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HOMEOMORPHISM AND CONTINUITY OF r-SPACES

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In [2] the author has introduced and studied a new class of spaces, called r-spaces, as a generalization of topological spaces and has shown several examples of such spaces.

In the present paper we define homeomorphic, continuous and other types of mappings from one r-space into another and we investigate some of their properties and some relations between them.

1. Notations and remarks

Throughout this paper we shall use the notations from [1] and 2^X will denote the class of all subsets of X. The notation $A \subseteq B$ means that A is a subset of B and if A is a proper subset of B, we write $A \subset B$.

Let X be a nonempty set and ϱ be a relation on 2^X satisfying:

R_1) for each subset A of X there is a subset B of X such that $A\varrho B$

R_2) $\emptyset\varrho\emptyset$

R_3) if $A\varrho B$, then $A \subseteq B$

R_4) if $A\varrho B$, then $B\varrho B$

R_5) if $A \subseteq B$ and $B\varrho B$, then there is a subset C of X such that $A\varrho C$ and $C \subseteq B$

R_6) if $A\varrho B$, then there is no subset C of X such that $C\varrho C$ and $A \subseteq C \subset B$.

Then ϱ is called a relation of the closure on 2^X. The pair (X, ϱ) is called an r-space if X is a nonempty set and ϱ is a relation of the closure on 2^X. If (X, ϱ) is an r-space and for subsets A, B of X we have $A\varrho B$, then we say that B is a closure of A. A set $A \subseteq X$ satisfying $A\varrho A$ is called a closed set. A subset A of X is said to be open if X-A is closed. A subset of X of the form $\{x\}\cup A$, where $x \in X$ and A is open, is said to be a preneighbourhood of x. By a neighbourhood of a point $x \in X$ we mean any open subset of X containing x.

To simplify the notation we often refer to the r-space as X instead of the more proper form (X, ϱ).

We now mention some results of [2] became of their relationship with the present work.

L_1 From several equivalent formulations of the definition of an r-space we give the following one:
If (X, ϱ) is an r-space, then the class $\mathcal{T}=\{A\subseteq X\colon A\varrho A\}$ of all closed subsets of X has the following properties
Ω_1: $\emptyset, X\in\mathcal{T}$
Ω_2: for each $A\subseteq X$ and each $B\in\mathcal{T}$ such that $A\subseteq B$ there is a minimal element C of the class $\{M\in\mathcal{T}\colon A\subseteq M\subseteq B\}$.
Now let X be a nonempty set and $\mathcal{T}$ be a class of subsets of X satisfying Ω_1 and Ω_2. Then, as shown in [2], the relation ϱ defined on 2^X by
(1_0) $A\varrho B$ iff B is a minimal element of $\{M\in\mathcal{T}\colon A\subseteq M\}$
has the properties R_1—R_6, (X, ϱ) is an r-space and $\mathcal{T}$ is the precise class of all closed subsets of (X, ϱ).

L_2 A subset A of an r-space X may have more than one closure.

L_3 Each closed subset A of an r-space X has only one closure, namely the set A.

L_4 Let X be an r-space and $\mathcal{T}$ be an arbitrary class of subsets of X. In the case of X being a finite set, $\mathcal{T}$ has always the property Ω_2. This means that if $\mathcal{T}$ contains $\emptyset$ and X, then determines uniquely an r-space (X, ϱ) and $\mathcal{T}$ is the class of all closed subsets of this r-space.

L_5 Let X be an r-space and $A\subseteq X$. Then A is an open set iff for each $x\in A$ and each preneighbourhood V of x such that $V\subseteq A$ there is a neighbourhood V_1 of x satisfying $V\subseteq V_1\subseteq A$.

2. Homeomorphism and continuity of r-spaces

Definition 1. *Let (X, ϱ_1) and (Y, ϱ_2) be r-spaces. A one-to-one mapping f of X onto Y is said to be a homeomorphism of X onto Y if the following conditions are satisfied:*

(1) *if* $M\varrho_2 N$, *then* $f^{-1}(M)\varrho_1 f^{-1}(N)$

(2) *if* $A\varrho_1 B$, *then* $f(A)\varrho_2 f(B)$.

Two r-spaces (X, ϱ_1) and (Y, ϱ_2) are said to be homeomorphic provided there exists a homeomorphism f of X onto Y.

It follows easily from the above definition that the identity map of an r-space onto itself is always a homeomorphism and the inverse of a homeomorphism is again a homeomorphism. It is also evident that the composition of two homeomorphisms is a homeomorphism. Consequently the collection of r-spaces can be divided into equivalence classes such that each r-space is homeomorphic to every member of its equivalence class and to these r-spaces only.

Now we give a useful characterization of homeomorphicity of r-spaces.

Theorem 1. *Let (X, ϱ_1) and (Y, ϱ_2) be r-spaces. Then (X, ϱ_1) and (Y, ϱ_2) are homeomorphic iff there is a one-to-one map f of X onto Y satisfying the following conditions*

(3) *if* $M\varrho_2 M$, *then* $f^{-1}(M)\varrho_1 f^{-1}(M)$

(4) *if* $A\varrho_1 A$, *then* $f(A)\varrho_2 f(A)$.

Proof. If (X, ϱ_1) and (Y, ϱ_2) are homeomorphic, then the conditions (3) and (4) follow immediately from (1) and (2). To prove the converse suppose that (3) and (4) hold. First we show that (2) is true. Let $A\varrho_1 B$. Then $A \subseteq B$ by R_3 and $B\varrho_1 B$ by R_4. Since $A \subseteq B$, then $f(A) \subseteq f(B)$ and from $B\varrho_1 B$ it follows $f(B)\varrho_2 f(B)$ by (4). We want to prove that $f(A)\varrho_2 f(B)$. Suppose this is not true. Then, by R_5 and R_6, there is a subset M of Y such that $M\varrho_2 M$ and $f(A) \subseteq M \subset f(B)$. From this, using the fact that f is a one-to-one mapping, we have $f^{-1}(f(A) \subseteq f^{-1}(M) \subset f^{-1}(f(B))$, which means that $A \subseteq f^{-1}(M) \subset B$. But this contradicts the condition R_6 of the definition of an r-space, since we know that $f^{-1}(M)\varrho_1 f^{-1}(M)$ and $A\varrho_1 B$ hold. It remains to be shown that (1) holds. The proof of this statement is omitted, since it is similar to that of (2).

Remark 1. *If f is a mapping of X into Y, we shall write f: $X \to Y$.*

Definition 2. *Let X and Y be r-spaces and $\mathcal{T}_1$ and $\mathcal{T}_2$ be classes of all closed subsets of X and Y respectively. Then a mapping f: $X \to Y$ is said to be continuous iff $f^{-1}(B) \in \mathcal{T}_1$ for each $B \in \mathcal{T}_2$.*

Theorem 2. *Let (X, ϱ_1) and (Y, ϱ_2) be r-spaces. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be classes of all closed subsets of X and Y respectively and let f: $X \to Y$ be a one-to-one and onto mapping. Then f is a homeomorphism of X onto Y iff f and f^{-1} are continuous.*

Proof. The proof of the theorem is an immediate consequence of L_1, Definition 2 and Theorem 2.

As stated above for a continuous mapping $f: X \to Y$, where X and Y are r-spaces, the inverse image of any closed set is a closed set again. An analogous assertion for images of closed sets is not generally true even if X and Y are topological spaces. Therefore it is natural to define the following notion.

Definition 3. *Let X and Y be r-spaces and f: $X \to Y$. Then f is said to be closed iff the image of each closed set is closed.*

If we consider the fact that for each one-to-one mapping of X onto Y $f = (f^{-1})^{-1}$ holds, then by Definition 3 and Theorem 2 we have the following immediate result

Corollary 1. *Let X and Y be r-spaces and f: $X \to Y$ be a one-to-one and onto mapping. Then f is a homeomorphism of X onto Y iff f is continuous and closed.*

From the preceding corollary and Definition 1 we see that if f is a one-to-one and onto mapping between two r-spaces, from continuity and closeness of f the properties (1) and (2) follow, and conversely. However, this equivalence is satisfied only if the mapping is supposed to be one-to-one and onto. We now give a discussion of those mappings which are not one-to-one.

Generally it is not true that a continuous and closed mapping satisfies the condition (2). The following example illustrates this fact.

Example 1. *Let A and B be any nonempty, disjoint and finite sets. Let x_0 be any point such that $x_0 \notin A \cup B$ and let*

$$X = A \cup B \cup \{x_0\}$$

$$Y = Y \cup \{x_0\}.$$

Let

$$\mathcal{T}_1 = \{\emptyset, A \cup B, B \cup \{x_0\}, X\}$$

and

$$\mathcal{T}_2 = \{\emptyset, \{x_0\}, Y\}$$

be classes of subsets of X and Y respectively.
Finally define a mapping f of X onto Y by

$$f(x) = \left\langle \begin{array}{ll} x & \text{for } x \in A \\ x_0 & \text{for } x \in X - A = B \cup \{x_0\}. \end{array} \right.$$

It is clear that X and Y are finite sets and both $\mathcal{T}_1$ and $\mathcal{T}_2$ contain $\emptyset$ and X resp. Y. Let (X, ϱ_1) and (Y, ϱ_2) be r-spaces such that $\mathcal{T}_1$ and $\mathcal{T}_2$ are classes of all closed subsets of (X, ϱ_1) and (Y, ϱ_2) respectively — see L_4. The relations ϱ_1 and ϱ_2 are given by the condition (1_0) in L_1.
By Definitions 2 and 3 it is evident that f is continuous and closed. On the other hand, from the definition of ϱ_1 it follows that for each nonempty subset M of B we have $M\varrho_1(A \cup B)$. However $f(M) = \{x_0\}$, $f(A \cup B) = Y$ and therefore $f(M)\,\varrho_2 f(A \cup B)$ does not hold.

Suppose now that (X, ϱ_1) and (Y, ϱ_2) are r-spaces. By L_1 and Definition 3 it is clear that a mapping $f\colon X \to Y$ is closed if it satisfies the condition (4). But if f satisfies the condition (2), it satisfies also the condition (4). We thus get the following result.

Corollary 2. *Each mapping satisfyig the condition (2) is closed.*

Now we show that the mappings satisfying the condition (2) which are onto mappings, are even continuous.

Theorem 3. *Let (X, ϱ_1) and (Y, ϱ_2) be r-spaces. Let $f\colon X \to Y$ be an onto mapping satisfying (2) i.e., there holds*

$$\text{if } A\varrho_1 B, \text{ then } f(A)\,\varrho_2 f(B).$$

Then f is continuous.

Proof. Suppose that B is closed in Y. We want to show that $f^{-1}(B)$ is closed in X. Suppose this is not true, i.e., $f^{-1}(B)$ is not closed in X. Then by R_1 there is at least one closure M of $f^{-1}(B)$ in X, i.e., $f^{-1}(B)\varrho_1 M$ holds. From the fact that $f^{-1}(B)$ is not closed and from R_6 we have $f^{-1}(B)\subset M$. Hence there is $x_0\in M$ such that $x_0\notin f^{-1}(B)$. Since f is an onto mapping, it follows that $B\subseteq f(M)$, $f(x_0)\notin f(f^{-1}(B))=B$ and $f(x_0)\in f(M)$. Hence $B\subset f(M)$. On the other hand we know that $f^{-1}(B)\varrho_1 M$. Therefore by (2) we have $f(f^{-1}(B))\varrho_2 f(M)$ and hence $B\varrho_2 f(M)$. This means that $f(M)$ is a closure of B. But this is impossible, since $B\subset f(M)$ and by L_3 the set B can have only one closure, namely the set B. This completes the proof.

Combining the results of Corollaries 1 and 2 and Theorem 3 we have the following result

Theorem 4. *Let* (X, ϱ_1) *and* (Y, ϱ_2) *be r-spaces and* f *be an one-to-one mapping of* X *onto* Y. *Then* f *is a homeomorphism of* X *onto* Y *iff* f *satisfies the condition* (2).

From the last theorem we see that for defining the notion of homeomorphism it is sufficient to consider besides the property to be one-to-one and onto only the condition (2).

We close this section with some results concerning the mapping satisfying condition (1).

Consider first the Example 1. We have seen that the mapping f described in this example is continuous and closed. By the definition of f it is clear that $A\varrho_2 Y$, $f^{-1}(A)=A$ and $f^{-1}(Y)=X$ hold. But $A\varrho_1 X$ does not hold and thus we have: continuous and closed mappings need not necessarily satisfy (1).

Suppose next $f\colon X\to Y$, where X and Y are r-spaces and let f satisfy (1). Then f satisfies (3) and by L_1 and Definition 2 it is evident that f is a continuous mapping. Thus we have

Corollary 3. *If* X *and* Y *are r-spaces, then each mapping* $f\colon X\to Y$ *satisfying* (1) *is continuous.*

On the other hand the mappings satisfying (1) are not necessarily closed, as shown by the following example.

Example 2. *Let* $X=\{x_1, x_2, x_3, x_4\}$, $Y=\{y_1, y_2, y_3\}$ *and let* $f\colon X\to Y$ *be given by:*

$$f(x_1)=y_1,\quad f(x_2)=y_1,\quad f(x_3)=y_2 \quad\text{and}\quad f(x_4)=y_3.$$

Next define the classes $\mathcal{T}_1$ and $\mathcal{T}_2$ of subsets of X and Y respectively in the following way

$$\mathcal{T}_1=\{\emptyset, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, X\}$$

and

$$\mathcal{T}_2=\{\emptyset, \{y_1\}, \{y_2\}, Y\}.$$

Finally let (X, ϱ_1) and (Y, ϱ_2) be r-spaces such that $\mathcal{T}_1$ and $\mathcal{T}_2$ are classes of all closed subsets of (X, ϱ_1) and (Y, ϱ_2) respectively. The existence of (X, ϱ_1) and (Y, ϱ_2) is an immediete consequence of L_4 and the fact that X, Y are finite sets and $\{\emptyset, X\} \subseteq \mathcal{T}_1$, $\{\emptyset, Y\} \subseteq \mathcal{T}_2$. The relations ϱ_1 and ϱ_2 are defined by (1_0) of L_1. From Figure 1 and condition (1_0) it is easy to see that f satisfies (1). It is also evident that $A = \{x_2, x_3\}$ is closed in X, but $f(A) = \{y_1, y_2\}$ is not closed in Y. Therefore f is not a closed mapping although it satisfies (1).

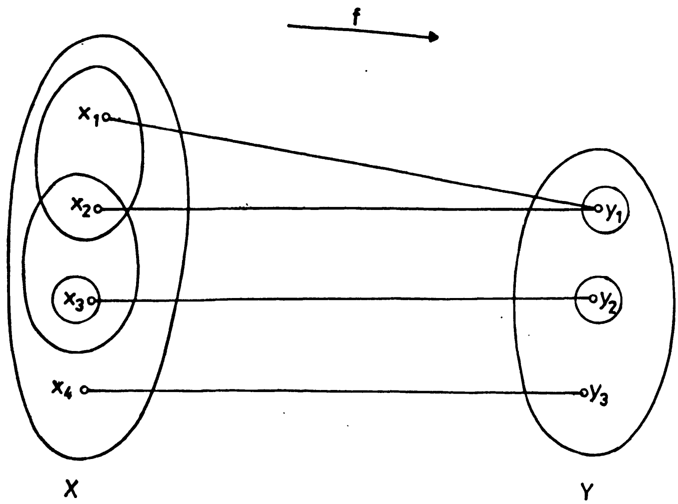

Fig. 1

Remark 2. From the previous example it is also evident that (1) is not a sufficient condition for (2). If we, e.g., put $A = \{x_2\}$ and $B = \{x_2, x_3\}$, then it is clear that $A\varrho_1 B$ holds, but $f(A)\varrho_2 f(B)$ is not true.

Now we shall prove that in the case of f being a one-to-one and onto mapping, the condition (1) is sufficient for f to be closed.

Lemma 1. *Let (X, ϱ_1) and (Y, ϱ_2) be r-spaces and $f\colon X \to Y$ be a one-to-one and onto mapping satisfying* (1). *Then f is closed.*

Proof. Let B be a closed subset of X. Suppose on the contrary that $f(B)$ is not closed. Then by R_1 there is $M \subseteq Y$ such that $f(B)\varrho_2 M$. From this by R_3 and R_6 it follows $f(B) \subset M$. Thus by (1) we have $f^{-1}(f(B))\varrho_1 f^{-1}(M)$ and $f^{-1}(f(B)) \subset f^{-1}(M)$. But since f is a one-to-one and onto mapping $B\varrho_1 f^{-1}(M)$ and $B \subset f^{-1}(M)$ hold, which contradicts L_3.

From this lemma and from the Corollaries 1 and 3 we have the following statement

Theorem 5. *If X and Y are r-spaces and f: $X \to Y$ is an one-to-one and onto mapping, then*

— *f is a homeomorphism of X onto Y iff f satisfies* (1)

— *the conditions* (1) *and* (2) *are equivalent.*

Finally we give an example showing that generally condition (2) need not necessarily imply (1).

Example 3. Let $X = \{x_1, x_2, x_3, x_4\}$, $Y = \{y_1, y_2\}$ and f: $X \to Y$ be given by:

$$f(x_1) = y_1, \quad f(x_2) = y_2, \quad f(x_3) = y_1 \quad \text{and} \quad f(x_4) = y_2.$$

Next define the classes $\mathcal{T}_1$ and $\mathcal{T}_2$ of subsets of X and Y respectively by

$$\mathcal{T}_1 = \{\emptyset, \{x_1, x_2\}, \{x_2, x_3, x_4\}, X\}$$

and

$$\mathcal{T}_2 = \{\emptyset, Y\}.$$

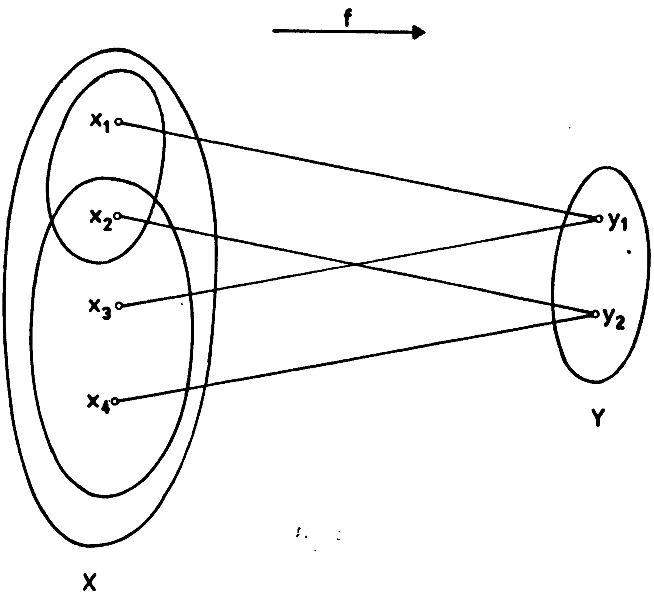

Fig. 2

From our assumptions and from L_4 it follows that there are r-spaces (x, ϱ_1) and (Y, ϱ_2) such that $\mathcal{T}_1$ and $\mathcal{T}_2$ are classes of all closed subsets of (X, ϱ_1) and (Y, ϱ_2) respectively.

Using (1_0) it is not hard to verify that f satisfies (2). If we put $M = \{y_2\}$ and $N = \{y_1, y_2\}$, then $M\varrho_2 N$. On the other hand $f^{-1}(M)\,\varrho_1 f^{-1}(N)$ does not hold, which means that (1) is not satisfied.

3. Some local properties of the continuity

From the importance of the notion of the homeomorphism taking into account Theorem 2 there follows the great importance of the continuity as well. Therefore we shall deal with some questions related to local properties of continuity.

It is known that if X and Y are arbitrary and nonempty sets, A is a subset of X and $f: X \to Y$ is a mapping, then

$$f^{-1}(Y-A)=X-f^{-1}(A).$$

As an immediate consequence of this fact and Definition 2 we have the following

Theorem 6. *Let X and Y be r-spaces. Let $\mathcal{D}_1$ and $\mathcal{D}_2$ be classes of all open subsets of X and Y respectively and let $f: X \to Y$. Then f is continuous iff $f^{-1}(A) \in \mathcal{D}_1$ for each $A \in \mathcal{D}_2$.*

As the continuity of a function in r-spaces (see Def. 2) can be defined in the same way as in topological spaces (see [1] p. 85), the notion of the continuity at a point can be introduced into the r-spaces in the way known in the theory of topological spaces.

Definition 4. *Let X and Y be r-spaces, $x_0 \in X$ and $f: X \to Y$. Then f is said to be continuous at x_0 iff for each neighbourhood V of $f(x_0)$ there is a neighbourhood U of x_0 such that $f(U) \subseteq V$.*

Now we shall show by an example that in the r-spaces the continuity at each point need not imply the continuity.

Example 4. *Let X be an infinite set. Define the classes $\mathcal{D}_1$ and $\mathcal{D}_2$ of subsets of X and Y as follows*

$\mathcal{D}_1$ consists of:
- *the empty set*
- *each subset of X having exactly $10.k$ elements, where $k=1, 3, 5, 7, \dots$*
- *all infinite subsets of X.*

$\mathcal{D}_2$ consists of:
- *the empty set*
- *each subset of X having exactly $10.k$ elements, where $k=1, 2, 4, 6, \dots$*
- *all infinite subsets of X.*

It is not hard to verify that both $\mathcal{D}_1$ and $\mathcal{D}_2$ satisfy the conditions Ω_1' and Ω_2' of theorem 5 of [2]. Therefore by Theorem 6 of [2] there are r-spaces (X, ϱ_1) and (X, ϱ_2) such that $\mathcal{D}_1$ and $\mathcal{D}_2$ are classes of all open subsets of (X, ϱ_1) and (X, ϱ_2) respectively.

Now let f be the identity mapping on X and let x_0 be any point of X. Since $f(x_0)=x_0$, then for each $V \in \mathcal{D}_2$ such that $f(x_0)=x_0 \in V$ there is at least one subset $U \subseteq V = f^{-1}(V)$ having exactly 10 elements and such that $x_0 \in U$ and $U=f(U) \subseteq V$. Since $U \in \mathcal{D}_1$ it follows that f is continuous at x_0.

On the other hand it is clear that if V is a subset of X containing exactly 20 elements, then $V \in \mathscr{D}_2$, but $f^{-1}(V) = V \notin \mathscr{D}_1$. This means that f is not continuous — see Theorem 6.

Definition 5. *Let X and Y be r-spaces and let f: $X \to Y$. If $x_0 \in X$, then f is said to be r-continuous at x_0 if*

(5) *for each neighbourhood V of $f(x_0)$ and each preneighbourhood U of x_0 such that $f(U) \subseteq V$ there is a neighbourhood U_1 of x_0 satisfying $U \subseteq U_1$ and $f(U_1) \subseteq V$.*

Theorem 7. *Let X and Y be r-spaces. Let $\mathscr{D}_1$ and $\mathscr{D}_2$ be classes of all open subsets of X and Y respectively and let f: $X \to Y$. Then f is continuous iff it is r-continuous at each $x_0 \in X$.*

Proof. Let first f be continuous. Suppose that $x_0 \in X$ and V is a neighbourhood of $f(x_0)$, i.e., $V \in \mathscr{D}_2$ such that $f(x_0) \in V$. Then it is clear that $x_0 \in f^{-1}(V)$ and by Theorem 6 we have $f^{-1}(V) \in \mathscr{D}_1$. Let U be a preneighbourhood of x_0 such that $f(U) \subseteq V$. Then $U \subseteq f^{-1}(V)$ and since $f^{-1}(V) \in \mathscr{D}_1$ the set $f^{-1}(V)$ is a neighbout-hood of x_0 satisfying $U \subseteq f^{-1}(V)$ and $f(f^{-1}(V)) \subseteq V$. This proves half the theorem.

To prove the converse suppose $A \in \mathscr{D}_2$ and the condition (5) holds. We want to prove $f^{-1}(A) \in \mathscr{D}_1$. Let $x_0 \in f^{-1}(A)$. This means that $f(x_0) \in f(f^{-1}(A)) \subseteq A$. If U is a preneighbourhood of x_0 such that $U \subseteq f^{-1}(A)$, then $f(U) \subseteq A$. But then according to the assumptions there exists a neighbourhood U_1 of x_0 such that $U \subseteq U_1$ and $f(U_1) \subseteq A$. This means that $U \subseteq U_1 \subseteq f^{-1}(A)$ and by L_5 it follows that $f^{-1}(A) \in \mathscr{D}_1$.

Now we are going to investigate the relation between the continuity and r-continuity at a point.

Theorem 8. *Let X and Y be r-spaces, let f: $X \to Y$ be a mapping and let $x_0 \in X$. If f is r-continuous at x_0, then it is also continuous at x_0.*

Proof. Since we know that for each $x \in X$ we have $\{x\} \cup \emptyset = \{x\}$, then by the definition of the preneighbourhood of a point it is clear that for each $x \in X$ the set $\{x\}$ is a preneighbourhood of x. Then by Definition 5 for each neighbourhood V of $f(x_0)$ and for the preneighbourhood $U = \{x_0\}$ of x_0 there is a neighbourhood U_1 of x_0 such that $\{x_0\} = U \subseteq U_1$ and $f(U_1) \subseteq V$. But by Definition 4 this means that f is continuous at x_0.

Finally we show that in topological spaces the r-continuity at a point is equivalent to the continuity at a point. But since topological spaces are special cases of r-spaces, then the r-continuity at a point by the last theorem implies the continuity at this point. Therefore it suffices to prove the converse statement.

Theorem 9. *Let X and Y be topological spaces and let $\mathscr{D}_1$ and $\mathscr{D}_2$ be classes of all open subsets of X and Y respectively. Then f: $X \to Y$ is a continuous mapping at $x_0 \in X$ iff f is r-continuous at x_0.*

Proof. Suppose that V is a neighbourhood of $f(x_0)$ and $U_1 = \{x_0\} \cup U$, where

$U \in \mathcal{D}_1$ is a preneighbourhood of x_0 such that $f(U_1) \subseteq V$. We want to find a neighbourhood U_2 of x_0 such that $U_1 \subseteq U_2$ and $f(U_2) \subseteq V$. Since f is continuous at x_0 there is a neighbourhood U_3 of x_0 such that $f(U_3) \subseteq V$. Now if we put $U_2 = U \cup U_3$ then it is clear that $x_0 \in U_2$, $U_1 \subseteq U_2$ and $f(U_2) \subseteq V$. But we deal with topological spaces and therefore $U_2 = U \cup U_3$ is in $\mathcal{D}_1$. This means that U_2 is a neighbourhood of x_0, which completes the proof.

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ГОМЕОМОРФИЗМ И НЕПРЕРЫВНОСТЬ r-ПРОСТРАНСТВ

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Резюме

r-пространства являются обобщением топологических пространств. Существуют тоже другие важные примери r-пространств. В настоящей работе изучаются некоторые свойства отображений между r-пространствами.