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Mathematica Slovaca, Vol. 32 (1982), No. 1, 63--73

Persistent URL: http://dml.cz/dmlcz/136285

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ON MULTILATTICES WITH ISOMORPHIC GRAPHS

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The aim of this paper is to obtain a generalization of a result of M. Kolibiar [2] concerning meet semilattices of locally finite length for the case of lower directed partially ordered sets of locally finite length.

A meet semilattice $\mathcal{S}=(S;\wedge)$ of a locally finite length is said to be a B-semilattice if it fulfils the following condition:

(1) *If $a, b, c \in S$, $a \neq b$ and if c covers both a, b, then both a, b cover $a \wedge b$.*

The following theorems (K_1), (K_2) were proved in paper [2].

(K_1) *Let $\mathcal{A}$, $\mathcal{S}$, $\mathcal{S}'$ be semilattices of a locally finite length, $\mathcal{B}$ a lattice of locally finite length and let $f\colon \mathcal{S}\to\mathcal{A}\times\mathcal{B}$, $g\colon \mathcal{S}'\to\mathcal{A}\times\tilde{\mathcal{B}}$ be subdirect representations of semilattices such that* $\operatorname{Im} f=\operatorname{Im} g$. *Then $g^{-1}/\operatorname{Im} g\circ f$ is an isomorphism of the graphs $G(\mathcal{S})$, $G(\mathcal{S}')$.*

(K_2) *Let $\mathcal{S}$, $\mathcal{S}'$ be B-semilattices and let $h\colon G(\mathcal{S})\to G(\mathcal{S}')$ be an isomorphism of graphs. Then there exist a semilattice $\mathcal{A}$ and a lattice $\mathcal{B}$ and subdirect representations of semilattices $f\colon \mathcal{S}\to\mathcal{A}\times\mathcal{B}$, $g\colon \mathcal{S}'\to\mathcal{A}\times\tilde{\mathcal{B}}$ such that* $\operatorname{Im} f=\operatorname{Im} g$ and $h=g^{-1}/\operatorname{Im} g\circ f$.

Let us recall some basic concepts and properties.

A partially ordered set $\mathcal{P}=(P;\leq)$ is said to be of a locally finite length if each bounded chain in $\mathcal{P}$ is finite, For the elements $a, b\in P$ we write $a \prec b$ (a is covered by b) if $a<b$ and there does not exist any element $c\in P$ such that $a<c<b$. In this case we say that $[a, b]$ is a prime interval. We denote by $\tilde{\mathcal{P}}$ the partially ordered set dual to $\mathcal{P}$.

A multilattice [1] *is a poset $\mathcal{M}=(M;\leq)$ in which the condition (i) and its dual (ii) are satisfied: (i) If $a, b, h\in M$, $a\leq h$, $b\leq h$, then there exists $v\in M$ such that* (a) *$v\leq h$, $v\geq a$, $v\geq b$* and (b) *$z\in M$, $z\geq a$, $z\geq b$, $z\geq v$ implies $z=v$.*

The symbol $(a\vee b)_h$ designates the set of all elements $v\in M$ satisfying (i); the symbol $(a\wedge b)_d$ has a dual meaning.

We denote $a\vee b=\bigvee\limits_{\substack{a\leq h\\ b\leq h}}(a\vee b)_h \quad a\wedge b=\bigvee\limits_{\substack{d\leq a\\ d\leq b}}(a\wedge b)_d.$

Recall that the sets $a\vee b$, $a\wedge b$ may be empty. It is evident that each partially ordered set of locally finite length is a multilattice.

A lower directed multilattice $\mathcal{M}$ of a locally finite length is said to be lower semimodular if it fulfils the following covering condition:

(σ) *Let $a, b, u, h \in M$, $a \prec h$, $b \prec h$, $a \neq b$, $u \in a \wedge b$. Then $u \prec a$, $u \prec b$.*

A multilattice $\mathcal{M}$ of a locally finite length is modular [1] *if the condition* (σ) *and its dual* (σ') *are satisfied in $\mathcal{M}$.*

By a graph $G(\mathcal{P})$ is meant an unoriented graph (without multiple edges and loops) whose vertices are elements of P; two vertices a, b are joined by the edge (a, b) iff either $a \prec b$ or $b \prec a$.

The set $\mathcal{S} = \{a, b, u, v\} \subset \mathcal{P}$ is said to be elementary square if a, b are uncomparable elements and $a \prec v$, $b \prec v$, $u \prec a$, $u \prec b$.

Let $\mathcal{P}_1$, $\mathcal{P}_2$ be partially ordered sets of a locally finite length and let φ be an isomorphism of the graph $G(\mathcal{P}_1)$ onto the graph $G(\mathcal{P}_2)$. We say that the elementary square $\mathcal{S} = \{a, b, u, v\} \subset \mathcal{P}_1$ is broken by the isomorphism φ if either $\varphi(u) \prec \varphi(a)$, $\varphi(u) \prec \varphi(b)$, $\varphi(v) \prec \varphi(a)$, $\varphi(v) \prec \varphi(b)$ or $\varphi(a) \prec \varphi(u)$, $\varphi(a) \prec \varphi(v)$, $\varphi(b) \prec \varphi(u)$, $\varphi(b) \prec \varphi(v)$.

Let $\mathcal{M}$ be a multilattice, $x_1, y_1, x_2, y_2 \in M$. We say that an interval $[y_1, x_1]$ is direct transposed [1] *with an interval $[y_2, x_2]$ iff $x_1 \in x_2 \vee y_1$, $y_2 \in x_2 \wedge y_1$.*

We say that an interval $[y_1, x_1]$ is transposed with an interval $[x_2, y_2]$ ($x_1, y_1, x_2, y_2 \in M$) iff there are intervals $[b_i, a_i]$ $b_i, a_i \in M$, $i = 0, 1, \ldots, r$, such that the interval $[b_i, a_i]$ is direct transposed with the interval $[b_{i+1}, a_{i+1}]$ for $i = 0, 1, \ldots, r-1$ and $[b_0, a_0] = [y_1, x_1]$, $[b_r, a_r] = [y_2, x_2]$.

Intervals $[y_1, x_1]$, $[y_2, x_2]$ are said to be lower T-transposed if there is an interval $[t, s]$ such that the intervals $[y_1, x_1]$, $[y_2, x_2]$ are transposed with the interval $[t, s]$.

The following theorem was proved in [1, 4.7].

(B) *Let $\mathcal{M}$ be a multilattice of locally finite length fulfilling the covering condition* (σ). *Let C_1, C_2 be maximal chains between $a, b \in M$. Then C_1, C_2 are of the same length and there exists a one-to-one mapping of the set of all prime intervals of the chain C_1 onto the set of all prime intervals of the chain B, such that the corresponding prime intervals are lower T-transposed.*

Multilattices $\mathcal{M}_1$, $\mathcal{M}_2$ are said to be isomorphic (denoted $\mathcal{M}_1 \sim \mathcal{M}_2$) if there exists a bijection f of M_1 onto M_2 satisfying:

$$x \leq y \text{ iff } f(x) \leq f(y) \quad (x, y \in M_1).$$

Let $\mathcal{M}$, $\mathcal{A}$, $\mathcal{B}$ be multilattices and let f be an isomorphism of $\mathcal{M}$ to the multilattice $\mathcal{A} \times \mathcal{B}$. We shall say that f is a subdirect representation of the multilattice $\mathcal{M}$ if the projection of the Im f *into A, B, resp., is the whole set A, B, resp.*

In the first part of this paper we shall prove the following assertion.

Theorem 1. *Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{M}$, $\mathcal{M}'$ be lower directed multilattices of a locally finite length and let $f: \mathcal{M} \to \mathcal{A} \times \mathcal{B}$, $g: \mathcal{M}' \to \mathcal{A} \times \tilde{\mathcal{B}}$ be subdirect representations of $\mathcal{M}$, $\mathcal{M}'$ such that* Im f = Im g. *Then the mapping $\varphi = g^{-1}/\text{Im}\, g \circ f$ is an isomorphism of* *the graph $G(\mathcal{M})$ onto the graph $G(\mathcal{M}')$,such that no elementary square $\mathcal{S} \subset \mathcal{M}$, $\mathcal{S}' \subset \mathcal{M}'$, resp., is broken by the isomorphism φ, φ^{-1}, respectively.*

Proof. Let $\mathcal{T}=(T, \leqslant)$, $\mathcal{T}'=(T; \subseteq)$ be images of the multilattices $\mathcal{M}, \mathcal{M}'$ under the isomorphisms f, g. We shall denote by $\cup, \cap$ the operations in the multilattice $\mathcal{T}'$. It is evident that $\mathcal{T}$, $\mathcal{T}'$ are lower directed multilattices. We shall show that the graphs $G(\mathcal{T})$, $G(\mathcal{T}')$ coincide. Let (a_1, a_2), $(b_1, b_2) \in T$ such that $(b_1, b_2) \prec (a_1, a_2)$ in $\mathcal{T}$. Then $b_1 \leqslant a_1$, $b_2 \leqslant a_2$. Since $a_1 \wedge b_1 = \{b_1\}$, $a_2 \vee b_2 = \{a_2\}$, we have $(a_1, a_2) \cap (b_1, b_2) = \{a_1 \wedge b_1, a_2 \vee b_2\} = \{(b_1, a_2)\} \in T$ because the multilattice $\mathcal{M}'$ is lower directed. Therefore $(b_1, b_2) \leqslant (b_1, a_2) \leqslant (a_1, a_2)$ hence either $b_1 = a_1$ or $a_2 = b_2$. Then in $\mathcal{T}'$ we have $(a_1, a_2) = (b_1, a_2) \subset (b_1, b_2)$ in the first case and $(b_1, b_2) \subset (a_1, b_2) = (a_1, a_2)$ in the second case. Let us suppose that there exists an element $(x_1, x_2) \in \mathcal{A} \times \tilde{\mathcal{B}}$ such that either $(b_1, a_2) \subseteq (x_1, x_2) \subseteq (b_1, b_2)$ or $(b_1, b_2) \subseteq (x_1, x_2) \subseteq (a_1, b_2)$. Then we have $b_1 = x_1$, $b_2 \leqslant x_2 \leqslant a_2$ in the first case and $x_2 = b_2$, $b_1 \leqslant x_1 \leqslant a_1$ in the second case. Hence in the multilattice $\mathcal{A} \times \mathcal{B}$ we have $(b_1, b_2) \leqslant (b_1, x_2) \leqslant (b_1, a_2) = (a_1, a_2)$ in the first case and $(b_1, a_2) \leqslant (x_1, b_2) \leqslant (a_1, b_2) = (a_1, b_2)$ in the second case. From the assumption it follows that in the first case we have either $x_2 = b_2$ or $x_2 = a_2$, in the second case either $x_1 = b_1$ or $x_1 = a_1$. We get that either $(x_1, x_2) = (b_1, b_2)$ or $(x_1, x_2) = (a_1, a_2)$. Thus either $(a_1, a_2) \prec (b_1, b_2)$ or $(b_1, b_2) \prec (a_1, a_2)$ in $\mathcal{A} \times \tilde{\mathcal{B}}$. Analogously we can prove that if $(y_1, y_2) \prec (x_1, x_2)$ in the multilattice $\mathcal{A} \times \tilde{\mathcal{B}}$, then in the multilattice $\mathcal{A} \times \mathcal{B}$ either $(y_1, y_2) \prec (x_1, x_2)$ or $(x_1, x_2) \prec (y_1, y_2)$ holds. From this it follows that the graphs $G(\mathcal{T})$, $G(\mathcal{T}')$ are the same and the mapping $\varphi = g^{-1}/\mathrm{Im}\, g \circ f$ is an isomorphism of the graph $G(\mathcal{M})$ onto the graph $G(\mathcal{M}')$.

Next we shall show that no elementary square of the multilattice $\mathcal{M}$ is broken by the isomorphism φ.

Let $\{a, b, u, v\} \subset \mathcal{M}$ be an elementary square such that either

(i) $\varphi(u) \prec \varphi(a)$, $\varphi(u) \prec \varphi(b)$, $\varphi(v) \prec \varphi(a)$, $\varphi(v) \prec \varphi(b)$

or

ii) $\varphi(a) \prec \varphi(u)$, $\varphi(b) \prec \varphi(u)$, $\varphi(a) \prec \varphi(v)$, $\varphi(b) \prec \varphi(v)$.

Let us consider the case (i) and let $f(v) = (v_1, v_2)$, $f(a) = (a_1, a_2)$, $f(b) = (b_1, b_2)$. Then $a_1 \leqslant v_1$, $b_1 \leqslant v_1$. From $\varphi(v) \prec \varphi(a)$, $\varphi(v) \prec \varphi(b)$ it follows that $g(\varphi(a)) \subset g(\varphi(v))$, $g(\varphi(b)) \subset g(\varphi(v))$. Hence $v_1 \leqslant a_1$, $v_1 \leqslant b_1$. This implies $a_1 = b_1 = v_1$. Analogously we get that $u_2 = a_2 = b_2$. Thus $f(a) = f(b)$ contradicting $f(a) \neq f(b)$. In the case (ii) we get the same conclusion. The assertion that no elementary square of the multilattice $\mathcal{M}'$ is broken by the isomorphism φ^{-1} can be proved analogously.

The aim of the next part of this paper is to prove that for lower semimodular multilattices $\mathcal{M}$, $\mathcal{M}'$ of a locally finite length also the converse assertion holds.

Now we shall suppose that $\mathcal{M}$, $\mathcal{M}'$ are lower semimodular multilattices of a locally finite length and that φ is an isomorphism of the graph $G(\mathcal{M})$ onto the graph $G(\mathcal{M}')$ such that no elementary square of $\mathcal{M}$, $\mathcal{M}'$, resp., is broken by φ, φ^{-1}, respectively. We shall denote $x' = \varphi(x)$ for $x \in M$.

Let $P_1 = \{(a, b) \in M \times M$: either $a < b$ and $a' < b'$, or $b < a$ and $b' < a'\}$, $P_0 = \{(a, b) \in M \times M$: either $a < b$ and $b' < a'$, or $b < a$ and $a' < b'\}$. $P'_i = \{(a', b') : (a, b) \in P_i\}$, $i \in \{0, 1\}$.

We shall say that a prime interval $[x, y]$, $x, y \in M$ is preserved (reversed) if $(x, y) \in P_1$ ($(x, y) \in P_0$). An interval $[u, v]$, $u, v \in M$ is preserved (reversed) if each prime interval of this interval is preserved (reversed); the interval (u, u) is simultaneously preserved and reversed.

Since the relation between the multilattices $\mathcal{M}$ and $\mathcal{M}'$ is symmetric, we may exchange the roles of these multilattices in each of the following lemmas. (E g , Lemma 1′ denotes the assertion that we obtain from Lemma 1 by exchanging $\mathcal{M}$ and $\mathcal{M}'$.)

Lemma 1. *Let $\{a, b, c, d\} \subset M$ be an elementary square. Then $(a, d) \in P_i$ iff $(c, b) \in P_i$, $i \in \{0, 1\}$.*

Proof. If $a' < d'$ and $b' < d'$ ($d' < a'$, $d' < b'$), then from the assumption that no elementary square of $\mathcal{M}$ is broken by the isomorphism φ it follows that $c' < b'$ ($b' < c'$).

It can be easily shown that if $a' < d' < b'$, then $a' < c' < b'$ and if $d' < a'$, $b' < d'$, then $b' < c' < a'$.

Lemma 2. *Let $a, b, c, d, u, v \in M$ such that $a < b$, $v \in b \wedge c$, $u \in a \wedge v$. Then either $u = v$ or $u < v$.*

Proof. If $v = b$ or $v \leq a$, the assertion is obvious. Let a, v be incomparable and let $b = b_0 > b_1 > \ldots > b_n = v$ be a maximal chain from v to b. If we assume that $n = 1$, then the assertion is valid by the condition (σ). Let us suppose that the assertion is valid when the length of a minimal chain from v to b is $n - 1$. Let $u_1 \in (a \wedge b_{n-1})_u$, $u_1 < b_{n-1}$. According to the condition (σ) for each element $u_2 \in u_1 \wedge v$ we have $u_2 < v$. Since $u \in v \wedge u_1$, we get $u < v$.

Lemma 3. *Let $a, b, c, u \in M$ such that $a < b$, $c < b$, $u \in a \wedge c$, $c \notin a \wedge c$. Then $[u, c]$ is a prime interval and it is preserved (reversed) iff the prime interval $[a, b]$ is preserved (reversed).*

Proof. According to Lemma 2 $[u, c]$ is a prime interval. Let $b = b_0 > b_1 > \ldots > b_n = c$ be a maximal chain from b to c. The proof is based on induction on n and Lemma 1.

Corollary 1. *Let prime intervals $[u, x]$, $[y, v]$ be lower T-transposed. Then the prime interval $[y, v]$ is preserved (reversed) iff the interval $[u, x]$ is preserved (reversed).*

Lemma 4. *Let $a, b, u, v \in M$ such that $a < b$, $v < b$, $u \in a \wedge v$, $v \notin a \wedge v$. If the interval $[u, a]$ is preserved (reversed), then the interval $[v, b]$ is preserved (reversed) as well.*

Proof. Let $a=a_0>a_1>\ldots>a_n=u$ be a maximal chain from u to a. If $n=1$, we shall show that $[v, b]$ is a prime interval. Let $v_1\in M$ such that $v<v_1<b$ and let $u_1\in(v_1\wedge a)_u$. Then either $u_1=a$ or $u_1=u$. Since $[a, b]$ is a prime interval we obtain $u_1\neq a$. According to Lemma 2 $[u, v]$, $[u, v_1]$ are prime intervals and hence $v_1=v$. From Lemma 1 it follows that if the interval $[u, a]$ is preserved (reversed), then the prime interval $[v, b]$ is preserved (reversed) as well. It suffices to apply induction on n to finish the proof.

Lemma 5. Let $x, y\in M$, $x<y$ and let $x=a_0<a_1<\ldots<a_n=y$ be a maximal chain such that $a'_{i-1}<a'_i$ $(a'_i<a'_{i-1})$ $(i=1, \ldots, n)$. If $x\leqslant a<b\leqslant y$, then the interval $[a, b]$ is preserved (reversed).

Proof. Let $[a, b]$ be a prime interval, $a, b\in[x, y]$. There exists a maximal chain R from x to y such that $a, b\in R$. According to Theorem (B) the prime interval $[a, b]$ is lower T-transposed with some prime interval $[a_{i-1}, a_i]$. From Corollary 1 and the assumption of the theorem it follows that the interval $[a, b]$ is preserved (reversed).

Lemma 6. *Let $a, b, u, v\in M$, $v\geqslant a$, $v\geqslant b$, $u\in a\wedge b$. If the interval $[a, v]$ is preserved (reversed), then the interval $[u, b]$ is preserved (reversed).*

Proof. Let $v=v_0>v_1>\ldots>v_n=a$ be a maximal chain from a to v. If $n=1$, then the assertion follows from Lemma 3. Let us suppose that the assertion is valid for $n-1$. Let $u_1\in(v_{n-1}\wedge b)_u$. Then the interval $[u_1, b]$ is preserved (reversed). Since $u\in a\wedge b$, we have $u\in a\wedge u_1$ and according to Lemma 2 either $u_1=u$ or $u<u_1$. If $u<u_1$, then the interval $[u, u_1]$ is preserved (reversed) according to Lemma 3. Thus the interval $[u, b]$ is preserved (reversed).

Lemma 7. *Let $x, y, u, v\in M$ such that $u\in x\wedge y$, $v\in x\vee y$ and let $[u, x]$ be a prime interval. Then $[y, v]$ is a prime interval.*

Proof. Suppose that $u\in x\wedge y$, $v\in x\vee y$ and $[u, x]$ is a prime interval. If $v=x$, then $u=y$ and the assertion is obvious. Let $x=x_0<x_1<\ldots<x_n=v$ be a maximal chain from x to v. We proceed by induction on n. If $n=1$, then it follows from Theorem (B) that all maximal chains from x to v are of the same length 2. Hence $[y, v]$ is a prime interval. Let the assertion hold if the maximal chain between the corresponding elements is of the length $m\leqslant n-1$. Choose $y_1\in(x_{n-1}\wedge y)_u$. Then $y_1<y$ by Lemma 2. It is obvious that $u\in x\wedge y_1$. Let $\bar{x}\in(x\vee y_1)_{x_{n-1}}$. By the induction assumption $[y_1, \bar{x}]$ is a prime interval. Since $y_1\in\bar{x}\wedge y$, $v\in\bar{x}\vee y$ and since the lengths of all maximal chains from $\bar{x}$ to v are less or equal to $n-1$, we infer that $[y, v]$ is a prime interval.

Let $i\in\{0, 1\}$. Let us denote by Θ_i (Θ'_i) the least equivalence relation on $\mathcal{M}$ $(\mathcal{M}')$ such that $P_i\subset\Theta_i$ $(P'_i\subset\Theta'_i)$.

Lemma 8. *Let $a, b\in M$ such that $(a, b)\in\Theta_1$ $((a, b)\in\Theta_0)$. Then there exists an element $u\in M$ such that $u\in a\wedge b$ and the intervals $[u, b]$ are preserved (reversed).*

Proof. Let $(a, b)\in\Theta_1$. Then there is a sequence $(p_0) a=c_0^0, c_1^0, \ldots, c_n^0=b$ $(n\geq 1)$ such that $(c_k, c_{k+1})\in P_1$ for each $k\in\{0, \ldots n-1\}$.

Let $0<i<n$. We shall say that an element c_i^0 has a property (p) if the elements c_{i-1}^0, c_{i+1}^0 are covered by c_i^0. Let us replace in the sequence (p_0) all elements c_i^0 with the property (p) by the elements $c_i^1\in c_{i-1}^0\wedge c_{i+1}^0$. We denote by c_k^1 those elements c_k^0 which have not the property (p). Then $(c_k^1, c_{k+1}^1)\in P_1$, $k=0, 1, \ldots, n-1$. After a finite number of analogous steps we get the sequence

(p_m) $a=c_0^m, c_1^m, \ldots, c_n^m=b$ $(n\geq 1)$ such that $(c_k^m, c_{k+1}^m)\in P_1$ for each $k=0, \ldots, n-1$ and in the sequence (p_m) there does not exist any element with the property (p). It means that there exists an element c_j^m such that $a=c_0^m > \ldots \geq c_j^m \leq \ldots \leq c_n^m=b$ $(n\geq 1)$. If we choose $u\in(a\wedge b)_{c_j^m}$, then the intervals $[u, a]$, $[u, b]$ are preserved by Lemma 5.

The proof for Θ_0 is analogous.

Corollary 2. *Let $a\leq b$ $(a, b\in M)$. Then $(a, b)\in\Theta_1$ $((a, b)\in\Theta_0)$ iff the interval $[a, b]$ is preserved (reversed).*

Lemma 9. *Let $a, b, c\in M$, $(a, b)\in\Theta_i$, $i\in\{0, 1\}$. Then there are elements $u, v\in M$ such that $u\in a\wedge c$, $v\in b\wedge c$, $(u, v)\in\Theta_i$.*

Proof. Let $(a, b)\in\Theta_1$. According to Lemma 8 there exists an element $x\in M$ such that $x\in a\wedge b$ and the intervals $[x, a]$, $[x, b]$ are preserved. Let us choose $d\in x\wedge c$, $u\in(a\wedge c)_d$, $v\in(b\wedge c)_d$. We shall show that $d\in u\wedge x$. Let $d_1\in(u\wedge x)_d$. Since $d_1\geq d$, $d_1\leq x$, $d_1\leq u$, $d_1\leq c$ and $d\in x\wedge c$, we have $d=d_1$. According to Lemma 6 the interval $[d, u]$ is preserved. By a similar argument, the interval $[u, v]$ is preserved as well. Thus $(u, v)\in\Theta_1$. For $(a, b)\in\Theta_0$ the proof if analogous.

Let the relation $\leq$ be defined on the set M/Θ_i (M'/Θ_i'), $i\in\{0, 1\}$ as follows: $[a]\Theta_i \leq [b]\Theta_i$ iff there exist $a_i\in[a]\Theta_i$, $b_1\in[b]\Theta_i$ such that $a_1\leq b_1$ $([a']\Theta_i' \leq [b']\Theta_i'$ iff there exist $a_i'\in[a']\Theta_i'$, $b_i'\in[b']\Theta_i'$ such that $a'\leq b')$.

Lemma 10. *Let $i\in\{0, 1\}$. Then $\mathcal{M}/\Theta_i = (M/\Theta_i, \leq)$ $(\mathcal{M}'/\Theta_i' = (M'/\Theta_i', \leq))$ are partially ordered sets.*

Proof. The reflexivity of the relation $\leq$ on the set M/Θ_1 is clear. We shall show that the relation $\leq$ is anti-symmetric on the set M/Θ_1. Let $a\leq b$, $\leq c$ $(a, b, c, d\in M)$, $(a, c)\in\Theta_1$, $(b, d)\in\Theta_1$. According to Lemma 8 there exist $z\in a\wedge c$, $t\in b\wedge d$ such that the intervals $[z, a]$, $[z, c]$, $[t, b]$, $[t, d]$ are preserved. Choose $y\in z\wedge t$. Then by Lemma 6 the interval $[y, z]$ is preserved because $b\geq z$, $b\geq t$. By the same argument the interval $[y, t]$ is preserved and consequently the intervals $[y, a]$, $[y, b]$ are preserved. Thus $(a, b)\in\Theta_1$.

Now we shall show that the relation $\leq$ is transitive on the set M/Θ_i. Let $a\leq b$, $d\leq c$ $(a, b, c, d\in M)$, $(b, d)\in\Theta_1$. From Lemma 8 it follows that there exists an element $z\in b\wedge d$ such that the intervals $[z, b]$, $[z, d]$ are preserved. Since $b\wedge a=\{a\}$, there exists an element $t\in z\wedge a$ such that $(t, a)\in\Theta_1$ by Lemma 9. According to Lemma 6 the interval $[t, a]$ is preserved. From Corollary 2 it follows that $(t, a) \in \Theta_1$. Since $(t, a) \in \Theta_1$, $t \leq a$, we get $[a]\Theta_1 = [t]\Theta_1 \leq [c]\Theta_1$. Thus the relation $\leq$ is transitive.

The assertion for Θ_0 (Θ'_i) can be proved analogously.

Lemma 11. $\Theta_0 \wedge \Theta_1 = \omega$ (the identity).

Proof. Let $(x, y) \in \Theta_0$, $(x, y) \in \Theta_1$ $(x, y \in M)$. From Lemma 8 it follows that there exists an element $u_1 \in x \wedge y$ such that the intervals $[u_1, y]$, $[u_1, x]$ are preserved. From the same lemma it follows that there exists $u_2 \in x \wedge y$ such that the intervals $[u_2, x]$, $[u_2, y]$ are reversed. Now we shall show that $u'_1 \in x' \wedge y'$. Choose $u'_3 \in (x' \wedge y')u'_1$. By Lemma 5′ the intervals $[u'_1, u'_3]$, $[u'_3, x']$, $[u'_3, y']$ are preserved and this yields $u_3 \in (x \wedge y)_{u_1}$. Consequently $u_3 = u_1$ and $u'_3 = u'_1$. Since $u'_2 > x'$, $u'_2 > y'$ according to Lemma 6′ we infer that the intervals $[u'_1, x']$, $[u'_1, y']$ are simultaneously preserved and reversed. Hence $x' = u' = y'$. This implies $x = y$.

Lemma 12. *Let $i \in \{0, 1\}$ and let $[a]\Theta_i$, $[b]\Theta_i \in M/\Theta_i$ such that $[a]\Theta_i < [b]\Theta_i$ in $\mathcal{M}/\Theta_i$. Then there exist elements $a_2 \in [a]\Theta_i$, $b_2 \in [b]\Theta_i$ such that $a_2 \prec b_2$ in $\mathcal{M}$.*

Proof. Let $[a]\Theta_i \prec [b]\Theta_i$ for some $i \in \{0, 1\}$. According to the definition of the relation $\leq$ on M/Θ_i there exist elements $a_1 \in [a]\Theta_i$, $b_1 \in [b]\Theta_i$ such that $a_1 < b_1$. Let $a_1 = c_0 < c_1 < \ldots < c_n = b_1$ $(n \geq 1)$ be a maximal chain from a_1 to b_1. Let $j = \min \{k : c_k \in [b]\Theta_i\}$. Then $a_2 = c_{j-1}$, $b_2 = c_j$ are the desired elements.

Lemma 13. *Let $i \in \{0, 1)$ and $[a_j]\Theta_i \in M/\Theta_i$ $(j = 1, 2, 3)$ such that $[a_1]\Theta_i \prec [a_3]\Theta_i$, $[a_2]\Theta_i \prec [a_3]\Theta_i$ in $\mathcal{M}/\Theta_i$. Then there exist elements $a \in [a_1]\Theta_i$, $b \in [a_2]\Theta_i$, $c \in [a_3]\Theta_i$ such that $a \prec c$, $b \prec c$ in the multilattice $\mathcal{M}$.*

Proof. Let $[a_1]\Theta_i \prec [a_3]\Theta_i$, $[a_2]\Theta_i \prec [a_3]\Theta_i$ for some $i \in \{0, 1\}$. Then by Lemma 12 there exist elements $c_1 \in [a_1]\Theta_i$, $c_2 \in [a_2]\Theta_i$, c_3, $c_4 \in [a_3]\Theta_i$ such that $c_1 \prec c_3$, $c_2 \prec c_4$ in $\mathcal{M}$. According to Lemma 8 there exists an element $c \in c_3 \wedge c_4$ such that $(c, a_3) \in \Theta_i$. Choose $a \in c_1 \wedge c$, $b \in c_2 \wedge c$. From Lemma 2 and Lemma 6 it follows that a, b, c have the required property.

Lemma 14. *Let $c, d \in M$, $d \prec c$ and for $i \in \{0, 1\}$, $[c]\Theta_i \neq [d]\Theta_i$. Then $[d]\Theta_i \prec [c]\Theta_i$ in $\mathcal{M}/\Theta_i$.*

Proof. Let $[c]\Theta_0 \neq [d]\Theta_0$. Suppose there is an element $e \in M$ such that $[d]\Theta_0 < [e]\Theta_0 < [c]\Theta_0$. Then there are elements e_1, $e_2 \in [e]\Theta_0$ such that $e_1 \leq c$, $d \leq e_2$. Since $(c, d) \notin \Theta_0$, we have $(c, d) \in \Theta_1$. According to Lemma 8 there is an element $u \in e_1 \wedge e_2$ such that the intervals $[u, e_1]$, $[u, e_2]$ are reversed. Let $z \in u \wedge d$. From Lemma 2 it follows that $[z, u]$ is a prime interval which is preserved by Lemma 3. According to Lemma 6 the interval $[z, d]$ is reversed and if we choose $t \in (e_1 \wedge d)_z$, then the interval $[t, d]$ is reversed as well. From Lemma 4 it follows that the interval $[e_1, c]$ is reversed. Hence $[e_1, c] \in \Theta_0$. This implies $[e]\Theta_0 = [c]\Theta_0$ contradicting $[e]\Theta_0 < [c]\Theta_0$. The assertion for Θ_1 can be proved in the same way.

Lemma 15. *Let $i \in \{0, 1\}$, $[a]\Theta_i$, $[b]\Theta_i$, $[c]\Theta_i$, $[d]\Theta_i \in M/\Theta_i$ and let $[b]\Theta_i \prec [c]\Theta_i$, $[a]\Theta_i \prec [c]\Theta_i$, $[a]\Theta_i \neq [b]\Theta_i$, $[d]\Theta_i < [a]\Theta_i$, $[d]\Theta_i < [b]\Theta_i$ in $\mathcal{M}/\Theta_i$. Then* *there exists* $[u]\Theta_i \in \mathcal{M}/\Theta_i$ *such that* $[u]\Theta_i \prec [a]\Theta_i$, $[u]\Theta_i \prec [b]\Theta_i$ *and* $[d]\Theta_i \leqq [u]\Theta_i$.

Proof. Let the assumptions of the lemma be fulfilled for Θ_0. Then there exist $b_2 \in [b]\Theta_0$, $a_2 \in [a]\Theta_0$, $d_1 \in [d]\Theta_0$, $d_2 \in [d]\Theta_0$ such that $d_2 < a_2$, $d_1 < b_2$. According to Lemma 13 there exist $c_1 \in [c]\Theta_0$, $a_1 \in [a]\Theta_0$, $b_1 \in [b]\Theta_0$ such that $b_1 \prec c_1$, $a_1 \prec c_1$. From Lemma 8 it follows that there exist $a_3 \in a_1 \wedge a_2$, $b_3 \in b_1 \wedge b_2$, $d_3 \in d_1 \wedge d_2$ such that the intervals $[d_3, d_1]$, $[d_3, d_2]$, $[b_3, b_2]$, $[a_3, a_2]$ are reversed. Choose $z_1 \in d_3 \wedge b_3$, $z_2 \in a_3 \wedge d_3$. Then the intervals $[z_1, d_3]$, $[z_2, d_3]$ are reversed by Lemma 6. If $d_4 \in z_2 \wedge z_1$, then the intervals $[d_4, z_2]$, $[d_4, z_1]$ are reversed as well. From this we get $(d_3, d_4) \in \Theta_0$. Moreover $d_4 < a_1$, $d_4 < b_1$. Choose $u \in (a_1 \wedge b_1)_{d_4}$. According to the condition (σ'), $u \prec a_1$, $u \prec b_1$. Since $(c_1, b_1) \in \Theta_1$, $(c_1, a_1) \in \Theta_1$, we infer that $(u, a_1) \in \Theta_1$, $(u, b_1) \in \Theta_1$ by Lemma 3. From Lemma 11 it follows that $(u, a_1) \notin \Theta_0$, $(u, b_1) \notin \Theta_0$ and therefore $[u]\Theta_0 \prec [a]\Theta_0$, $[u]\Theta_0 \prec [b]\Theta_0$ according to Lemma 14. Then $[u]\Theta_0 \geqslant [d]\Theta_0$ because $u \geqslant d_4$.

Lemma 16. *Let* $i \in \{0, 1\}$, $[a]\Theta_i, [b]\Theta_i \in M/\Theta_i$, $[a]\Theta_i \prec [b]\Theta_i$ *in* $\mathcal{M}/\Theta_i$. *If there exist two finite maximal chains in* $\mathcal{M}/\Theta_i$ *from* $[a]\Theta_i$ *to* $[b]\Theta_i$, *then they are of the same length.*

Proof. Let R_1, R_2 be two maximal chains from $[a]\Theta_0$ to $[b]\Theta_0$ and let R_1 be of the length $n \geqslant 1$. We produced by induction on n. If $n = 1$, then $R_1 = R_2$. Suppose that $[b_{n-1}]\Theta_0 \in R_1$, $[b_{n-1}]\Theta_0 \prec [b]\Theta_0$. According to the induction assumption, all finite maximal chains from $[a]\Theta_0$ to $[b_{n-1}]\Theta_0$ have the same lengths $n-1$. If $[b_{n-1}]\Theta_0 \in R_2$, then card R_1 = card R_2. Let $[b_{n-1}]\Theta_0 \notin R_2$. Then there exists $[c]\Theta_0 \in R_2$ such that $[c]\Theta_0 \prec [b]\Theta_0$ and $[b]\Theta_0$, $[b_{n-1}]\Theta_0$ are incomparable. By Lemma 15 there exists $[u]\Theta_0$ such that $[u]\Theta_0 \prec [c]\Theta_0$, $[u]\Theta_0 \prec [b_{n-1}]\Theta_0$ and moreover $[a]\Theta_0 \leqslant [u]\Theta_0$. Let R_3 be a maximal finite chain from $[a]\Theta_0$ to $[u]\Theta_0$. As the chain R_3 is of the length $n-2$, the chain $R_3 \cup \{[c]\Theta_0\}$ is of the length $n-1$. Since by the induction assumption all finite maximal chains from $[a]\Theta_0$ to $[c]\Theta_0$ are of the same length $n-1$, the chain R_2 is of the length n because $[c]\Theta_0 \prec [b]\Theta_0$.

Lemma 17. *Let* $i \in \{0, 1\}$. *The partially ordered set* $\mathcal{M}/\Theta_i = (M/\Theta_i, \leqslant)$ *is of locally finite length.*

Proof. Let $[a]\Theta_i < [b]\Theta_i$. We may suppose that $a < b$. There exists a maximal chain $a = c_0 \prec c_1 \prec \dots \prec c_m = b$, $m \geqq 1$ in the multilattice $\mathcal{M}$. From Lemma 14 it follows that in $\mathcal{M}/\Theta_i$ either $[c_j]\Theta_i = [c_{j+1}]\Theta_i$ or $[c_j]\Theta_i \prec [c_{j+1}]\Theta_i$ $(0 \leqslant j \leqslant m)$. Hence there exists a finite maximal chain R from $[a]\Theta_i$ to $[b]\Theta_i$. If R is of the length $m = 1$, then all maximal chains from $[a]\Theta_i$ to $[b]\Theta_i$ are of the length 1. Suppose that if there is a maximal chain of the length $m-1$ between two elements in $\mathcal{M}/\Theta_i$, then all maximal chains between the same elements are finite. Let R' be a maximal chain from $[a]\Theta_i$ to $[b]\Theta_i$, $R' \neq R$. If all elements of the chain R' are comparable with the element $[c_{m-1}]\Theta_i$, then card R = card R'. Suppose that there exists $[c]\Theta_i \in R'$ such that $[c]\Theta_i$, $[c_{m-1}]\Theta_i$ are incomparable. According to Lemma 15 there exists $[u]\Theta_i \in M/\Theta_i$ such that $[u]\Theta_i \prec [c]\Theta_i$, $[u]\Theta_i < [c_{m-1}]\Theta_i$. From the induction assumption and Lemma 16 it follows that all maximal chains from $[a]\Theta_i$ to $[u]\Theta_i$ are finite and they are of the length $m-2$ at most. This yields that all chains from $[a]\Theta_i$ to $[c]\Theta_i$ are finite and they are of the length $m-1$ at most. Now we show that the maximal chains from $[c]\Theta_i$ to $[b]\Theta_i$ are finite. Let $[c_{m-1}]\Theta_i = [e_0]\Theta_i > [e_1]\Theta_i > \ldots > [e_n]\Theta_i = [u]\Theta_i$ be a maximal chain of the length n. First we show that if $n=1$, then $[c]\Theta_i \prec [b]\Theta_i$. Let $[x]\Theta_i \in M/\Theta_i$ be such that $[c]\Theta_i < [x]\Theta_i < [b]\Theta_i$. According to Lemma 15 there exists $[u_1]\Theta_i \in M/\Theta_i$ such that $[u_1]\Theta_i \geqslant [u]\Theta_i$ and $[u_1]\Theta_i \prec [x]\Theta_i$. Then $[u_1]\Theta_i = [u]\Theta_i$. This yields $[u]\Theta_i \prec [x]\Theta_i$ contradicting $[u]\Theta_i \prec [c]\Theta_i$. Let $n>1$ and $[y]\Theta_i \in M/\Theta_i$ be such that $[c]\Theta_i < [y]\Theta_i < [b]\Theta_i$. Assume that the assertion is valid if some chain between $[c_{m-1}]\Theta_i$ and $[u]\Theta_i$ is of the lenth $p \leqslant n-1$. Since $[u]\Theta_i < [y]\Theta_i$, according to Lemma 15 there exists $[u_2]\Theta_i \in M/\Theta_i$ such that $[u_2]\Theta_i > [u]\Theta_i$ and $[u_2]\Theta_i \prec [y]\Theta_i$. All maximal chains between $[u_2]\Theta_i$, $[c_{m-1}]\Theta_i$ or $[u]\Theta_i$, $[u_2]\Theta_i$, respectively, are of the length $n-1$ at most, hence all maximal chains between $[y]\Theta_i$, $[b]\Theta_i$ or $[c]\Theta_i$, $[y]\Theta_i$, respectively, are finite.

From Lemmas 15, 17 and from the definition of the relation $\leqslant$ on the set M/Θ_i, $i \in \{0, 1\}$ it follows that the partially ordered set $\mathcal{M}/\Theta_i$ ($\mathcal{M}'/\Theta_i'$) is a lower semimodular multilattice.

The following Lemmas 18, 19, 20 can be proved in the same way as the Lemmas 3.6.7, 3.6.8, 3.6.9 in [2].

Lemma 18. a) $[y]\Theta_0 \prec [x]\Theta_0$ *in* $\mathcal{M}/\Theta_0$ *iff* $[y']\Theta_0' \prec [x']\Theta_0'$ *in* $\mathcal{M}'/\Theta_0'$. b) $[y]\Theta_1 \prec [x]\Theta_1$ *in* $\mathcal{M}/\Theta_1$ *iff* $[x']\Theta_1' \prec [y']\Theta_1'$ *in* $\mathcal{M}'/\Theta_1'$.

Lemma 19. $\mathcal{M}'/\Theta_0' \sim \mathcal{M}/\Theta_0$, $\mathcal{M}'/\Theta_1' \sim \mathcal{M}^{\sim}/\Theta_1$.

Lemma 20. *The multilattice* $\mathcal{M}/\Theta_1$ *is directed and modular.*

Let f be a mapping $\mathcal{M} \to \mathcal{M}/\Theta_0 \times \mathcal{M}/\Theta_1$, g' be a mapping $\mathcal{M}' \to \mathcal{M}'/\Theta_0' \times \mathcal{M}'/\Theta_1'$ such that $f(x) = ([x]\Theta_0, [x]\Theta_1)$ for each $x \in M$ and $g'(x') = ([x']\Theta_0', [x']\Theta_1')$ for each $x' \in M'$.

Lemma 21. *Let* $x, y \in M$. *If* $f(x) \prec f(y)$ *in the multilattice* $\mathcal{M}/\Theta_0 \times \mathcal{M}/\Theta_1$, *then* $x \prec y$ *in the multilattice* $\mathcal{M}$.

Proof. From the assumption of the assertion it follows that either

$$(1) \qquad [x]\Theta_0 \prec [y]\Theta_0 \quad \text{and} \quad [x]\Theta_1 = [y]\Theta_1$$

or

$$(2) \qquad [x]\Theta_1 \prec [y]\Theta_1 \quad \text{and} \quad [x]\Theta_0 = [y]\Theta_0.$$

Let the case (1) be valid. Then $(x, y) \in \Theta_1$, $(x, y) \notin \Theta_0$. According to Lemma 12 there exist $x_1 \in [x]\Theta_0$, $y_1 \in [y]\Theta_0$ such that $x_1 \prec y_1$ and $(x_1, y_1) \notin \Theta_0$. This implies $(x_1, y_1) \in \Theta_1$. By Lemma 8 there exists $z_1 \in y \wedge y_1$ such that the intervals $[z_1, y_1]$,

$[z_1, y]$ are reversed. Choose $z_2 \in x_1 \wedge z_1$. From Lemma 3 it follows that $[z_2, z_1]$ is a prime interval which is preserved and by Lemma 6 the interval $[z_2, z_1]$ is reversed Then $z_2' \prec z_1'$ and $y' \leq z_1'$ in $\mathcal{M}'$. Choose $u' \in z_2' \wedge y'$. According to Lemma 3 the interval $[u', y']$ is preserved and by Lemma 6 it is reversed. Since the interval $[x_1', z_2']$ is reversed, we get $(u', x_1') \in \Theta_0'$, $(u', y') \in \Theta_1'$. This implies $(u, x_1) \in \Theta_0$, $(u, y) \in \Theta_1$. We have $(y, x) \in \Theta_1$, hence $(u, x) \in \Theta_1$. From the transitivity of Θ_0 we obtain $(u, x) \in \Theta_0$, because $(x_1, x) \in \Theta_0$. According to Lemma 11, $u = x$. Thus $x \prec y$.

In the case (2) there exist elements $y_1 \in [y]\Theta_1$, $x_1 \in [x]\Theta_1$ such that $x_1 \prec y_1$ according to Lemma 12. Since $(x_1, y_1) \notin \Theta_1$, we have $(x_1, y_1) \in \Theta_0$ and $y_1' \prec x_1'$ in $\mathcal{M}$. From Lemma 8′ it follows that there exists $z_1' \in x' \wedge x_1'$ such that the intervals $[z_1', x_1']$, $[z_1', x']$ are preserved. Choose $z_2' \in y_1' \wedge z_1'$. Then $[z_2', z_1']$ is a prime interval which is reversed by Lemma 3′ and the interval $[z_2', y']$ is preserved according to Lemma 6′. From Lemma 8′ it follows that there exists $t_1' \in y' \wedge z_2'$ such that the intervals $[t_1', z_2']$, $[t_1', y']$ are reversed because $(z_2', y') \in \Theta_1'$. Choose $u' \in (x' \wedge y')_{t_1}$. By Lemma 5′ the interval $[u', y']$ is preserved. From $(x, y) \in O_0$ it follows that there exists $v \in x \wedge y$ such that the intervals $[v, x]$, $[v, y]$ are reversed. Hence $v' \geq x'$, $v' \geq y'$ and the intervals $[x', v']$, $[y', v']$ are reversed. The intervals $[u', y']$, $[u', x']$ are reversed by Lemma 6′. This yields $u' = y'$ and the interval $[y', x']$ is reversed. Hence $x \prec y$. Now we show that $t' \in z_1' \wedge y'$. Choose $t_2' \in (z_1' \wedge y')_{t_1'}$. The interval $[t_1', t_2']$ is preserved by Lemma 5′ because the interval $[t_1', y']$ is preserved. Since the interval $[z_2', z_1']$ is reversed, the interval $[t_1', t_2']$ is reversed by Lemma 6′. Hence $t_2' = t_1'$ and $t_1' \in z_1' \wedge y'$. Using Lemma 6′ we get that the interval $[t_1', z_2']$ is reversed. This yields $t_1' = z_2'$. We have that $x' \in z_1' \vee y'$ (in fact, if $r' \in (z_1' \vee y')_{x'}$, then the interval $[r', x']$ is simultaneously preserved and reversed, hence $r' = x'$). From this we obtain that $[y', x']$ is a prime interval by Lemma 7.

Lemma 22. *Let $\mathcal{T} = (T; \leq)$, $\mathcal{T}' = (T, \subseteq)$ be the images of the multilattices $\mathcal{M}, \mathcal{M}'$ under the mappings f, g'. Then f, g' are isomorphisms of the multilattices $\mathcal{M}, \mathcal{M}'$ onto the multilattices $\mathcal{T}, \mathcal{T}'$.*

Proof. According to Lemma 17 the multilattice $\mathcal{M}/\Theta_0 \times \mathcal{M}/\Theta_1$ is of locally finite length. Hence the multilattice $\mathcal{T}$ is of locally finite length. Let $x, y \in M$ such that $x \prec y$. From Lemma 11 it follows that either $(x, y) \notin \Theta_0$, or $(x, y) \notin \Theta_1$. Let $(x, y) \notin \Theta_0$. Then $[x]\Theta_0 \prec [y]\Theta_0$ in $\mathcal{M}/\Theta_0$ by Lemma 14. Since $x \prec y$ and $(x, y) \notin \Theta_0$ we have $(x, y) \in \Theta_1$. Hence $f(x) = ([x]\Theta_0, [x]\Theta_1) \prec f(y) = ([y]\Theta_0, [y]\Theta_1)$ in $\mathcal{M}/\Theta_0 \times \mathcal{M}/\Theta_1$. If we assume $(x, y) \notin \Theta_1$, we arrive at the same conclusion.

Let us assume that $f(x) \prec f(y)$ in $\mathcal{M}/\Theta_0 \times \mathcal{M}/\Theta_1$. From Lemma 21 it follows that $x \prec y$ in $\mathcal{M}$. Since the ordering relations on $\mathcal{T}$ and $\mathcal{M}$ are uniquely determined by the corresponding covering relations, we have $\mathcal{M} \sim \mathcal{T}$. Analogously we can prove $\mathcal{M}' \sim \mathcal{T}'$.

Let $k_0 : [x']\Theta_0' \mapsto [x]\Theta_0$, $k_1 : [x']\Theta_1' \mapsto [x]\Theta_1$ be the isomorphisms from Lemma 19. Denote $\mathcal{M}/\Theta_0 = \mathcal{A}$, $\mathcal{M}/\Theta_1 = \mathcal{B}$, $\mathcal{M}'/\Theta_0' = \mathcal{A}'$, $\mathcal{M}'/\Theta_1' = \mathcal{B}'$. Then $k_0 \times k_1$ is an isomorphism of $\mathscr{A}' \times \mathscr{B}'$ onto $\mathscr{A} \times \tilde{\mathscr{B}}$. If we denote $g = (k_0 \times k_1) \circ g'$, then the mapping g is an isomorphism from $\mathscr{M}'$ to $\mathscr{A} \times \tilde{\mathscr{B}}$ and $\operatorname{Im} f = \operatorname{Im} g$.

From the definition of the mappings f, g it follows that the projection of $\operatorname{Im} f$ ($\operatorname{Im} g$) to the set $A(B)$ is the whole set $A(B)$.

By summarizing, we obtain the following theorem:

Theorem 2. Let $\mathscr{M}$, $\mathscr{M}'$ be lower semimodular multilattices of a locally finite length and let φ be an isomorphism of the graph $G(\mathscr{M})$ onto the graph $G(\mathscr{M}')$ such that no elementary square of the multilattice $\mathscr{M}$, $\mathscr{M}'$ respectively is broken by φ, φ^{-1} respectively. Then there exist a lower semimodular multilattice $\mathscr{A}$, a modular multilattice $\mathscr{B}$ ($\mathscr{A}$, $\mathscr{B}$ of a locally finite length) and subdirect representations $f: \mathscr{M} \to \mathscr{A} \times \mathscr{B}$, $g: \mathscr{M}' \to \mathscr{A} \times \tilde{\mathscr{B}}$ such that $\operatorname{Im} f = \operatorname{Im} g$.

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Received January 21, 1980

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МУЛЬТИСТРУКТУРЫ С ИЗОМОРФНЫМИ ГРАФАМИ

Мария Томкова

Резюме

В этой статье обобщены две теоремы М. Колибиара, кассающиеся пар полуструктур. Если M, M', A, B снизу направленные мультиструктуры локально конечной длины и $f: M \to A \times B$, $g: M' \to A \times \bar{B}$ являются полупрямыми представлениями такими, что $Im\, f = \operatorname{Im} g$, то графы $G(M)$, $G(M')$ изоморфны. Найдено условие, при котором справедливо обратное утверждение в случае снизу модулярныцх мультиструктур M, M'.