann operator N restricted to $(0,1)$-forms with holomorphic coefficients is equivalent to compactness of the commutator $[P,M]$ defined on the whole $L^2(\Omega)$. In addition we derive a formula for the $\overline{\partial}$-Neumann operator restricted to $(0,1)$ forms with holomorphic coefficients expressed by commutators of the Bergman projection and the multiplications operators by z and $\overline{z}$.

2. Commutators of the Bergman projection and multiplication operators

Let Ω be a bounded domain in $\mathbb{C}^n$ and $B(z,w)$ the Bergman kernel of Ω. We define the following operator

$$T\colon\ L^2_{(0,1)}(\Omega) \longrightarrow L^2(\Omega),$$

by

$$Tf(z) = \int_\Omega B(z,w)\langle f(w), z-w\rangle\,\mathrm{d}\lambda(w),$$

where $f = \sum\limits_{k=1}^n f_k\,\mathrm{d}\overline{z}_k$ and $\langle f(w), z-w\rangle = \sum\limits_{k=1}^n f_k(w)(\overline{z}_k - \overline{w}_k)$.

Let $\mathscr{P}\colon\ L^2_{(0,1)}(\Omega) \longrightarrow A^2_{(0,1)}(\Omega)$ be the orthogonal projection on the space of $(0,1)$-forms with holomorphic coefficients. We claim that

$$Tf = T\mathscr{P}f, \quad f \in L^2_{(0,1)}(\Omega).$$

Note that

$$\mathscr{P}f = \sum_{k=1}^n P(f_k)\,\mathrm{d}\overline{z}_k,$$

where $P\colon\ L^2(\Omega) \longrightarrow A^2(\Omega)$ is the usual Bergman projection. So we get

$$\begin{aligned}
T\mathscr{P}f(z) &= \int_\Omega B(z,w)\langle \mathscr{P}(f), z-w\rangle\,\mathrm{d}\lambda(w) \\
&= \sum_{k=1}^n \int_\Omega B(z,w)Pf_k(w)\,(\overline{z}_k - \overline{w}_k)\,\mathrm{d}\lambda(w) \\
&= \sum_{k=1}^n \int_\Omega B(z,w)\int_\Omega B(w,\zeta)f_k(\zeta)\,\mathrm{d}\lambda(\zeta)\,(\overline{z}_k - \overline{w}_k)\,\mathrm{d}\lambda(w) \\
&= \sum_{k=1}^n \int_\Omega \int_\Omega B(z,w)B(w,\zeta)(\overline{z}_k - \overline{w}_k)\,\mathrm{d}\lambda(w) f_k(\zeta)\,\mathrm{d}\lambda(\zeta) \\
&= \sum_{k=1}^n \int_\Omega \left[\int_\Omega B(\zeta,w)B(w,z)(z_k - w_k)\,\mathrm{d}\lambda(w)\right]^{-} f_k(\zeta)\,\mathrm{d}\lambda(\zeta) \\
&= \sum_{k=1}^n \int_\Omega B(z,\zeta)(\overline{z}_k - \overline{\zeta}_k)\,f_k(\zeta)\,\mathrm{d}\lambda(\zeta) = Tf(z),
\end{aligned}$$

where we used the reproducing property of the Bergman kernel for the holomorphic function $w \mapsto B(w,z)(z_k - w_k)$. For another proof of this fact see Remark b) from below.

Now, let $\mathscr{S}$ denote the canonical solution operator to $\overline{\partial}$ restricted to $A^2_{(0,1)}(\Omega)$. From [9] we have for $f \in L^2_{(0,1)}(\Omega)$

$$\mathscr{S}(\mathscr{P}f) = T(\mathscr{P}f) = Tf.$$

Hence we have proved the following

Theorem 1. *If $f \in L^2_{(0,1)}(\Omega)$, then $T(\mathscr{P}f) = Tf$. The operator $\mathscr{S}$ is compact as an operator from $A^2_{(0,1)}(\Omega)$ to $L^2(\Omega)$, if and only if the operator T is compact as an operator from $L^2_{(0,1)}(\Omega)$ to $L^2(\Omega)$.*

Remarks.

a) The operator T can be written as a sum of commutators

$$Tf = \sum_{k=1}^{n} [\overline{M_k}, P] f_k, \quad f = \sum_{k=1}^{n} f_k \, \mathrm{d}\overline{z}_k$$

where $\overline{M_k} v(z) = \overline{z}_k v(z),\ v \in L^2(\Omega),\ k = 1, \ldots, n$.

b) If $g \perp A^2_{(0,1)}(\Omega)$, then $Tg = T\mathscr{P}g = 0$. This also follows from the direct calculation

$$\begin{aligned} Tg(z) &= \sum_{k=1}^{n} P\overline{M_k} g(z) = \sum_{k=1}^{n} \int_\Omega B(z,w) \overline{w}_k g_k(w) \, \mathrm{d}\lambda(w) \\ &= \sum_{k=1}^{n} \int_\Omega g_k(w) \ \ [B(w,z) w_k]^- \ \mathrm{d}\lambda(w) = 0, \end{aligned}$$

because $w \mapsto B(w,z) w_k$ is holomorphic and $g_k \perp A^2(\Omega)$, for $k = 1, \ldots, n$.

c) The adjoint operator $T^* \colon \ L^2(\Omega) \longrightarrow L^2_{(0,1)}(\Omega)$ is given by

$$T^*(g) = \sum_{k=1}^{n} [P, M_k] \, g \, \mathrm{d}\overline{z}_k, \ \ g \in L^2(\Omega),$$

where $M_k v(z) = z_k v(z)$. Here we have

$$T^*(I - P)(g) = T^*(g),$$

since

$$[P, M_k] \, Pg = PM_k Pg - M_k Pg = 0.$$

In a similar way the following results can be proved.

Lemma 1.

a) $PM_j P = M_j P$,

b) $P\overline{M_j} P = P\overline{M_j}$.

Let

$$B^2_{(0,1)}(\Omega) = \{ f \in L^2_{(0,1)}(\Omega) \colon \ f \in \ker \overline{\partial} \}.$$

Now suppose that Ω is bounded pseudoconvex domain in $\mathbb{C}^n$. The $\overline{\partial}$-Neumann operator N can be viewed as an operator from $B^2_{(0,1)}(\Omega)$ to $B^2_{(0,1)}(\Omega)$. The operator

$$\overline{\partial}^* N \colon \ B^2_{(0,1)}(\Omega) \longrightarrow A^2(\Omega)^\perp$$

is the canonical solution operator to $\overline{\partial}$ (see [4]).

Theorem 2. *If* $f = \sum\limits_{k=1}^{n} f_k \,\mathrm{d}\overline{z}_k \in B^2_{(0,1)}(\Omega)$, *then*

$$\mathscr{P} N \mathscr{P} f = \sum_{k=1}^{n} \left(\sum_{j=1}^{n} (P M_k \overline{M}_j P f_j - M_k P \overline{M}_j f_j) \right) \mathrm{d}\overline{z}_k.$$

If $f = \sum\limits_{k=1}^{n} f_k \,\mathrm{d}\overline{z}_k \in A^2_{(0,1)}(\Omega)$, *then*

$$\mathscr{P} N f = \sum_{k=1}^{n} [P, M_k] \left(\sum_{j=1}^{n} \overline{M}_j f_j \right) \mathrm{d}\overline{z}_k.$$

Proof. First we observe that for $f \in B^2_{(0,1)}(\Omega)$ we have

$$N \overline{\partial} \overline{\partial}^* N f = N(I - \overline{\partial}^* \overline{\partial} N) f = N f,$$

where we used the fact that

$$N\colon\ B^2_{(0,1)}(\Omega) \longrightarrow B^2_{(0,1)}(\Omega).$$

If $f \in A^2_{(0,1)}(\Omega)$, then by Theorem 1 it follows that

$$\overline{\partial}^* N f = T f.$$

Let $f \in A^2_{(0,1)}(\Omega)$ and $g \in B^2_{(0,1)}(\Omega)$ with orthogonal decompostion $g = h + \tilde{h}$, where $h \in A^2_{(0,1)}(\Omega)$ and $\tilde{h} = (I - \mathscr{P}) g$, then

$$\begin{aligned}(g, N \overline{\partial} \overline{\partial}^* N f) &= (\overline{\partial}^* N (h + \tilde{h}), T f) = (\overline{\partial}^* N h, T f) + (\overline{\partial}^* N \tilde{h}, T f) \\ &= (T h, T f) + (\overline{\partial}^* N \tilde{h}, T f) = (T g, T f) + (\overline{\partial}^* N \tilde{h}, T f) \\ &= (g, T^* T f) + (\overline{\partial}^* N \tilde{h}, T f).\end{aligned}$$

Since

$$(\overline{\partial}^* N \tilde{h}, T f) = (N \tilde{h}, \overline{\partial} T f) = (N \tilde{h}, f) = (\tilde{h}, N f),$$

we obtain

$$\begin{aligned}(g, N f) &= (g, N \overline{\partial} \overline{\partial}^* N f) = (g, T^* T f) + (\tilde{h}, N f) \\ &= (g, T^* T f) + ((I - \mathscr{P}) g, N f) = (g, T^* T f) + (g, (I - \mathscr{P}) N f).\end{aligned}$$

Now, since $g \in B^2_{(0,1)}(\Omega)$ was arbitrary, we get

$$N f = T^* T f + N f - \mathscr{P} N f,$$

and therefore

$$\mathscr{P} N f = T^* T f.$$

If we take into account, that for $f \in B^2_{(0,1)}(\Omega)$ we have $Tf = T\mathscr{P} f$, we can now apply the last formula to $\mathscr{P} f$ and get

$$\mathscr{P} N \mathscr{P} f = T^* T f.$$

It remains to compute T^*T. If $f \in B^2_{(0,1)}(\Omega)$, then

$$\begin{aligned} T^*Tf &= \sum_{k=1}^n [P, M_k]\left(\sum_{j=1}^n [\overline{M}_j, P] f_j\right) \mathrm{d}\overline{z}_k \\ &= \sum_{k=1}^n \left(\sum_{j=1}^n (PM_k\overline{M}_jP - M_kP\overline{M}_jP - PM_kP\overline{M}_j + M_kP\overline{M}_j)f_j\right) \mathrm{d}\overline{z}_k \\ &= \sum_{k=1}^n \left(\sum_{j=1}^n (PM_k\overline{M}_jPf_j - M_kP\overline{M}_jf_j)\right) \mathrm{d}\overline{z}_k, \end{aligned}$$

where we used Lemma 1.

If $f \in A^2_{(0,1)}(\Omega)$, then

$$Pf_j = f_j$$

and we obtain the second formula in Theorem 2. □

Using the last results we get the criterion for compactness of the commutators $[P, M_k]$:

Theorem 3. *Let Ω be a bounded pseudoconvex domain in $\mathbb{C}^n$. Then the following conditions are equivalent:*

(1) $N|_{A^2_{(0,1)}(\Omega)}$ *is compact;*

(2) $\overline{\partial}^* N|_{A^2_{(0,1)}(\Omega)}$ *is compact;*

(3) $[P, M_k]$ *is compact on* $L^2(\Omega)$ *for* $k = 1, \ldots, n$;

(4) $(I - P)\overline{M}_k P$ *is compact on* $L^2(\Omega)$ *for* $k = 1, \ldots, n$;

(5) $[M_\varphi, P]$ *is compact on* $L^2(\Omega)$ *for each continuous function* φ *on* $\bar{\Omega}$.

P r o o f. Let $\mathscr{S}_1 = \overline{\partial}^* N_1 \colon B^2_{(0,1)}(\Omega) \longrightarrow A^2(\Omega)^\perp$ be the canonical solution opertor to $\overline{\partial}$ and similarly $\mathscr{S}_2 = \overline{\partial}^* N_2 \colon B^2_{(0,2)}(\Omega) \longrightarrow B^2_{(0,1)}(\Omega)^\perp$, then

$$N_1 = \mathscr{S}_1^* \mathscr{S}_1 + \mathscr{S}_2 \mathscr{S}_2^*.$$

(see for instance [4] or [7]). Since $\mathscr{S}_2^*|_{A^2_{(0,1)}(\Omega)} = 0$, we have

$$N_1|_{A^2_{(0,1)}(\Omega)} = \mathscr{S}_1^* \mathscr{S}_1|_{A^2_{(0,1)}(\Omega)},$$

and (1) is equivalent to (2).

Now suppose that (2) holds. Then, since the restriction of $\overline{\partial}^* N$ to $A^2_{(0,1)}(\Omega)$ is of the form

$$\overline{\partial}^* N f = \sum_{k=1}^n [\overline{M_k}, P] f_k,$$

where $f = \sum_{k=1}^n f_k \, dz_k \in A^2_{(0,1)}(\Omega)$, then by Theorem 1 it follows that the operators $[\overline{M_k}, P]$ are compact on $L^2(\Omega)$. Since $[\overline{M_k}, P]^* = [P, M_k]$, we obtain property (3).

It is also clear by Theorem 1 that (3) implies (2).

Now suppose that (3) holds. It follows that $[\overline{M_k}, P]P$ is also compact, and since

$$[\overline{M_k}, P]P = \overline{M_k}P - P\overline{M_k}P = (I-P)\overline{M_k}P,$$

the Hankel operators $(I-P)\overline{M_k}P$ are compact. So we have shown that (3) implies (4).

Suppose that (4) holds. The Hankel operators $H_{z_j \overline{z_k}}$ with symbol $z_j \overline{z_k}$ can be written in the form

$$H_{z_j \overline{z_k}} = (I-P)M_j(P + (I-P))\overline{M_k}P = (I-P)M_j(I-P)\overline{M_k}P,$$

hence it follows that $H_{z_j \overline{z_k}}$ is compact. Similarly one can show that for any polynomial

$$p(z, \overline{z}) = \sum_{|\alpha| \leqslant N} \lambda_\alpha z^{\alpha_1} \overline{z}^{\alpha_2},$$

where $\alpha = (\alpha_1, \alpha_2)$ in a multiindex in $\mathbb{N}^{2n}$, the corresponding Hankel operator $H_p = (I-P)M_pP$ is compact. Now let $\varphi \in \mathscr{C}(\overline{\Omega})$. Then, by the Stone-Weierstraß Theorem, there exists a polynomial p of the above form such that

$$\|\varphi - p\|_\infty < \varepsilon.$$

Hence

$$\|H_\varphi - H_p\| = \|(I-P)M_{\varphi - p}P\| \leqslant \|\varphi - p\|_\infty.$$

Since the compact operators form a closed twosided ideal in the operator norm and since for $g = g_1 + g_2$ where $g_1 \in A^2(\Omega)$ and $g_2 \in A^2(\Omega)^\perp$ we have

$$[M_\varphi, P]g = -H^*_{\bar{\varphi}} g_2 + H_\varphi g_1,$$

it follows that $[M_\varphi, P]$ is compact. □

Remark. If Ω is a bounded convex domain, then compactness of $\overline{\partial}^* N|_{A^2_{(0,1)}(\Omega)}$ implies already compactness of $\overline{\partial}^* N$ on all of $L^2_{(0,1)}(\Omega)$ (see [6]), hence, in this case property (1) of Theorem 3 can be replaced by N being compact on $L^2_{(0,1)}(\Omega)$ and property (2) of Theorem 3 can be replaced by $\overline{\partial}^* N$ being compact on $L^2_{(0,1)}(\Omega)$.

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Author's address: Friedrich Haslinger, Institut für Mathematik, Universität Wien, Nordbergstraße 15, A-1090 Wien, Austria, e-mail: friedrich.haslinger@univie.ac.at.