Mathematica Slovaca

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Mathematica Slovaca, Vol. 44 (1994), No. 4, 483--487

Persistent URL: http://dml.cz/dmlcz/136619

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SEVENTY YEARS OF PROFESSOR JÁN JAKUBÍK

Professor Ján Jakubík, a prominent Slovak mathematician, reached seventy years of age on 8 October 1993.

Ján Jakubík was born in Dudince (Slovakia). After having completed his secondary school education at Banská Štiavnica, he studied Mathematics and Physics at the Comenius University in Bratislava. Then he worked at the Slovak University of Technology in Bratislava (1948-1952) and at the University of Technology in Košice (1952-1985). Since 1985, he has been a member of the Mathematical Institute of the Slovak Academy of Sciences. He was appointed Associate Professor (1956) and Full Professor (1963). He was elected corresponding member of the Slovak Academy of Sciences (1964) and of the Czechoslovak Academy of Sciences (1965). In 1977 he was elected ordinary member both of the Slovak Academy of Sciences and the Czechoslovak Academy of Sciences.

His work was already appraised in Czechoslovak Mathematical Journal [h] and in Mathematica Slovaca [b] ten years ago. In the present article we will focus our attention only on the results published between 1984 and 1993.

J. Jakubík scientific research during the considered period can be divided into the following three domains: (a) ordered sets; (b) ordered groups; (c) convergences in ordered structures. The main results concern:(a) directed products, intervals and antichains; (b) radical classes, torsion classes and retracts; (c) sequential convergences in Boolean algebras and in l-groups.

Let us briefly characterize some typical results in the mentioned directions:

To the direction (a):

Let X be a finite partially ordered set. We denote by $MA(X)$ the system of all maximal antichains in X. This system is considered to be partially ordered by the set-theoretical inclusion. Then $MA(X)$ is a lattice. G. Behrendt [a] proposed the question to find an interval characterization of those partially ordered sets for which the lattice $MA(X)$ is distributive or modular, respectively. In [154] and [146], the mentioned questions are solved; the desired characterizations consist in giving certain types of convex subsets in X which are forbidden.

For a partially ordered set P, we denote by $\operatorname{Int} P$ the system of all intervals of P, including the empty set. The system $\operatorname{Int} P$ is partially ordered by the set-theoretical inclusion. If P is a lattice, then $\operatorname{Int} P$ is a lattice as well. V. I. Igoshin [f] proposed the problem whether there exists an infinite lattice L such that $\operatorname{Int} L$ is selfdual. In [144], it was shown that the answer is negative. Namely, the following result was proved: let P be a partially ordered set with card $P > 4$. Then the partially ordered system Int P is not selfdual.

Let C_o be the class of all lattices of finite length which are determined up to isomorphisms by their graphs. In [111], the following results are proved: the class C_o is closed with respect to weak direct products; a lattice L belongs to C_o if and only if all directly indecomposable direct factors of L belong to C_o; each lattice can be embedded into a lattice belonging to C_o. For a lattice L, we denote by $T(L)$ the set of all nonisomorphic types of lattices whose graphs are isomorphic to the graph of L. Using weak direct product decomposition, it is proved that for each cardinal α there exists a lattice L with $\operatorname{card} T(L) \geqslant \alpha$.

To the direction (b):

In 1977, J. Jakubík [g] introduced the concept of a radical class, which is wider than the concept of a torsion class. M. Darnel [c] has proved that for any radical class there exist unique radical classes R^s and R^h, which are closed with respect to l-subgroups and l-homomorphic images, respectively, and contain R. He asked the question whether or not the relation $R^{sh} = R^{hs}$ is valid for each radical class R. The negative answer is obtained in [117]. Let R_1 be the collection of all radical classes for which the above equation fails to hold. It is shown that the collection R_1 is large: there exists an injective mapping of the class of all cardinals into the collection R_1.

Results concerning the lattice $R(G)$ of all radical subgroups of a complete lattice ordered groups are presented in [114]; we mention some of them. For each cardinal $\alpha > 0$ there is a proper class A_α of mutually nonisomorphic complete lattice ordered groups such that for each $G \in A_\alpha$, $R(G)$ is isomorphic to the Boolean algebra 2^α; for each complete lattice ordered group G there exists a complete lattice ordered group G_1 such that $G \in R(G_1)$ and G is covered by G_1 in the lattice $R(G_1)$. Further it is proved that the lattice of all radical classes of a complete lattice ordered group is a Stone lattice.

Retracts of partially ordered sets were investigated, e.g., by D. Duffus and I. Rival [d]. Retracts of abelian lattice ordered groups were dealed with in [130] and [135]. Let an abelian l-group G be an internal direct product of its l-subgroups $G_i (i \in I)$. Relations between retracts of G and retracts of G_i were described in [130]. In [135], some properties of the collection R of all retract varieties of abelian l-groups, were established, e.g.,: R behaves as a complete lattice; R is a Brouwer lattice; the collection of all principal retract varieties is an ideal of R; R has a large collection of atoms but no dual atom.

To the direction (c):

Let $\operatorname{Conv} G$ be the partially ordered set of all convergences of an abelian lattice ordered group G in the sense of [e]. If B is a Boolean algebra, then $\operatorname{Conv} B$ is defined analogously. In general, neither $\operatorname{Conv} G$ nor $\operatorname{Conv} B$ need be a lattice. $\operatorname{Conv} G$ ($\operatorname{Conv} B$) is a lattice if and only if it possesses a greatest element and in such a case it is a complete lattice. In [134], it was shown that the existence of the greatest element in $\operatorname{Conv} G$ depends merely on the lattice properties of G, and that the class of all lattice ordered groups having the largest convergence is a radical class. The following results were proved in [125] and [126]: if G is a completely distributive archimedean lattice ordered group, then $\operatorname{Conv} G$ is a complete lattice; if B is a completely distributive Boolean algebra, then $\operatorname{Conv} B$ is a complete lattice. In [140], the author found further sufficient conditions for $\operatorname{Conv} G$ ($\operatorname{Conv} B$) to be a complete lattice; these conditions are expressed by means of higher degrees of distributivity of a lattice ordered group G (Boolean algebra B). Let α be a cardinal, and let $G(\alpha)$ be the free lattice ordered group with α free generators. In [151], it is proved that if $\alpha \geqslant 2$, then $G(\alpha)$ admits a nontrivial convergence. If $\alpha = 1$, then $G(\alpha)$ has no nontrivial convergence.

The activity of Prof. Jakubík includes many reports and reviews. He took a considerable share in organizing science in Slovakia and in Czechoslovakia, while acting in several scientific committees and boards of Academies.

He founded and led a seminar on algebra in which mathematicians from Departments of Mathematics in Košice have been working for more than 30 years. He educated several specialists in algebra who continue with research of ordered structures.

During his activity at the University of Technology in Košice he paid a great attention to pedagogical work. His lectures were appreciated for clear and logical structure. His students remember that period with gratitude.

Prof. J. Jakubík is a distinguished companion in personal meetings. All who know him appreciate his modesty, deliberation and his sensitive humane approach.

The 70th anniversary of Prof. J. Jakubík encounters him in full creative potential. The authors of this article thank him in the name of the whole mathematical community for his considerable contribution to the development of algebra and they wish him good health and happiness in his personal life and many results in his further mathematical research.

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SUPLEMENT TO THE LIST OF SCIENTIFIC PAPERS OF J. JAKUBÍK[1]

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