Kybernetika

Hansheng Wu

Decentralized robust tracking control of uncertain large scale systems with multiple delays in the interconnections

Kybernetika, Vol. 45 (2009), No. 1, 121--136

Persistent URL: http://dml.cz/dmlcz/140001

KYBERNETIKA — VOLUME *45* (2009), NUMBER 1, PAGES 121 – 136

DECENTRALIZED ROBUST TRACKING CONTROL OF UNCERTAIN LARGE SCALE SYSTEMS WITH MULTIPLE DELAYS IN THE INTERCONNECTIONS

Hansheng Wu

The problem of the decentralized robust tracking and model following is considered for a class of uncertain large scale systems including time-varying delays in the interconnections. On the basis of the Razumikhin-type theorem and the Lyapunov stability theory, a class of decentralized memoryless local state feedback controllers is proposed for robust tracking of dynamical signals. It is shown that by employing the proposed decentralized robust tracking controllers, one can guarantee that the tracking error between each time-delay subsystem and the corresponding local reference model without time-delay decreases uniformly asymptotically to zero. In this paper, it is assumed that the time-varying delays are any continuous and bounded nonnegative functions, and the proposed decentralized robust tracking controllers are independent of the delays. Therefore, the results obtained in the paper are applicable to large scale systems without exact knowledge of the delays, i. e. large systems with perturbed delays.

Keywords: large scale systems, time delay, decentralized control, robust tracking

AMS Subject Classification: 93A14, 93A15, 93B52, 93D21, 34K06

1. INTRODUCTION

In the practical control problems, it is important to consider the robust tracking and model following problem of dynamical systems with significant uncertainties. Therefore, such a problem has been widely discussed, and some approaches for an actual system to tracking dynamical signals of a given reference model have been developed (see, e. g. [6, 11, 15] and the references therein).

It is well known that except for significant uncertainties, some delays are often encountered in various engineering systems to be controlled, such as chemical processes, hydraulic, and rolling mill systems, economic systems, and the existence of the delays is frequently a source of instability. Therefore, the problem of robust stabilization of uncertain time-delay systems has received considerable attention of many researchers, and many solution approaches have been developed (see, e. g. [7, 12, 19, 20] and the references therein). Furthermore, in [8, 10, 16], the robust tracking and model following problem for uncertain composite systems with time-delay is also considered, and some types of robust tracking controllers are proposed.

In recent years, there also are some works in which the problem of decentralized robust tracking and model following is considered for uncertain large scale systems. In [9], for example, the problem of decentralized robust tracking and model following for large scale interconnected systems with uncertainties is considered, and a class of continuous (nonlinear) decentralized state feedback controllers is proposed. It is also shown in [9] that the proposed decentralized robust tracking controllers can guarantee that the tracking error between each subsystem and the corresponding local reference model decreases asymptotically to zero. However, because of its complexity, few efforts were made to consider the decentralized robust tracking and model following problem of uncertain large scale time-delay systems.

In this paper, we consider the problem of the decentralized robust tracking and model following for a class of uncertain large scale systems including time-varying delays in the interconnections. By combining the Razumikhin-type theorem with the Lyapunov stability theory, we propose a class of decentralized local state feedback controllers for robust tracking of dynamical signals. We also show that by employing the proposed decentralized robust tracking controllers, one can guarantee that the tracking error between each time-delay subsystem and the corresponding local reference model without time-delay decreases uniformly asymptotically to zero. That is, we can make it possible that the output of each controlled uncertain time-delay subsystem tracks exactly the output of the corresponding local reference model without time-delay.

The paper is organized as follows. In Section 2, the decentralized model following problem to be tackled is stated and some standard assumptions are introduced. In Section 3, we propose a class of decentralized memoryless robust tracking controllers. The paper is concluded in Section 4 with a brief discussion of the results.

2. PROBLEM FORMULATION AND ASSUMPTION

We consider a large scale time-delay system S composed of N interconnected subsystems $S_i, i = 1, 2, \cdots, N$, described by the following differential equations:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \Big[A_i + \Delta A_i(\upsilon_i, t) \Big] x_i(t) + \Big[B_i + \Delta B_i(\xi_i, t) \Big] u_i(t) \tag{1a}$$

$$y_i(t) = C_i x_i(t) \tag{1b}$$

where $t \in \mathbb{R}^+$ is the time, $x_i(t) \in \mathbb{R}^{n_i}$ is the current value of the state, $u_i(t) \in \mathbb{R}^{m_i}$ is the input (or control) vector, and $y_i(t) \in \mathbb{R}^{l_i}$ is the output vector. Each dynamical subsystem is interconnected as

$$u_i(t) = \sum_{j=1}^{N} A_{ij}(\zeta_i, t) x_j(t - h_{ij}(t)) + w_i(\nu_i, t), \qquad i = 1, 2, \ldots, N. \tag{2}$$

In (1) and (2), for each $i \in \{1, 2, \ldots, N\}$, A_i, B_i, C_i are known constant matrices of appropriate dimensions. In particular, the matrix $A_{ij}(\cdot)$ stands for the extent of interconnection between S_i and S_j, and are assumed to be continuous in all their arguments; $\Delta A_i(\cdot)$, $\Delta B_i(\cdot)$ represent the uncertainties of the systems, $w_i(\cdot)$ is the external disturbance vector, and are also assumed to be continuous in all their arguments.

Moreover, the uncertain parameters $(\upsilon_i,\ \xi_i,\ \zeta_i,\ \nu_i) \in \Psi_i \subset \mathbb{R}^{L_i},\ \ i \in \{1, 2, \ldots, N\}$, are Lebesgue measurable and take values in a known compact bounding set Ω_i; the time delays $h_{ij}(t)$, $i, j = 1, 2, \ldots, N$, are assumed to be any bounded, and continuous functions, i.e. $0 \leq h_{ij}(t) \leq \bar{h}_{ij}$ where $\bar{h}_{ij}$ are any nonnegative constants. Here, the time-varying delays and their bounds are not required to be known for the system designer. In this paper, $x(\cdot) \in \mathbb{R}^n$ denotes $\begin{bmatrix} x_1^\top(\cdot) & x_2^\top(\cdot) & \cdots & x_N^\top(\cdot) \end{bmatrix}^\top$, where $n = n_1 + n_2 + \cdots + n_N$.

The initial condition for each subsystem with time delays is given by

$$x_i(t) \ = \ \chi_i(t), \qquad t \in [t_0 - \bar{h}_i,\ t_0] \tag{3}$$

where $\chi_i(t)$ is a continuous function on $[t_0 - \bar{h}_i,\ t_0]$, and $\bar{h}_i$ is defined as follows.

$$\bar{h}_i \ := \ \max\{\ \bar{h}_{ij},\ \ j = 1, 2, \ldots, N\ \}.$$

For this class of input-interconnected large scale systems including delayed state perturbations in the interconnections, we introduce a decentralized local memoryless state feedback controller $\bar{u}_i(t)$ given by

$$\bar{u}_i(t) \ = \ p_i(x_i(t),\ t), \qquad i = 1, 2, \ldots, N \tag{4}$$

for each subsystem which modifies (2) to

$$u_i(t) \ = \ \bar{u}_i(t) + \sum_{j=1}^{N} A_{ij}(\zeta_i, t) x_j(t - h_{ij}(t)) + w_i(\nu_i, t), \qquad i = 1, 2, \ldots, N \tag{5}$$

where $p_i(\cdot) : \mathbb{R}^{n_i} \times \mathbb{R}^+ \to \mathbb{R}^{m_i}$ is a continuous function which will be proposed later.

On the other hand, for each $i \in \{1, 2, \cdots, N\}$, the reference sign $\hat{y}_i(t)$, which should be followed by the output $y_i(t)$ of each subsystem S_i, is assumed to be the output of a reference model $\hat{S}_i$ described by the differential equation of the form:

$$\frac{\mathrm{d}\hat{x}_i(t)}{\mathrm{d}t} \ = \ \hat{A}_i \hat{x}_i(t) + \hat{B}_i r_i(t) \tag{6a}$$

$$\hat{y}_i(t) \ = \ \hat{C}_i \hat{x}_i(t) \tag{6b}$$

where $\hat{x}_i(t) \in \mathbb{R}^{\hat{n}_i}$ is the state vector of the reference model, $\hat{y}_i(t) \in \mathbb{R}^{\hat{l}_i}$ is the output vector of the reference model, $r_i(t) \in \mathbb{R}^{\hat{m}_i}$ is the input vector of the reference model, and $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$ are known constant matrices of appropriate dimensions. Here, $\hat{y}_i(t)$ has the same dimension as $y_i(t)$, i.e. $\hat{l}_i = l_i$. Furthermore, in order to guarantee that the tracking of dynamical signals is practically meaningful, we require that the model state must be bounded, i.e. for each reference model $\hat{S}_i$, $i \in \{1, 2, \ldots, N\}$, there exists a finite positive constant M_i such that for all $t \geq t_0$,

$$\|\hat{x}_i(t)\| \leq M_i, \qquad i \in \{1, 2, \cdots, N\}.$$

In addition, the input vector of each reference model is similarly assumed to be bounded, i.e. for all $t \geq t_0$,

$$\|r_i(t)\| \leq \bar{r}_i, \qquad i \in \{1, 2, \cdots, N\}$$

where $\bar{r}_i$ is any positive constant.

As pointed out in [6], not all models of the form given in (6) can be tracked by a corresponding subsystem given in (1) with a feedback controller. Similar to [6], in this paper, the requirement for the developed decentralized local controller to track the model described by (6) is the existence of the matrices $G_i \in \mathbb{R}^{n_i \times \hat{n}_i}$, $H_i \in \mathbb{R}^{m_i \times \hat{n}_i}$, $F_i \in \mathbb{R}^{m_i \times \hat{m}_i}$, such that for each $i \in \{1, 2, \ldots, N\}$, the following matrix algebraic equation holds.

$$\begin{bmatrix} A_i & B_i & 0 \\ 0 & 0 & B_i \\ C_i & 0 & 0 \end{bmatrix} \begin{bmatrix} G_i \\ H_i \\ F_i \end{bmatrix} = \begin{bmatrix} G_i \hat{A}_i \\ G_i \hat{B}_i \\ \hat{C}_i \end{bmatrix}. \tag{7}$$

For each $i \in \{1, 2, \ldots, N\}$, if a solution cannot be found to satisfy this algebraic matrix equation, a different model or output matrix C_i must be chosen. In particular, the approach to finding the solution of the algebraic matrix equation similar to (7) is also discussed in detail in [1,8].

Now, the question is how to synthesize a decentralized local state feedback controller $\bar{u}_i(t)$ such that the output $y_i(t)$ of each time-delay subsystem follows the output $\hat{y}_i(t)$ of the corresponding reference model without time-delay.

Remark 2.1. For the model following problem of uncertain composite dynamical systems, some robust state (or output) feedback tracking controllers are presented in the control literature (see, e. g. [6, 11, 15] for uncertain systems without time-delay, and [8, 10, 16] for uncertain time-delay systems, and the references therein). In particular, in a recent paper [9], the model following problem of uncertain large scale interconnected systems has been discussed. However, few efforts are made to consider the problem of decentralized robust tracking and model following for uncertain large scale systems with time-delay, because of its complexity. In this paper, we will consider the problem of robust tracking and model following for a class of large scale systems with uncertainties, delayed state perturbations, and external disturbances, and want to propose decentralized robust tracking controller.

Before proposing our decentralized robust tracking controllers, we introduce for system (1) the following standard assumptions.

Assumption 2.1. The pairs (A_i, B_i), $i = 1, 2, \cdots, N$, given in system (1) are completely controllable.

Assumption 2.2. For all $(x, t) \in \mathbb{R}^n \times \mathbb{R}$, there exist some continuous and bounded matrix functions $N_i(\cdot)$, $E_i(\cdot)$ of appropriate dimensions such that

$$\begin{aligned} \Delta A_i(v_i, t) &= B_i N_i(v_i, t), \qquad i = 1, 2, \ldots, N \\ \Delta B_i(\xi_i, t) &= B_i E_i(\xi_i, t), \qquad i = 1, 2, \ldots, N. \end{aligned}$$

Remark 2.2. It is obvious that Assumption 2.2 defines the matching condition about the uncertainties of the isolated subsystems, and is a rather standard assumption for robust control problem (see, e. g. [1, 2, 6, 8, 9, 10, 11, 13, 14, 16, 17], and the references therein). For a dynamical system with matched uncertainties, one can always design some types of state (or output) feedback controllers such that the stability of the system can be guaranteed. It is well known that these matching conditions restrict the structure of each subsystem by stipulating that all uncertainties and interconnections should fall into the range space of the control vector B_i. However, this fact is true for a large class of systems, particularly mechanical systems.

For convenience, we now introduce the following notations which represent the bounds of the uncertainties and external disturbances.

$$\begin{aligned}
\rho_i(t) &:= \max_{\upsilon_i} \big\| N_i(\upsilon_i, t) \big\| \\
\kappa_i(t) &:= \max_{\xi_i} \big\| E_i(\xi_i, t) \big\| \\
\mu_i(t) &:= \min_{\xi_i} \left[\frac{1}{2} \lambda_{\min} \left(E_i(\xi_i, t) + E_i^\top(\xi_i, t) \right) \right] \\
\tilde{w}_i(t) &:= \max_{\nu_i} \big\| w_i(\nu_i, t) \big\| \\
\rho_{ij}(t) &:= \max_{\zeta_i} \big\| A_{ij}(\zeta_i, t) \big\|, \qquad j = 1, 2, \ldots, N
\end{aligned}$$

where $i \in \{1, 2, \ldots, N\}$, $\|\cdot\|$ is the spectral norm of a matrix, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the matrix, respectively. Here, the functions $\rho_i(t)$, $\kappa_i(t)$, $\mu_i(t)$, $\tilde{w}_i(t)$, $\rho_{ij}(t)$ are assumed, without loss of generality, to be uniformly continuous with respect to time.

By employing the notations given above, we introduce for uncertain large scale system (1) the following standard assumption.

Assumption 2.3. For every $t \geq t_0$, $\mu_i(t) > -1$, $i \in \{1, 2, \ldots, N\}$.

Remark 2.3. It is worth pointing out that for the uncertain large scale interconnected system described by (1) and (5), Assumption 2.3 is standard. It is well known that the assumption mentioned in Assumption 2.3 is a necessary condition for robust stability of uncertain dynamical systems (see, e. g., [6, 8, 9, 10, 15, 16, 18] and the references relative to robust stabilization of uncertain systems).

On the other hand, it follows from Assumption 2.1 that for any given symmetric positive definite matrix $Q_i \in \mathbb{R}^{n_i \times n_i}$, there exists an unique symmetric positive definite matrix $P_i \in \mathbb{R}^{n_i \times n_i}$ as the solution of the algebraic Riccati equation of the form

$$A_i^\top P_i + P_i A_i - \eta_i P_i B_i B_i^\top P_i = -Q_i, \qquad i = 1, 2, \ldots, N \tag{8}$$

where η_i, $i \in \{1, 2, \ldots, N\}$ is any given positive constant. Moreover, we introduce the following notations:

$$\sigma_i \;:=\; \sqrt{\lambda_{\max}(P_i)/\lambda_{\min}(P_i)}, \qquad i = 1, 2, \ldots, N$$

which will be used in the control law proposed later.

In the remainder of this section, we introduce for time-delay system the following lemma (see, e. g. Theorem 4.3 of [5], Chapter 5]) which will be used in the subsequent sections.

Lemma 2.1. (Hale [4], Hale and Lunel [5]) Consider the retarded functional differential equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} \;=\; f(t,\, x_t) \tag{9}$$

with the initial condition

$$x(t) \;=\; \psi(t), \quad t \in [t_0 - \bar{h},\, t_0].$$

Suppose that the functions $\gamma_i(\cdot)$, $i = 1, 2, 3$, are of K-class. If there is a continuous function $V(\cdot) :\ [t_0 - \bar{h},\, \infty) \times \mathbb{R}^n \to \mathbb{R}^+$ such that

i) for any $t \in [t_0 - \bar{h},\, \infty)$ and $x \in \mathbb{R}^n$,

$$\gamma_1(\|x\|) \;\leq\; V(t, x) \;\leq\; \gamma_2(\|x\|);$$

ii) there is a continuous non-decreasing function $p(s) > s$ for $s > 0$, such that

$$\frac{\mathrm{d}V(t, x)}{\mathrm{d}t} \;\leq\; -\gamma_3(\|x\|)$$

if for any $\xi \in [t - \bar{h},\, t]$ and $t \geq t_0$,

$$V(\xi, x(\xi)) \;<\; p[V(t, x(t))],$$

then the solutions to (9) are uniformly asymptotically stable.

3. DECENTRALIZED ROBUST TRACKING CONTROLLERS

In this section, we propose a class of decentralized local memoryless state feedback controllers which can guarantee that the output $y_i(t)$ of each time-delay subsystem follows the output $\hat{y}_i(t)$ of the corresponding local reference model without time-delay and the tracking error decreases asymptotically to zero. For this, let the tracking error between each subsystem and the corresponding local reference model be defined as

$$e_i(t) \;=\; y_i(t) - \hat{y}_i(t), \qquad i \in \{1, 2, \ldots, N\} \tag{10}$$

then the decentralized state feedback tracking control laws can be constructed as

$$\bar{u}_i(t) \;=\; H_i\hat{x}_i(t) + F_i r_i(t) + \tilde{p}_i(t), \qquad i \in \{1, 2, \ldots, N\} \tag{11}$$

where $H_i \in \mathbb{R}^{m_i \times \hat{n}_i}$ and $F_i \in \mathbb{R}^{m_i \times \hat{m}_i}$ are assumed to satisfy the matrix equation described by (7), and $\tilde{p}_i(t)$ is auxiliary control function which will be given later.

Here, we first define for each subsystem a new state vector $z_i(t)$, called the auxiliary state, as follows:

$$z_i(t) \;:=\; x_i(t) - G_i\hat{x}_i(t) \tag{12}$$

where $G_i \in \mathbb{R}^{n_i \times \hat{n}_i}$ is still assumed to satisfy matrix equation (7).

From (7) and (12) we can obtain the relationship between the tracking error $e_i(t)$ and the auxiliary state vector $z_i(t)$ as follows.

$$e_i(t) \;=\; C_i z_i(t), \qquad i \in \{1, 2, \ldots, N\}. \tag{13}$$

Then, from (13) we can obtain that for any $i \in \{1, 2, \ldots, N\}$, $\|e_i(t)\| \le \|C_i\|\,\|z_i(t)\|$. Since $\|C_i\| < \infty$, it follows that for any $i \in \{1, 2, \ldots, N\}$,

$$\big\|z_i(t)\big\| \to 0 \quad \text{implies} \quad \big\|e_i(t)\big\| \to 0.$$

Therefore, it is sufficient to only consider the stability of $z_i(t),\ i = 1, 2, \ldots, N$.

For each subsystem, applying (11) to (1a) and (5) yields an auxiliary subsystem $\hat{S}_i,\ i \in \{1, 2, \ldots, N\}$, of the form:

$$\begin{aligned}\frac{\mathrm{d}z_i(t)}{\mathrm{d}t} \;=\;& \Big[A_i + \Delta A_i(\upsilon_i, t)\Big] z_i(t) + \Big[B_i + \Delta B_i(\xi_i, t)\Big]\tilde{p}_i(t) \\ &+ \Big[B_i + \Delta B_i(\xi_i, t)\Big] \sum_{j=1}^{N} A_{ij}(\zeta_i, t) z_j(t - h_{ij}(t)) \\ &+ g_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t)\end{aligned} \tag{14}$$

where

$$\begin{aligned}g_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t) :=& \Big[\Delta A_i(\upsilon_i, t)G_i + \Delta B_i(\xi_i, t)H_i\Big]\hat{x}_i(t) + \Delta B_i(\xi_i, t)F_i r_i(t) \\ &+ \Big[B_i + \Delta B_i(\xi_i, t)\Big] \left\{ \sum_{j=1}^{N} A_{ij}(\zeta_i, t) G_j \hat{x}_j(t - h_{ij}(t)) + w_i(\nu_i, t) \right\}.\end{aligned} \tag{15}$$

Then, by making use of the matching condition (see Assumption 2.2), (15) can be readily reduced to

$$g_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t) \;=\; B_i f_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t) \tag{16}$$

where

$$\begin{aligned}f_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t) :=& \Big[N_i(\upsilon_i, t)G_i + E_i(\xi_i, t)H_i\Big]\hat{x}_i(t) + E_i(\xi_i, t)F_i r_i(t) \\ &+ \Big[I_i + E_i(\xi_i, t)\Big] \left\{ \sum_{j=1}^{N} A_{ij}(\zeta_i, t) G_j \hat{x}_j(t - h_{ij}(t)) + w_i(\nu_i, t) \right\}.\end{aligned} \tag{17}$$

Furthermore, we introduce for (17) the following notation.

$$\beta_i(t) \;:=\; \max\Big\{\big\|f_i(\upsilon_i,\xi_i,\zeta_i,\nu_i,r_i,\hat{x}_i,t)\big\| \;:\; (\upsilon_i,\xi_i,\zeta_i,\nu_i)\in\Psi_i,\\ \big\|r_i(t)\big\|\le\bar{r}_i,\;\; \big\|\hat{x}_i(t)\big\|\le M_i,\;\; t\in\mathbb{R}^+\Big\}.$$

Here, the function $\beta_i(t)$, $i\in\{1,2,\dots,N\}$, is still assumed to be uniformly continuous with respect to time.

Now, we give the auxiliary control function $\tilde{p}_i(t)$ as follows.

$$\tilde{p}_i(t) \;=\; p_{i1}(z_i(t),t)+p_{i2}(z_i(t),t), \qquad i=1,2,\dots,N \tag{18a}$$

where $p_{i1}(\cdot)$ and $p_{i2}(\cdot)$ are given by the following functions:

$$p_{i1}(z_i(t),t) \;=\; -\frac{1}{2}\,k_{i1}(t)B_i^\top P_i z_i(t), \tag{18b}$$

$$p_{i2}(z_i(t),t) \;=\; -\frac{k_{i2}(t)B_i^\top P_i z_i(t)}{\big\|B_i^\top P_i z_i(t)\big\|\,\beta_i(t)\;+\;\varepsilon_i\big\|z_i(t)\big\|^2} \tag{18c}$$

and where the control gain functions $k_{i1}(t)$ and $k_{i2}(t)$ are given by

$$k_{i1}(t) \;=\; \frac{\eta_i+\delta_i^2\rho_i^2(t)+(1+\kappa_i(t))^2\sum_{j=1}^N\alpha_j^2\rho_{ij}^2(t)}{1+\mu_i(t)} \tag{18d}$$

$$k_{i2}(t) \;=\; \frac{\beta_i^2(t)}{1+\mu_i(t)} \tag{18e}$$

where ε_i, δ_i, α_j, are positive constants, and δ_i, α_j, are selected such that the following conditions are satisfied.

$$\frac{1}{\delta_i^2}+\frac{\sigma_i^2 N}{\alpha_i^2} \;<\; \lambda_{\min}(Q_i)-2\varepsilon_i. \tag{18f}$$

Here, ε_i has been chosen such that $0<2\varepsilon_i<\lambda_{\min}(Q_i)$.

Remark 3.1. For any $i\in\{1,2,\dots,N\}$, the decentralized memoryless state feedback controller described by (18) consists of two parts, $\tilde{p}_{i1}(\cdot)$ and $\tilde{p}_{i2}(\cdot)$. Here, $\tilde{p}_{i1}(\cdot)$ is linear in the auxiliary state, and $\tilde{p}_{i2}(\cdot)$ is continuous (nonlinear) controller which is employed to compensate for the uncertain $g_i(\cdot)$ including external disturbance to produce an asymptotic stability results for tracking error $e_i(t)$ between uncertain large scale time-delay subsystem and the local reference model.

Remark 3.2. It has been assumed that the matrix functions $N_i(\cdot)$, $E_i(\cdot)$, $w_i(\cdot)$, $A_{ij}(\cdot)$ are continuous in all their arguments. In addition, their bounds $\rho_i(t)$, $\kappa_i(t)$, $\mu_i(t)$, $\tilde{w}_i(t)$, $\rho_{ij}(t)$, as well as $\beta_i(t)$ are also assumed to be uniformly continuous with respect to time. Thus, it is obvious that the nonlinear auxiliary control function

$\tilde{p}_{i2}(z_i(t),\, t)$ described by (18c) is uniformly continuous in with respect to time. Moreover, by noting that for any $(z_i,\, t) \in \mathbb{R}^{n_i} \times \mathbb{R}^+$,

$$\left\|B_i^\top P_i z_i(t)\right\| \beta_i(t) \leq \left\|B_i^\top P_i z_i(t)\right\| \beta_i(t) + \varepsilon_i \left\|z_i(t)\right\|^2$$

we can obtain from (18c) that for any $(z_i,\, t) \in \mathbb{R}^{n_i} \times \mathbb{R}^+$,

$$\left\|\tilde{p}_{i2}(z_i(t),\, t)\right\| \;\leq\; \frac{\beta_i(t)}{1 + \mu_i(t)}$$

which shows the boundedness of the function $\tilde{p}_{i2}(z_i,\, t)$. Therefore, it can be concluded that the auxiliary control function described by (18) is uniformly continuous with respect to time, and and uniformly bounded with respect to z_i.

Remark 3.3. It is worth pointing out that at the origin $z_i = 0$, both numerator and denominator of the control described by (18c) vanish. This implies that the existence of the solutions to the closed-loop auxiliary subsystem given in (14) and (18) may not be guaranteed at the origin $z_i = 0$ in the usual sense. However, by employing the method similar to the one presented in [18], we can easily prove that the right-hand side of the closed-loop auxiliary subsystem given in (14) and (18) is upper semicontinuous on $(z,\, t) \in \mathbb{R}^n \times \mathbb{R}^+$. Therefore, as a generalized dynamical system (GDS), the existence of the solutions to the closed-loop auxiliary subsystem can be well guaranteed. This implies that the limit of control (18c) as z_i approaches zero exists.

Thus, we can obtain the following theorem which shows that by employing the auxiliary controller described in (18), one can guarantee the uniform asymptotic stability of the auxiliary subsystems, described by (14), in the presence of the uncertain parameters and multiple delayed state perturbations in the interconnections.

Theorem 3.1. Consider the auxiliary subsystems, described in (14). Suppose that Assumptions 2.1 to 2.3 are satisfied. Then, by employing the auxiliary decentralized state feedback controllers given in (18), one can guarantee the uniform asymptotic stability of each auxiliary subsystem. That is, for any $t \in \mathbb{R}^+$,

$$\lim_{t\to\infty} \left\|z_i(t)\right\| \;=\; 0, \qquad i = 1, 2, \ldots, N.$$

P r o o f. Applying the controller given in (18) to (14) yields the following closed-loop auxiliary subsystems.

$$\begin{aligned}
\frac{\mathrm{d}z_i(t)}{\mathrm{d}t} \;=\;& \left[A_i - \frac{1}{2}k_{i1}(t) B_i B_i^\top P_i\right] z_i(t) + \Delta A_i(\upsilon_i, t) z_i(t) \\
& -\frac{1}{2}k_{i1}(t)\Delta B_i(\xi_i, t) B_i^\top P_i z_i(t) \\
& +\Big[\, B_i + \Delta B_i(\xi_i, t)\,\Big] p_{i2}(z_i(t), t)
\end{aligned}$$

$$+\Big[B_i + \Delta B_i(\xi_i, t)\Big] \sum_{j=1}^{N} A_{ij}(\zeta_i, t) z_j(t - h_{ij}(t))$$
$$+g_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t). \tag{19}$$

For each nominal subsystem (i. e. the subsystem in the absence of the uncertain parameters, delayed state perturbations, and interconnections) of closed-loop auxiliary subsystem (19), we first define a positive definite function of the form

$$V_i(z_i(t), t) \;=\; z_i^\top(t) P_i z_i(t), \tag{20}$$

where $P_i \in \mathbb{R}^{n_i \times n_i}$ is the solution of algebraic Riccati equation (8).

Let $z_i(t)$ be the solution of the closed-loop auxiliary subsystems described by (19) for $t \geq t_0$; and let the Lyapunov function, described by (20), of the nominal subsystem be a candidate of the Lyapunov function of subsystem (19). Then, we can obtain that for any $t \geq t_0$,

$$\begin{aligned}
\frac{\mathrm{d}V_i(z_{it},\, t)}{\mathrm{d}t} \;=\;& z_i^\top(t)\Big[A_i^\top P_i + P_i A_i - k_{i1}(t) P_i B_i B_i^\top P_i\Big] z_i(t) \\
&+2 z_i^\top(t) P_i \Delta A_i(\upsilon_i, t) z_i(t) \\
&-k_{i1}(t) z_i^\top(t) P_i \Delta B_i(\xi_i, t) B_i^\top P_i z_i(t) \\
&+2 z_i^\top(t) P_i \Big[B_i + \Delta B_i(\xi_i, t)\Big] p_{i2}(z_i(t), t) \\
&+2 z_i^\top(t) P_i \Big[B_i + \Delta B_i(\xi_i, t)\Big] \sum_{j=1}^{N} A_{ij}(\zeta_i, t) z_j(t - h_{ij}(t)) \\
&+2 z_i^\top(t) P_i g_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t).
\end{aligned} \tag{21}$$

From Assumption 2.2, (16), and (18), we can obtain that for any $t \geq t_0$,

$$\begin{aligned}
\frac{\mathrm{d}V_i(z_{it},\, t)}{\mathrm{d}t} \;=\;& z_i^\top(t)\Big[A_i^\top P_i + P_i A_i - k_{i1}(t) P_i B_i B_i^\top P_i\Big] z_i(t) \\
&+2 z_i^\top(t) P_i B_i N_i(\upsilon_i, t) z_i(t) \\
&-k_{i1}(t) z_i^\top(t) P_i B_i \left[\frac{1}{2}\big(E_i(\xi_i, t) + E_i^\top(\xi_i, t)\big)\right] B_i^\top P_i z_i(t) \\
&-\frac{2 k_{i2}(t) z_i^\top(t) P_i B_i \Big[I_i + E_i(\xi_i, t)\Big] B_i^\top P_i z_i(t)}{\big\|B_i^\top P_i z_i(t)\big\|\, \beta_i(t) \;+\; \varepsilon_i \big\|z_i(t)\big\|^2} \\
&+2 z_i^\top(t) P_i B_i \Big[I_i + E_i(\xi_i, t)\Big] \sum_{j=1}^{N} A_{ij}(\zeta_i, t) z_j(t - h_{ij}(t)) \\
&+2 z_i^\top(t) P_i B_i f_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t) \\
\leq\;& z_i^\top(t)\Big[A_i^\top P_i + P_i A_i - k_{i1}(t)\big(1 + \mu_i(t)\big) P_i B_i B_i^\top P_i\Big] z_i(t)
\end{aligned}$$

$$
\begin{aligned}
&+2\rho_i(t)\left\|B_i^\top P_i z_i(t)\right\|\left\|z_i(t)\right\| \\
&-\frac{2k_{i2}(t)\big(1+\mu_i(t)\big)z_i^\top(t)P_iB_iB_i^\top P_i z_i(t)}{\left\|B_i^\top P_i z_i(t)\right\|\beta_i(t)\ +\ \varepsilon_i\left\|z_i(t)\right\|^2} \\
&+2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N}\rho_{ij}(t)\left\|B_i^\top P_i z_i(t)\right\|\left\|z_j(t-h_{ij}(t))\right\| \\
&+2\beta_i(t)\left\|B_i^\top P_i z_i(t)\right\|. \qquad (22)
\end{aligned}
$$

Then, from (8), (18d), and (18e) we can further obtain that for any $t \geq t_0$,

$$
\begin{aligned}
\frac{\mathrm{d}V_i(z_{it},\,t)}{\mathrm{d}t} \leq\ & -z_i^\top(t)Q_i z_i(t) \\
& -\left(\delta_i^2\rho_i^2(t)+(1+\kappa_i(t))^2\sum_{j=1}^{N}\alpha_j^2\rho_{ij}^2(t)\right)\left\|B_i^\top P_i z_i(t)\right\|^2 \\
& +2\rho_i(t)\left\|B_i^\top P_i z_i(t)\right\|\left\|z_i(t)\right\| \\
& -\frac{2\beta_i^2(t)\left\|B_i^\top P_i z_i(t)\right\|^2}{\left\|B_i^\top P_i z_i(t)\right\|\beta_i(t)\ +\ \varepsilon_i\left\|z_i(t)\right\|^2}+2\beta_i(t)\left\|B_i^\top P_i z_i(t)\right\| \\
& +2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N}\rho_{ij}(t)\left\|B_i^\top P_i z_i(t)\right\|\left\|z_j(t-h_{ij}(t))\right\| \\
=\ & -z_i^\top(t)Q_i z_i(t)-(1+\kappa_i(t))^2\sum_{j=1}^{N}\alpha_j^2\rho_{ij}^2(t)\left\|B_i^\top P_i z_i(t)\right\|^2 \\
& -\delta_i^2\rho_i^2(t)\left\|B_i^\top P_i z_i(t)\right\|^2+2\rho_i(t)\left\|B_i^\top P_i z_i(t)\right\|\left\|z_i(t)\right\| \\
& -\frac{2\beta_i^2(t)\left\|B_i^\top P_i z_i(t)\right\|^2}{\left\|B_i^\top P_i z_i(t)\right\|\beta_i(t)\ +\ \varepsilon_i\left\|z_i(t)\right\|^2}+2\beta_i(t)\left\|B_i^\top P_i z_i(t)\right\| \\
& +2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N}\rho_{ij}(t)\left\|B_i^\top P_i z_i(t)\right\|\left\|z_j(t-h_{ij}(t))\right\| \\
=\ & -z_i^\top(t)Q_i z_i(t)-(1+\kappa_i(t))^2\sum_{j=1}^{N}\alpha_j^2\rho_{ij}^2(t)\left\|B_i^\top P_i z_i(t)\right\|^2 \\
& -\left[\delta_i\rho_i(t)\left\|B_i^\top P_i z_i(t)\right\|-\frac{1}{\delta_i}\left\|z_i(t)\right\|\right]^2+\frac{1}{\delta_i^2}\left\|z_i(t)\right\|^2 \\
& +2\frac{\left\|B_i^\top P_i z_i(t)\right\|\beta_i(t)\ \cdot\ \varepsilon_i\left\|z_i(t)\right\|^2}{\left\|B_i^\top P_i z_i(t)\right\|\beta_i(t)\ +\ \varepsilon_i\left\|z_i(t)\right\|^2}
\end{aligned}
$$

$$
\begin{aligned}
& +2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N}\rho_{ij}(t)\,\big\|B_i^\top P_i z_i(t)\big\|\,\big\|z_j(t-h_{ij}(t))\big\| \\
\le\; & -z_i^\top(t)Q_i z_i(t)+\frac{1}{\delta_i^2}\big\|z_i(t)\big\|^2 \\
& -(1+\kappa_i(t))^2\sum_{j=1}^{N}\alpha_j^2\rho_{ij}^2(t)\,\big\|B_i^\top P_i z_i(t)\big\|^2 \\
& +2\frac{\big\|B_i^\top P_i z_i(t)\big\|\,\beta_i(t)\;\cdot\;\varepsilon_i\big\|z_i(t)\big\|^2}{\big\|B_i^\top P_i z_i(t)\big\|\,\beta_i(t)\;+\;\varepsilon_i\big\|z_i(t)\big\|^2} \\
& +2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N}\rho_{ij}(t)\,\big\|B_i^\top P_i z_i(t)\big\|\,\big\|z_j(t-h_{ij}(t))\big\|. \qquad (23)
\end{aligned}
$$

Therefore, it follows from (23) and from the inequality

$$0\;\le\;\frac{ab}{a+b}\;\le\;a,\qquad \forall\,a,\;b\;>\;0$$

that for any $t\ge t_0$,

$$
\begin{aligned}
\frac{\mathrm{d}V_i(z_{it},\,t)}{\mathrm{d}t}\;\le\; & -z_i^\top(t)Q_i z_i(t)+\left(2\varepsilon_i+\frac{1}{\delta_i^2}\right)\big\|z_i(t)\big\|^2 \\
& -(1+\kappa_i(t))^2\sum_{j=1}^{N}\alpha_j^2\rho_{ij}^2(t)\,\big\|B_i^\top P_i z_i(t)\big\|^2 \\
& +2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N}\rho_{ij}(t)\,\big\|B_i^\top P_i z_i(t)\big\|\,\big\|z_j(t-h_{ij}(t))\big\|. \qquad (24)
\end{aligned}
$$

In the light of the Razumikhin-type theorem (see, e.g. Lemma 2.1), for each subsystem we assume that, for any positive number $q_i>1$, the following inequality holds.

$$V_i(z_i(\xi),\xi)\;<\;q_i^2V_i(z_i(t),t),\qquad \xi\in[t-\bar h_i,\;t]$$

where $i\in\{1,2,\dots,N\}$. Then, it follows from (20) and the property of the matrix P_i that

$$\big\|z_i(\xi)\big\|\;<\;q_i\sigma_i\big\|z_i(t)\big\|,\qquad \xi\in[t-\bar h_i,\;t]. \qquad (25)$$

By substituting (25) into (24) we can obtain that for any $t\ge t_0$,

$$
\begin{aligned}
\frac{\mathrm{d}V_i(z_{it},\,t)}{\mathrm{d}t}\;\le\; & -z_i^\top(t)Q_i z_i(t)+\left(2\varepsilon_i+\frac{1}{\delta_i^2}\right)\big\|z_i(t)\big\|^2 \\
& -(1+\kappa_i(t))^2\sum_{j=1}^{N}\alpha_j^2\rho_{ij}^2(t)\,\big\|B_i^\top P_i z_i(t)\big\|^2
\end{aligned}
$$

$$
\begin{aligned}
& +2\big(1+\kappa_i(t)\big)\sum_{j=1}^{N} q_j\sigma_j\rho_{ij}(t)\,\big\|B_i^\top P_i z_i(t)\big\|\,\big\|z_j(t)\big\| \\
\le\quad & -\lambda_{\min}(Q_i)\,\big\|z_i(t)\big\|^2 + \left(2\varepsilon_i + \frac{1}{\delta_i^2}\right)\big\|z_i(t)\big\|^2 \\
& +\sum_{j=1}^{N}\frac{\big(q_j\sigma_j\big)^2}{\alpha_j^2}\big\|z_j(t)\big\|^2. \qquad (26)
\end{aligned}
$$

Letting

$$
V(z_t,\, t) \;:=\; \sum_{i=1}^{N} V_i(z_{it},\, t)
$$

form (26) we obtain that for any $t \ge t_0$,

$$
\frac{\mathrm{d}V(z_t,\, t)}{\mathrm{d}t} \;\le\; -\sum_{i=1}^{N}\left(\lambda_{\min}(Q_i) - 2\varepsilon_i - \frac{1}{\delta_i^2}\right)\big\|z_i(t)\big\|^2 + \sum_{i=1}^{N}\frac{N\big(q_i\sigma_i\big)^2}{\alpha_i^2}\big\|z_i(t)\big\|^2. \quad (27)
$$

It is obvious from the definition of $V(z_t,\, t)$ that, if an inequality of the form

$$
\frac{\mathrm{d}V_i(z_{it},\, t)}{\mathrm{d}t} \;\le\; -\left(\lambda_{\min}(Q_i) - 2\varepsilon_i - \frac{1}{\delta_i^2} - \frac{N\big(q_i\sigma_i\big)^2}{\alpha_i^2}\right)\big\|z_i(t)\big\|^2 \qquad (28)
$$

holds for any $i \in \{1, 2, \ldots, N\}$, the inequality described by (27) is also satisfied. Therefore, from (28), for each $i \in \{1, 2, \ldots, N\}$, one can obtain that for any $t \ge t_0$,

$$
\frac{\mathrm{d}V_i(z_{it},\, t)}{\mathrm{d}t} \;\le\; -\gamma_i(q_i)\big\|z_i(t)\big\|^2, \qquad i = 1, 2, \ldots, N \qquad (29)
$$

where $\gamma_i(q_i)$ is defined by

$$
\gamma_i(q_i) \;:=\; \lambda_{\min}(Q_i) - 2\varepsilon_i - \left[\frac{1}{\delta_i^2} + \frac{\big(q_i\sigma_i\big)^2 N}{\alpha_i^2}\right]. \qquad (30)
$$

If the parameters δ_i and α_i, $i = 1, 2, \ldots, N$, are selected such that (18f) is satisfied, then a sufficiently small $q_i^* > 1$ exists such that $\gamma_i(q_i^*) > 0$, $i \in \{1, 2, \ldots, N\}$. Thus, according to Lemma 2.1, each closed-loop auxiliary subsystem $\hat{S}_i$, $i \in \{1, 2, \ldots, N\}$, described by (14) and (18), is uniformly asymptotically stable. That is, the auxiliary state $z_i(t)$, $i \in \{1, 2, \ldots, N\}$, tends asymptotically to zero. □

Thus, from Theorem 3.1 we can obtain the following theorem which shows that by employing the decentralized local memoryless state feedback controllers described in (11) with (18), one can guarantee the zero-tracking errors between each subsystem with time-delay and the local reference model without time-delay.

Theorem 3.2. Consider the model following problem of uncertain large scale system (1) with (5) satisfying Assumptions 2.1 to 2.3. Then, by using the decentralized local memoryless state feedback controllers $\bar{u}_i(t)$ described in (11) with (18), one can guarantee that the tracking error $e_i(t),\ i \in \{1, 2, \ldots, N\}$, between each subsystem and local reference model, decreases uniformly asymptotically to zero.

P r o o f. From Theorem 3.1, we have shown that each closed-loop auxiliary subsystem described by (14) with (18) is uniformly asymptotically stable. That is, for the auxiliary state $z_i(t)$ of each subsystem, we can obtain that

$$\lim_{t\to\infty} \|z_i(t)\| = 0 \qquad i \in \{1, 2, \ldots, N\}.$$

Then, it can easily be obtained from the relationship between $e_i(t)$ and $z_i(t)$, i.e. $e_i(t) = C_i z_i(t)$, that each local tracking error $e_i(t),\ i \in \{1, 2, \ldots, N\}$, also decreases uniformly asymptotically to zero. □

Remark 3.4. Similar to [6], we can give a procedure for constructing the decentralized state feedback controller described by (11) with (18) as follows.

(i) Find the solutions G_i, H_i, F_i of algebraic matrix equation (7). If no solution exists, then a different choice of the local reference model or the output matrix C_i must be made.

(ii) Solve, for any given positive constant η_i and positive definite matrix Q_i, algebraic Riccati equation (8) for P_i, $i \in \{1, 2, \ldots, N\}$.

(iii) Evaluate the bounds of the uncertain $N_i(\cdot)$, $E_i(\cdot)$, $w_i(\cdot)$, $A_{ij}(\cdot)$, to obtain $\rho_i(\cdot)$, $\kappa_i(\cdot)$, $\mu_i(\cdot)$, $\tilde{w}_i(\cdot)$, $\rho_{ij}(\cdot)$, as well as $\beta_i(\cdot)$.

(iv) Select a set of the control parameters ε_i, δ_i, α_i, such that the inequality described by (18f) holds.

(v) Form the decentralized memoryless state feedback tracking controller described by (11) with (18).

4. CONCLUDING REMARKS

The problem of the decentralized robust tracking and model following has been considered for a class of uncertain large scale systems including time-varying delays in the interconnections. A class of decentralized memoryless local state feedback controllers has been proposed for robust tracking of dynamical signals. In the light of the auxiliary control functions, the proposed decentralized robust tracking controller consists of two parts, i.e. linear and nonlinear. The nonlinear controller is continuous and bounded, and is used to compensate for the uncertainty including the external disturbance of the systems to produce an asymptotic tracking result. That is, by employing the proposed decentralized robust tracking controllers, one can guarantee that the tracking error between each time-delay subsystem and the corresponding local reference model decreases uniformly asymptotically to zero.

Moreover, since the time-varying delays have been assumed to be any continuous and bounded nonnegative functions, and the proposed decentralized robust tracking controllers are independent of the delays, the results obtained in the paper are applicable to large scale systems without exact knowledge of the delays, i. e. large systems with perturbed delays. Therefore, our results may be expected to have some applications to practical decentralized robust tracking and model following problems of uncertain large scale time-delay systems.

(Received March 7, 2008.)

REFERENCES

[1] B. Brogliato and A. Trofino Neto: Practical stabilization of a class of nonlinear systems with partially known uncertainties. Automatica *31* (1995), 145–150.

[2] C. H. Choi and H. S. Kim: Adaptive regulation for a class of uncertain systems with partial knowledge of uncertainty bounds. IEEE Trans. Automat. Control *38* (1993), 1246–1250.

[3] S. Gutman: Uncertain dynamical systems – A Lyapunov min-max approach. IEEE Trans. Automat. Control *24* (1979), 437–443.

[4] J. K. Hale: Theory of Functional Differential Equations. Springer-Verlag, New York 1977.

[5] J. K. Hale and S. M. V. Lunel: Introduction to Functional Differential Equations. Springer-Verlag, New York 1993.

[6] T. H. Hopp and W. E. Schmitendorf: Design of a linear controller for robust tracking and model following. ASME J. Dyn. Syst. Measurement Control *112* (1990), 552–558.

[7] V. B. Kolmanovskii and V. R. Nosov: Stability of Functional Differential Equations. Academic Press, London 1986.

[8] S. Oucheriah: Robust tracking and model following of uncertain dynamic delay systems by memoryless linear controllers. IEEE Trans. Automat. Control *44* (1999), 1473–1577.

[9] S. Shigemaru and H. Wu: Decentralized robust tracking and model following for uncertain large-scale interconnected systems. J. Optim. Theory Appl. *110* (2001), 35–52.

[10] K. K. Shyu and Y. C. Chen: Robust tracking and model following for uncertain time-delay systems. Internat. J. Control *62* (1995), 589–600.

[11] T. Sugie and K. Osuka: Robust model following control with prescribed accuracy for uncertain nonlinear systems. Internat. J. Control *58* (1993), 991–1009.

[12] H. Wu: Eigenstructure assignment based robust stability conditions for uncertain systems with multiple time-varying delays. Automatica *33* (1997), 97–102.

[13] H. Wu: Continuous adaptive robust controllers guaranteeing uniform ultimate boundedness for uncertain nonlinear systems. Internat. J. Control *72* (1999), 115–122.

[14] H. Wu: Adaptive stabilizing state deedback controllers of uncertain dynamical systems with multiple time delays. IEEE Trans. Automat. Control *45* (2000), 1697–1701.

[15] H. Wu: Robust tracking and model following control with zero-tracking error for uncertain dynamical systems. J. Optim. Theory Appl. *107* (2000), 169–182.

[16] H. Wu: Design of memoryless controllers for robust tracking and model following of uncertain systems with multiple time delays. IEEJ Trans. Electronics, Informat. Systems *121* (2001), 1268–1276.

[17] H. Wu: Decentralized adaptive robust control for a class of large-scale systems including delayed state perturbations in the interconnections. IEEE Trans. Automat. Control *47* (2002), 1745–1751.

[18] H. Wu: Adaptive robust tracking and model following of uncertain dynamical systems with multiple time delays. IEEE Trans. Automat. Control *49* (2004), 611–616.

[19] H. Wu and K. Mizukami: Robust stability criteria for dynamical systems including delayed perturbations. IEEE Trans. Automat. Control *40* (1995), 487–490.

[20] H. Wu and K. Mizukami: Linear and nonlinear stabilizing continuous controllers of uncertain dynamical systems including state delay. IEEE Trans. Automat. Control *41* (1996), 116–121.

Hansheng Wu, Department of Information Science, Prefectural University of Hiroshima, Hiroshima City, Hiroshima 734–8558. Japan.
e-mail: hansheng@pu–hiroshima.ac.jp