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EIGHTY YEARS OF PROFESSOR ŠTEFAN SCHWARZ

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A prominent Slovak mathematician, Professor Štefan Schwarz, Doctor of Science, an outstanding specialist in the theory of semigroups and in number theory, reached eighty years of age on May 18, 1994 and continues his creative work in mathematics.

He was born at Nové Mesto nad Váhom. After having completed his studies at Charles University in Prague in 1936, he became Assistant Professor at the Mathematical Institute of the Faculty of Science, Charles University, and later in 1939, he became a member of the staff of the newly established Slovak Technical University.

In 1944, he was deported and imprisoned in a German concentration camp, from where he was liberated in 1945. Two of his sisters died in such camps.

In 1946 he was appointed Associate Professor at the Faculty of Science in Bratislava and since 1947 he has been Full Professor at the Slovak Technical University. He was elected corresponding member and ordinary member (Academician) of the Czechoslovak Academy of Sciences in 1952 and 1960, respectively. In 1953 he also became a member of the then established Slovak Academy of Sciences. Since 1966 until 1988 he has been Head of the Mathematical Institute of this Academy. In the years 1965–1970, he was President of the Slovak Academy of Sciences and Vice-President of the Czechoslovak Academy of Sciences.

The scientific papers of Štefan Schwarz concern the following regions:

a) theory of semigroups and its applications in other fields of mathematics;
b) theory of finite fields;
c) number theory;
d) non-negative matrices and Boolean matrices.

The results of Š t e f a n S c h w a r z have been incorporated and reproduced in detail in several monographs; in particular, cf. [1a] and [2a] for the case of algebraic semigroups, [3a] for topological semigroups, [4a] for Boolean matrices and [5a] for the theory of finite fields.

His work was appraised twenty years ago and ten years ago in the Czechoslovak Mathematical Journal, Časopis pro pěstování matematiky, Matematický časopis and Mathematica Slovaca; cf., e.g., [1c], [2c], [3c].

The purpose of the present article is to characterize the papers of Štefan Schwarz which have been published after 1984.

For a semigroup S having a minimal left ideal let K be the union of all minimal left ideals of S. Schmetterer proposed the problem of characterizing those semigroups for which Ka is a minimal left ideal of S for each $a \in S$. In [94], Š. Schwarz finds such a characterization; a semigroup has this property if and only if it can be obtained by a right composition of a special type of semigroups (called U_ℓ-semigroups). He also obtains several results concerning the converse problem: to decide whether a given family of U_ℓ-semigroups admits at least one right composition.

The results of the paper [96] concern the structure of semigroups S such that

(i) S contains a universally minimal left ideal L (i.e., the least left ideal L) which is a left group, and
(ii) S has an L-homomorphism.

In the paper [92], the author continues the study of the semigroup of binary relations on a finite set which was begun in [72] and [73]. At the same time he shows that there exists a close connection of the question under consideration with the theory of Boolean matrices and the theory of directed graphs. Let $B_n(V)$ be the semigroup of all binary relations on a finite V with n elements, where $n \geqq 2$. Further, let M_n be the set of all $n \times n$ matrices over the Boolean algebra $\{0, 1\}$; M_n is a semigroup under Boolean matrix multiplication. Finally, let $G_n(V)$ be the set of all directed graphs having the set V as the set of all vertices with allowable loops and simple directed edges. There exists a natural one-to-one correspondence between the sets $B_n(V)$, M_n and $G_n(V)$. By applying this correspondence and by a masterful synthesis of methods belonging to semigroup theory, theory of Boolean matrices, and graph theory, Š. Schwarz managed to solve a combinatorial problem which was proposed by A. Paz in his monograph [6a] on probabilistic automata. A series of further results in the field under consideration is deduced in [92].

Using the usual notation, let $GF(q)$ be a finite field with q elements and let S_n be the multiplicative semigroup of all $n \times n$ matrices over $GF(q)$. I. B. Marshall [2b] (1941), I. Niven [3b] (1948), and V. Klein [1b] (1980) dealt with the identity $A^{\varkappa} = A^{\varkappa+\delta}$, where A runs over the set of all regular matrices belonging to S_n, and the integers $\varkappa$ and δ are as small as possible. In [93], Š. Schwarz proved the following theorem concerning singular matrices (by using semigroup-theoretical methods):

For each $n \times n$ matric A over $GF(q)$ with $1 \leqq \operatorname{rank}(A) \leqq h \leqq n-1$ the relation

$$A^{h+1} = A^{h+1+\lambda(h,q)}$$

is valid, and this result is the best possible. Here, $\lambda(h, q)$ is the least common multiple of h and q.

The powers of non-negative matrices have been investigated by several authors (see, e.g., the monographs [4a] and [7a]). In particular, the following question has been considered: Suppose that a non-negative $n \times n$ matrix P satisfies the condition

(*) some power of P has a positive column.

What is the least integer k such that P^k has a positive column?

D. Isaacson and P. Madsen [4b] (1974) proved that $k \leqq n^2-3n+3$; the same result is contained in a paper of L. T. Kou [5b] (1984). In his article [95] Š. Schwarz proved a stronger result, namely:

Let P be a non-negative $n \times n$ matrix satisfying the condition (*). Put $L = n^2 - 3n + 3$ and $K = n^2 - 5n + 8$.

(i) If P is primitive, then P^L contains a positive column. For each $n \geqq 2$ there are primitive $n \times n$ matrices for which the number L cannot be replaced by a smaller one.

(ii) If P not primitive, then P^K has a positive column. For each $n \geqq 3$ there are $n \times n$ matrices for which the number K cannot be replaced by a smaller one.

In the paper [97], the author gives a new proof of the existence of a normal basis for cyclic extensions over any field. This proof is shorter and more transparent than those given until now in the literature. Next, an effective method to find all normal bases is described.

Pei, Wang, and Omura [6b] studied normal bases of finite fields having the form $GF(2^m)$ in connection with coding theory. In [98], Š. Schwarz considers the following question. Let $f(x)$ be an irreducible polynomial of degree n over $F_q = GF(q)$ and let α be a fixed root of $f(x)$; the problem is to find an effective method for deciding whether the roots of $f(x) = 0$ represent a normal basis for the cyclic extension $F_q(\alpha)$ over F_q. A criterion giving such an effective method is described in *Schwarz's theorem* which is the main result of [98]. This theorem widely generalizes the result of Pei, Wang and Omura; also, the method of proof is different.

Eighty years of Professor Schwarz present an occasion to look back to his contribution to the development of Slovak mathematics and Slovak culture. During the last 50 years Professor Schwarz played an outstanding role in the development of mathematics as in as well scientific and pedagogical activities.

For many years Professor Schwarz was leading scientific seminars on the theory of semigroups and he educated dozens of mathematicians lecturing now in universities and technical universities. He was a leading figure in the foundation and the formation of the journal Mathematica Slovaca.

For a long period Professor Schwarz was noted as one of the most important person in the organization of scientific life in Slovakia. In particular he gave vital perspective to the conception of the Mathematical Institute of the Slovak Academy of Sciences. He realized this conception in such measure that the institute can play an important role even in the present complicated conditions.

The pedagogical activity of Professor Schwarz presents an original and inspiring contribution. For many years he gave special lectures on mathematics at the Slovak Technical University in Bratislava, which were attended by many teachers. His personal contact with the professors on the Electrotechnical Faculty made it possible to expend the mathematical knowledge in engineering fields. We may say that he acquainted engineers with deeper mathematical tools.

Among the undergraduate students, there was a popular slogan: "If you do not understand the lectures of Prof. Schwarz, go away and try to study something else than technical subjects".

It is a pleasant fact that Professor Schwarz meets his anniversary in good health and full activity manifested e.g. in his work at the Mathematical Institute and in his scientific production and social attitude. The whole Slovak mathematical community congratulates Professor Štefan Schwarz on his anniversary. We wish him all the best for the next years, good health, further deep and nice mathematical results. We thank him for his scientific, pedagogical and organizational work.

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