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ON CONVERGENCE GRUPOIDS

JÁN ŠIPOŠ

The present paper deals with some type of convergence grupoids. As a result we obtain a generalization of some basic facts which are valid for compact topological semigroups, for not necessarily compact semigroups, and for grupoids of a certain type.

The idea of studying such grupoids has been inspired by the usefulness of some non-associative algebras.

§ 1. Preleminaries

A grupoid is a set S together with a binary operation (i.e. a function from the Cartesian product $S \times S$ into S), which in the following will be denoted multiplicatively.

A grupoid S is a quasigroup iff each of the equations

$$ax = b \quad \text{and} \quad ya = b$$

(a and b are from S) have unique solution with respect to the unknows x and y.

A grupoid is a semigroup provided the multiplication is associative, i.e., if $a(bc) = (ab)c$ for all a, b and c in S.

A subset T of a grupoid is called subgrupoid iff

$$TT \subset T.$$

If A is a subset of a grupoid, the intersection of all grupoids including A is called the subgrupoid generated by A. It consists of all finite products of elements of A.

In this paper we shall deal only with grupoids which are "almost" associative in the following sense: Let a and b be arbitrary elements of S. Let us assume that all posible products in S which one can construct by the help of these two elements are independent of the way in which brackets are used. For example, this means that

$$(ab)b = a(bb)$$
$$(ab)(ba) = a(b(ba)),$$

and so on. Such grupoids will be called *alternative* grupoids. In other words, the grupoid S is alternative iff every its subgrupoid, generated by two elements, is a semigroup. There is another important class of grupoids we shall deal with. We say that the grupoid S has *associative powers* iff every its one element generated subgrupoid is a semigroup.

In such grupoids the power a^n of an element a is unambiguously defined.

We say that an element e of a grupoid S is an idempotent iff $ee = e$. If e and f are idempotents of S we put $e \leqq f$ iff $ef = fe = e$. The set of all idempotents will be denoted by E. An idempotent e from S is called a primitive idempotent if there exists no idempotent $f \in S$, $f \neq e$ ($f \neq$ zero if S contains a zero element) for which $e \leqq f$ holds.

A *convergence space* F is a set F with a distinct class of sequences $\{a_n\}$ $(a_n \in F)$ which are called convergent. We assume that to each convergent sequence there corresponds a unique element a of F called the limit of the sequence and denoted by $a = \lim_n a_n$ (sometimes we write simply $a_n \to a$) such that $\lim_n a_n = a$ if $a_n = a$ for $n = 1, 2, \ldots$.

We assume also that if $a_n \to a$, then $a_{n_k} \to a$, where $\{a_{n_k}\}$ is a subsequence of $\{a_n\}$.

We do not assume that this convergence is determined by a topology.

The closure vA of a set $A \subset F$ is a set of all limits of all convergent sequences $\{a_n\}$ taking their values in A (i.e. $a_n \in A$).

A is called closed if $vA = A$. By $\bar{A}$ we denote the smallest closed set containing A.

Let $\langle 0, \Omega \rangle$ be the set of all countable ordinals and the first uncountable ordinal Ω. We put $v^0 A = A$, $v^1 A = vA$, $v^\xi A = vv^{\xi-1}A$ or $v^\xi A = \bigcup_{\eta<\xi} v^\eta A$ according to whether $\xi - 1$ exists or not. It is a well-known fact that $v^\Omega A = \bar{A}$. We note that the closure operation $A \to \bar{A}$ defines a topology for F in the usual way.

A *convergence grupoid* is a grupoid S provided with a convergence structure in which multiplication is continuous, i.e., if $a_n \to a$ and $b_n \to b$, then $a_n b_n \to ab$ (the elements a_n, b_n, a and b being in S).

A convergence grupoid S is called *sequentially point compact* iff S is with associative powers and iff every subsequence of $\{a^n\}$ contains a convergent subsequence for every a in S.

A convergence grupoid S is called *sequentially compact* iff every sequence $\{a_n\}$ of elements from S contains a convergent subsequence.

§2. Examples

We give examples to present objects we are interested in.

Example 1. Let S be a sequentially compact or compact topological semigroup. Clearly S is a sequentially point compact grupoid.

Example 2. Let S be the family of all real functions f defined on a space X, which takes only rational values from $\langle -1, 1\rangle$. We define fg by $(fg)(x)=f(x)g(x)$, the convergence being pointwise. S is clearly a sequentially point compact convergence semigroup. The only idempotents of this semigroup are the characteristic functions of subsets of X. Note that S is not sequentially compact.

Example 3. Let $\mathscr{C}$ be the unit ball of Cayley numbers (hypercomplex numbers of real dimension 8). It is known that with respect to the multiplication $\mathscr{C}$ is a compact alternative grupoid. Let us denote by $\mathscr{C}_1$ the set of all Cayley numbers with absolute value one, then clearly $\mathscr{C}_1$ is an alternative quasigroup which is compact.

Example 4. Let $\mathscr{F}$ be the following subset of $L_1\langle 0,1\rangle$. $\mathscr{F}=\{f;\ |f|\leqq 1\}$. Define the multiplication as in Example 2.

Then $\mathscr{F}$ is a commutative semigroup. $\mathscr{F}$ is a convergence semigroup with respect to the almost everywhere convergence.

Example 5. S is a finite grupoid with respect to the trivial convergence (convergent sequences are exactly the constant sequences).

Example 6. Let S be a torsion grupoid (i.e. a grupoid with associative powers in which every element is of finite order). Then S is a sequentially point compact grupoid with respect to the trivial convergence.

§ 3. Basic results

Lemma 7. *Let S be a convergence grupoid. Let $T\subset S$ be a subgrupoid of S. Then*

(i) vT is a grupoid;

(ii) If T is commutative, then vT is also commutative;

(iii) If T is a semigroup, then vT is also a semigroup;

(iv) If T is a quasigroup and S is sequentially compact, then vT is a quasigroup.

Proof. *(i)* and *(ii)*. Let a, $b\in vT$. Then there exist a_n, $b_n\in T$ such that $a_n\to a$ and $b_n\to b$. Clearly $a_n\cdot b_n\in T$ and $a_n\cdot b_n\to ab$. If now T is commutative, then

$$ab=\lim_n a_nb_n=\lim_n b_na_n=ba.$$

(iii) Let T be a semigroup. By *(i)* of this lemma vT is a grupoid. We must show the associativity of vT. Let a, b and c be in vT. Choose a_n, b_n and c_n from T with $a_n\to a$, $b_n\to b$ and $c_n\to c$. We have

$$a(bc)=\lim_n a_n(b_nc_n)=\lim_n (a_nb_n)c_n=(ab)c.$$

(iv) We show that the equations $ax=b$ and $ya=b$ have solutions in vT if a and b are in vT. Let $a_n\to a$ and $b_n\to b$, a_n, $b_n\in T$. Then, since T is a quasigroup, there exists an $x_n\in T$ such that $a_nx_n=b_n$. Let x_{n_k} be a convergent subsequence of $\{x_n\}$ with $\lim_k x_{n_k}=x$. Then clearly $x\in vT$ and $ax=b$. The argumentation for the solvability of $ya=b$ is similar.

Lemma 8. *Let S be a convergence grupoid. Let T be a subgrupoid of S; then*

(i) *$\bar{T}$ is a grupoid;*

(ii) *If T is commutative, then $\bar{T}$ is also commutative;*

(iii) *If T is a semigroup, then $\bar{T}$ is also a semigroup;*

(iv) *If T is a quasigroup and S is sequentially compact, then $\bar{T}$ is a quasigroup.*

Proof. Let $\{T_\alpha;\ \alpha \in A\}$ be a family of grupoids (commutative grupoids, semigroups, quasigroups) which is directed by inclusion. Then clearly $\cup\{T_\alpha;\ \alpha \in A\}$ is also a grupoid (commutative grupoid, semigroup, quasigroup). The proof of the lemma follows now from the preceding lemma and the fact that $\bar{T} = v^{\Omega}T = \bigcup_{\xi<\Omega} v^{\xi}T$.

Lemma 9. *Let S be a sequentially point compact grupoid. Let a be in S. Let $A(a)$ be the set of all limits of subsequences of $\{a^n\}$. Then $A(a)$ is a commutative subgroup of S.*

Proof. Let $B = \{a, a^2, \ldots, a^n, \ldots\}$. $\bar{B}$ is clearly a commutative subsemigroup of S. Since $A(a) \subset \bar{B}$, we have that $A(a)$ is also a commutative semigroup. We shall show that for every $x,\ y \in A(a)$ there exists a $z \in A(a)$ such that $xz = y$.

Let $a^{n_k} \to x$ and $a^{m_k} \to y$. We may assume that the sequence $\{m_k - n_k\}$ is increasing and that the sequence $\{a^{m_k - n_k}\}$ is convergent. Denote the limit of the last mentioned sequence by z. Then obviously $z \in A(a)$ and $xz = y$.

An immediate consequence of the last lemma is the following theorem which generalizes the existence theorem of idempotents (see Nukamura [1] and Schwarz [2]).

Theorem 10. *Every sequentially point compact grupoid contains at least one idempotent.*

Let S be a sequentially point compact grupoid. We say that an element $a \in S$ belongs to an idempotent $e \in S$ iff there exists an increasing sequence $\{n_k\}$ such that $a^{n_k} \to e$. We denote by K_e the set of elements from S belonging to e. Since every sequence has at most one limit, we obtain that every element of S belongs exactly to one idempotent, and so the following theorem holds true.

Theorem 11. *Every sequentially point compact grupoid may be written as a disjoint union of its subsets*

$$S = \{K_e;\ e \in E\},$$

where E is a set of all idempotents from S.

§ 4. Maximal semigroups and maximal groups

Let e be an idempotent of a sequentially point compact grupoid S. We say that $P \subset S$ is a maximal semigroup belonging to e iff P is a semigroup which contains only one idempotent e and P is a maximal semigroup with this property.

Lemma 12. *Let e be an idempotent of a sequentially point compact grupoid S. There exist at least one maximal semigroup belonging to e.*

Proof. The proof is a standard application of the Hausdorff maximality principle.

It is obvious that every element of P belongs to the same idempotent. We note that the idempotent e need not be a unit element of the semigroup P. It is also true that two maximal semigroups P_e and P_f belonging to the idempotents $f \neq g$ are disjoint.

By Lemma 9 there exists in every sequentially point compact grupoid at least one group. Using again the Hausdorff maximality principle it is easy to see that every subgroup of S is included in a maximal one. The question arrises: Which elements may be covered by a subgroup of S?

We say that an element $a \in K_e$ is regular iff $ae = ea = a$. We shall show that every regular element (and only these elements) can be covered by a subgroup of S.

Lemma 13. *An element $a \in K_e$ is regular iff the closure of the set $(a, a^2, \ldots, a^n, \ldots\}$ is a group.*

Proof. If the set $v\{a, a^2, \ldots, a^n, \ldots\}$ is a group then a is clearly regular, since in this case e is a unit of this group and so $ae = ea = a$. Let now a be a regular element from K_e. By Lemma 9 $A(a)$ the set of all cluster points of $\{a, a^2, \ldots, a^n, \ldots\}$ is a group. We show that $v\{a, a^2, \ldots, a^n, \ldots\} = A(a)$. Let $a^{n_k} \to e$. Then $a \cdot a^{n_k} \to e \cdot a = a$ by the regularity of a and so a is in $A(a)$. It is now clear that $a^n \in A(a)$ and so $A(a) = v\{a, a^2, \ldots, a^n, \ldots\}$, which proves the assertion, since $A(a)$ is a group.

Using again the Hausdorff maximality principle we get:

Lemma 14. *Every regular element $a \in S$ of a sequentially point compact grupoid S is contained in a maximal subgroup of S.*

The following is also obvious.

Lemma 15. *The sequentially point compact grupoid S is a union of its subgroups iff every its element is regular.*

Let us denote by H_e the set of all regular elements from K_e, then H_e is a union of all subgroups containing the idempotent e.

Lemma 16. $K_e \cdot e = e \cdot K_e = H_e$.

Proof. Let $a \in H_e$; then $a \cdot e = a$ and so $a \in K_e \cdot e$. Let now $a \in K_e$. By Lemma 7 *(iii)* $v\{a, a^2, \ldots, a^n, \ldots\}$ is a semigroup which contains a and e, so

$$(a\cdot e)\cdot e=a\cdot(e\cdot e)=a\cdot e.$$

Since $v\{a, a^2, \ldots, a^n, \ldots\}$ is commutative, we get that $a\cdot e$ is a regular element, i.e., $a\cdot e\in H_e$. The argumentation for $e\cdot K_e=H_e$ is similar.

The question arises whether H_e is a grupoid? The answer is negative even if S is a finite commutative grupoid with associative powers as the following example shows.

Example 17. Let $S=\{e, a, b, x, 0\}$ be the grupoid with the multiplication table

	e	a	b	x	0
e	e	a	b	x	0
a	a	e	x	x	0
b	b	x	e	x	0
x	x	x	x	0	0
0	0	0	0	0	0.

In this example $K_e=H_e=\{e, a, b\}$, which is not a grupoid. Since $a\cdot b=x$ and x is not a regular element, we get that the product of two regular elements belonging to the same idempotent need not be even regular. Observe that the grupoid S is not alternative since $a\cdot(a\cdot b)=a\cdot x=x$ and $(a\cdot a)\cdot b=e\cdot b=b$.

§ 5. The alternative case

We turn now our attention to the sequentially point compact alternative grupoids. We shall show that in this case the structure of S is much similar to the case when S is a compact semigroup. In what follows we give a necessary and sufficient condition for K_e being a grupoid if S is alternative.

Lemma 18. *Let S be an alternative sequentially point compact grupoid. Let the elements x and y from S belong to the idempotent e and let xy belong to the idempotent f. Then $ef=fe=e$ and xye belongs to e.*

Proof. Denote by P the subgrupoid of S generated by the elements x and y. P is a subsemigroup of S and so by Lemma 8 $\bar{P}$ is also a subsemigroup which contains the elements x and y. Let G_e be a unique maximal subgroup of $\bar{P}$ containing the idempotent e. By the assumption

$$x^{n_k}\to e,\quad y^{m_k}\to e\quad\text{and}\quad (xy)^{l_k}\to f$$

for suitable sequences $\{n_k\}$, $\{m_k\}$ and $\{l_k\}$. Since $x^{n_k+1}\to xe$ and $y^{m_k+1}\to ye$, we have $xe\in A(x)$ and $ye\in A(y)$ (see Lemma 9). $A(x)$ and $A(y)$ are groups containing the idempotent e and so they must be included in the maximal group G_e. Thus xe and ye are in G_e and by this we get

$$(xe)(ye) = xye \in G_e \subset K_e,$$

(note that e commutes with all elements from K_e) and so

$$(xye)^{l_k} = (xy)^{l_k} \cdot e \to fe.$$

Similarly

$$(exy)^{l_k} \to ef$$

but $exy = xye$ and so $fe = ef$. It is now clear that ef is an idempotent. It must coincide with e since every element (hence xye too) from G_e belongs to e. So we have $ef = fe = e$. This completes the proof, since the second assertion of the lemma has been established above.

We say that an idempotent e is maximal iff $ef = fe = e$ implies $f = e$. As a corollary of the last lemma it follows:

Lemma 19. *If e is a maximal idempotent of a sequentially point compact alternative grupoid (especially if e is a unit of S), then K_e is a grupoid.*

Lemma 20. *Let S be a sequentially point compact alternative grupoid which satisfies the following condition: If x and y are from S, x belongs to the idempotent e and if xy belongs to the idempotent f, then f commutes with x. Then $ef = f$.*

Proof. Let $\bar{P}$ be the same as in the last lemma. Let G_f be the maximal subgroup of $\bar{P}$ containing f. Clearly $x \cdot yf \in G_f$ and so $xyG_f = xy(fG_f) = (xyf)G_f = G_f$. Thus there exists an $a \in G_f$ with $xyfa = f$. Put $yfa = t$. Then $f = xt$ and

$$f = f^2 = f(xt) = xft = xxtt = x^2 \cdot t^2.$$

Similarly

$$f^{n_k} = x^{n_k} \cdot t^{n_k}.$$

We may assume that $t^{n_k} \to b$ and so $f = eb$. Multiplying the last identity from the left by e we have

$$ef = e(eb) = eb = f$$

and the lemma is proved.

Theorem 21. *Let S be a sequentially point compact alternative grupoid. The following condition is necessary and sufficient for K_e being a grupoid for every idempotent e in S:*

If x and y belong to the same idempotent, e and xy belong to the idempotent f. Then $xf = fx$.

Proof. The necessity is trivial. If now the condition of theorem is valid, then $ef = e$ by Lemma 18 and $ef = f$ by the last lemma, and so $e = f$.

We shall need some other notions. A grupoid S is said to be *normal* iff $xS = Sx$ for every x in S. S is said *totally noncommutative* iff E contains at least two elements and $ef \neq fe$ for every e and f in E with $e \neq f$ (E denotes the set of all idempotents from S).

Theorem 22. *Let S be a sequentially point compact alternative grupoid. Each of the following conditions implies that the sets K_e are grupoids:*

(i) S is totally non-commutative;

(ii) E is contained in a centre;

(iii) S is commutative;

(iv) S is normal.

Proof. *(i)* Let S be totally non-commutative. Let $x, y \in K_e$, and let $xy \in K_f$. By Lemma 18 $ef = fe$, and so $e = f$. *(ii)* is a clear consequence of the last theorem. *(iii)* follows from *(ii)* of this theorem. *(iv)* Let S be normal. We show that E is contained in a centre. Let x be from S and e from E. By $eS = Se$ there exists an element $u \in S$ with $ex = ue$, and an element $v \in S$ with $xe = ev$. Now $ex = ue$ implies $(ex)e = (ue)e = ue = ex$, and $xe = ev$ implies $e(xe) = ev = xe$. Hence $ex = xe$ by the alternativity of S, which proves that e is contained in a centre. The assertion now follows by *(ii)* of this theorem.

§ 6. The structure of regular elements

Recall that an element a in K_e is called regular iff $ae = ea = a$. In his paper [2] Schwarz proved that the set of all regular elements belonging to e forms a maximal group included in K_e. This is not true in a general alternative grupoid. In fact the grupoid of all unit Cayley numbers demonstrates a situation when the unit is contained in more than one maximal subgroup. In spite of this fact we are able to give a theorem which completely describes the structure of all regular elements belonging to the same idempotent.

Lemma 23. *Let S be a sequentially point compact alternative grupoid. Let H_e be the set of all regular elements from K_e. Then H_e is a union of all maximal subgroups containing e, and H_e is a quasigroup, (the only maximal quasigroup in S containing e).*

Proof. Using the Hausdorff maximality principle it is easy to see that every group is included in a maximal group and that every quasigroup is included in a maximal quasigroup. A union of all maximal subgroups containing e is clearly a subset of H_e. If now $a \in K_e$ is regular, then by Lemma 13 a is contained in a subgroup and so in a maximal subgroup of K_e. Thus we have proved that H_e is a quasigroup. Let $a, b \in H_e$. Let P be the grupoid generated by the elements a and b. By the alternativness of S and by Lemma 8. $\bar{P}$ is a semigroup. Clearly a, a^{-1}, b and b^{-1} are in $\bar{P}$ (where a^{-1} is the inverse element of a with respect to the idempotent e (the meaning of b^{-1} is similar)). Let G be the subgroupoid of S, generated by the elements a, b, a^{-1} and b^{-1}. Then G is a group which contains e. Hence by the first part of the proof $ab \in G \subset H_e$. The equations $ax = b$ and $ya = b$ have always a solution in G and hence also in H_e.

As an immediate consequence of the last theorem we obtain:

Theorem 24. *A sequentially point compact grupoid S is a disjoint union of its sub-quasigroups iff every element of S is regular.*

The following lemma is an interesting consequence of the last theorem.

Lemma 25. *If a sequentially point compact grupoid has a unit and contains only one idempotent, then it is a quasigroup.*

The equivalence relation connected with the partition of S into sets K_e need not be a congruence in general. Hence a question arises under which condition the following is valid.

To every pair e and f of idempotents there exists an idempotent g such that $K_e \cdot K_f \subset K_g$.

We give now a sufficient condition.

Theorem 26. *Let S be a sequentially point compact alternative grupoid in which the set of all idempotents E is contained in a centre. Then $x \in K_e$ and $y \in K_f$ implies $xy \in K_{ef}$.*

Proof. Denote by P the subgrupoid of S generated by the elements x and y. By Lemma 8 $\bar{P}$ is a subsemigroup of S.

Let $x^{n_k} \to e$, $y^{m_k} \to f$ and $(xy)^{l_k} \to g$. Then $(xf)^{n_k} = x^{n_k} \cdot f \to ef$ and $(ey)^{m_k} = e \cdot y^{m_k} \to ef$. (The idempotents e, f and g are in $\bar{P}$). And so xef and yef are in H_{ef}. Since H_{ef} is a quasigroup $(xef)(yef) = xyef$ is in H_{ef} also but

$$(xyef)^{l_k} \to gef.$$

H_{ef} contains only one idempotent, hence $gef = ef$. Now by Lemma 20 we have that $eg = g$. Similarly one can get $gf = g$ and so $ef = efg = eg \cdot fg = g \cdot g = g$, which completes the proof.

§ 7. The sequentially compact case

Lemma 27. *If S is a sequentially compact grupoid, then every its maximal quasigroup is closed.*

Proof. Let H be a maximal quasigroup of S. Then by Lemma 7 vH is also a quasigroup of S and so $vH = H$, since H is maximal.

Combining this result with Lemma 23 we have:

Theorem 28. *If S is a sequentially compact alternative grupoid, then the sets H_e are closed.*

For the rest of this paper S will be a sequentially compact alternative grupoid.

Now we will study the sets K_e with respect to the convergence. It is obvious that K_e need not be closed. In this case, as we shall show, $\bar{K}_e$ contains an idempotent different from e.

Lemma 29. *Let K_e be non closed. Then v^2K_e contains an idempotent f different from e.*

Proof. Let $a \in vK_e - K_e$. Then $a^n \in vK_e$ for all n. According to lemma 9 there exists an idempotent f for which $a^{n_k} \to f$ holds. Clearly $f \in v^2K_e$ and $f \neq e$, since a does not belong to e.

It is true that $v^2K_e \subset \bar{K}_e$ and so we have:

Theorem 30. *If K_e is not closed, then $\bar{K}_e$ contains an idempotent different from e.*

Lemma 31. *Let ξ be a countable ordinal. Let $v^\xi K_e \cap K_f \neq \emptyset$; then $f \in v^{\xi+1}K_e$.*

Proof. Let $a \in v^\xi K_e \cap K_f$: Then $a^n \in v^\xi K_e \cap K_f$. Let $a^{n_k} \to f$; then clearly $f \in v^{\xi+1}K_e$.

The consequence of this lemma is the following:

Theorem 32. *Let $\bar{K}_e \cap K_f \neq \emptyset$; then $f \in \bar{K}_e$.*

Theorem 33. $e \cdot K_e = K_e \cdot e = H_e$.

Proof. Let $a \in vK_e$; then there exists a sequence $a_n \in K_e$ with $a_n \to a$. Obviously $a_n \cdot e \in H_e$ and $a_n \cdot e \to a \cdot e$. By Theorem 28 H_e is closed and so $ae \in H_e$. We get

$$(vK_e) \cdot e = H_e.$$

Similarly one can get that

$$(v^\xi K_e) \cdot e = e \cdot (v^\xi K_e) = H_e$$

for every countable ordinal ξ, and so

$$K_e \cdot e = e \cdot K_e = H_e.$$

Theorem 34. *Let f be an idempotent with $f \in \bar{K}_e$. Then $ef = fe = e$, i.e. $e \leqq f$.*

Proof. $ef \in e \cdot \bar{K}_e = H_e$, and so ef is in the quasigroup H_e. Since S is alternative and e is a unit of H_e,

$$ef \cdot ef = ((ef)e)f = ef \cdot f = e \cdot ff = ef,$$

and so ef is an idempotent of the quasigroup H_e, it must coincide with the unit of H_e. We have $ef = e$. The argumentation for $fe = e$ is similar.

It is now interesting whether $\bar{K}_e \cap K_f \neq \emptyset$ implies $K_f \subset \bar{K}_e$. We are able only to give a partial solution of this problem.

Lemma 35. *Let E be included in the centre of S. Then $a \in K_f$ and $b \in \bar{K}_e$ implies $ab \in \bar{K}_{ef}$.*

Proof. Let $a \in K_f$ and $b \in vK_e$. Then there exists a sequence $\{b_n\}$ in K_e such that $b_n \to b$. By theorem 26 $ab_n \in K_{ef}$ for $n = 1, 2, 3, \ldots$. Hence $ab \in vK_{ef}$ because $ab_n \to ab$. By transfinite induction one can prove that $a \in K_f$ and $b \in v^\xi K_e$ implies $ab \in v^\xi K_{ef}$ and so $a \in K_f$ and $b \in \bar{K}_e$ implies $ab \in \bar{K}_{ef}$.

Theorem 36. *Let E be included in the centre of S. Then* $\bar{K}_e \cap K_f \neq \emptyset$ *implies* $H_f \subset \bar{K}_e$.

Proof. By Theorem 32 $f \in \bar{K}_e$. Let $a \in H_f$; then $a \in K_f$ and $f \in \bar{K}_e$ implies by the last lemma that $a = af \in \bar{K}_{ef}$ (since a is regular). By Theorem 34 $ef = e$ which completes the proof.

REFERENCES

[1] NUMAKURA, K.: On bicompact semigroups. Math. J. of Okayama Univ., Vol. 1, 1952, 99—108.
[2] SCHWARZ, Š.: On Hausdorff bicompact semigroups. Czech. Math. J., 5, 80, 1955, 1—23.

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ГРУППОИДЫ СХОДИМОСТИ

Йан Шипош

Резюме

Целью этой статьи является в основном перенесение некоторых результатов о строении хаусдорфовых бикомпактных полугрупп на специальные группоиды сходимости.