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ON THE CARATHÉODORY METHOD OF THE EXTENSION OF MEASURES AND INTEGRALS

BELOSLAV RIEČAN

Everybody knows the Carathéodory method of the extension of a measure by using the induced outer measure and its restriction to the family of all measurable sets. Sometimes the idea of the method is used also for the extension of the Daniell integral (see e.g. [7] and [13]). In [13] both processes (for the measure and for the integral) are presented, the latter following the former, so suggestively that we have tried to construct a general theory including these special cases. Here we present the result of our investigation: an extension theory resembling the Carathéodory method for real-valued functions $J: S \to R$ defined on a sublattice S of a given lattice.

Of course, Topsoe works with an inner measure (a lower integral on non-negative functions, resp.) instead of an outer one. Therefore we follow two ways: the first is "upper" and the second is "lower". Unfortunately, these two ways are not symmetric. So in the first part of the article we present a generalization of the usual Carathéodory method and in the second part a generalization of the Topsoe considerations.

Recall that similar extension theories unifying the measure theory and the integration theory were constructed in [1], [2], [4], [8], [9], [11] and [12]. A review of the field of investigations with references is contained in [10].

1

First some notations. If $(x_n)_{n=1}^{\infty}$ is a sequence of elements of a lattice H such that $x_n \leqq x_{n+1}$ $(n = 1, 2, \dots)$ and $x = \bigvee_{n=1}^{\infty} x_n$, then write $x_n \nearrow x$. If $x_n \geqq x_{n+1}$ $(n = 1, 2, \dots)$ and $x = \bigwedge_{n=1}^{\infty} x_n$, then we write $x_n \searrow x$.

Now the assumptions. There is given a distributive, relatively σ-complete, σ-continuous lattice H with the least element O. Here σ-continuity means that $x_n \nearrow x$, $y_n \nearrow y$ (or $x_n \searrow x$, $y_n \searrow y$, resp.) implies $x_n \wedge y_n \nearrow x \wedge y$ $(x_n \vee y_n \searrow x \vee y$, resp.)

Relative σ-completness means that every countable bounded subset of H has the least upper bound and the greatest lower bound.

On the lattice H there are given two binary operations $+$, satisfying the following conditions:

1) $+$ is commutative.
2) If $x \leqq y$ and $z \in H$, then $x + z \leqq y + z$, $z \backslash x \geqq z \backslash y$, $y \backslash z \geqq x \backslash z$.
3) $x = (x \wedge y) + (x \backslash y)$ for all $x, y \in H$.
4) $(x \backslash y) \vee (x \backslash z) = x \backslash (y \wedge z)$ for all $x, y, z \in H$.
5) $(x \backslash y) \wedge (x \backslash z) = x \backslash (y \vee z)$ for all $x, y, z \in H$.
6) If $y_n \searrow y$ and $x \in H$, then $x \backslash y_n \nearrow x \backslash y$.

The basic examples of the presented structure are the following two: 1) A ring H of sets (or more general a Boolean algebra H) with $+$ as the set-theoretic union and $\backslash$ as the set-theoretic difference. 2) A positive cone H of real-valued functions (or more general the set of all positive elements of an Abelian lattice ordered group) with $+$ as the sum of two functions and $f \backslash g = f - \min(f, g)$($f \backslash g$ must be a positive function).

But let us go on. For constructing the Carathéodory measurability process we need an initial function. Denote it by J_0 and its domain by B. Hence we have a sublattice B of the lattice H closed under the operation $+$. As regards B we assume further that to any $x \in H$ there is such a $b \in B$ that $b \geqq x$. Finally, we assume that there is given a mapping $J_0 : B \to R \cup \{\infty\}$ satisfying the following conditions:

(*i*) $J_0(O) = 0$.
(*ii*) If $x \leqq y$, $x, y \in B$, then $J_0(x) \leqq J_0(y)$.
(*iii*) $J_0(x) + J_0(y) \geqq J_0(x \vee y) + J_0(x \wedge y)$ for every $x, y \in B$.
(*iv*) If $x_n \nearrow x$, $x_n \in B$, $x \in H$ $(n = 1, 2, \ldots)$, then $x \in B$ and $J_0(x) = \lim_{n \to \infty} J_0(x_n)$.
(*v*) $J_0(x + y) \leqq J_0(x) + J_0(y)$ for every $x, y \in B$.

Definition 1.1 *For every $x \in H$ we put*

$$J^*(x) = \inf\{J_0(b);\ b \geqq x,\ b \in B\}.$$

What is the meaning of J^* in our classical examples? In the measure theory J^* is essentially the induced outer measure. If μ is a non-negative measure defined on a ring A, then the induced outer measure is given by the equality

$$\mu^*(x) = \inf\left\{\sum_{i=1}^{\infty} \mu(a_i);\ x \subset \bigcup_{i=1}^{\infty} a_i,\ a_i \in A\right\}.$$

The same effect can be obtained if we put first $B = \{b;\ \exists a_n \in A,\ a_n \nearrow b\}$ and $J_0(b) = \lim_{n \to \infty} \mu(a_n)$ and then $\mu^*(x) = \inf\{J_0(b);\ b \supset x,\ b \in B\}$. By the way, the modification of the Carathéodory method is used in the known book [6] by Neveu on the probability theory.

In our second example J^* can be called an upper integral. This construction (from an elementary integral $J: A \to R$ through B, J_0 to H, J^*) was used in many papers and books, e.g. in [1], [8], [9] or in Krickeberg's well-known book [5] on the probability theory.

Here we are not interested in the construction of B and J_0, we determine them axiomatically.

Now some results.

Lemma 1.1 *J^* is an extension of J_0, J^* is non decreasing and $J^*(x+y) \leqq J^*(x)+J^*(y)$, $J^*(x)+J^*(y) \geqq J^*(x \vee y)+J^*(x \wedge y)$, $J^*(y) \leqq J^*(x \wedge y)+J^*(y \backslash x)$ for every $x, y \in H$.*

Proof. The two first assertions are obvious. The third assertion follows from Axioms 2 and (v), the fourth assertion follows from (iii) and the last from 2, 3 and (v).

Definition 1.2. *Denote by M the set of all $x \in H$ satisfying the following condition:*

$$J^*(y)=J^*(y \wedge x)+J^*(y \backslash x)$$

for every $y \in H$.

Lemma 1.2. *An element x belongs to M if and only if*

$$J^*(a) \geqq J^*(x \wedge a)+J^*(a \backslash x)$$

for every $a \in B$.

Proof. It follows from the inequality $J^*(y) \leqq J^*(x \wedge y)+J^*(y \backslash x)$ and Axiom 2.

Theorem 1.1. *M is a sublattice of the lattice H.*

Proof. Let $x, y \in M$, $a \in B$. By the second inequality in Lemma 1.1 and the distributive law we have

$$J^*(a \wedge x \wedge y)=J^*((a \wedge x) \wedge (a \wedge y)) \leqq$$

$$\leqq J^*(a \wedge x)+J^*(a \wedge y)-J^*(a \wedge (x \vee y)).$$

By the same inequality, Axioms 4 and 5 we obtain

$$J^*(a \backslash (x \wedge y))=J^*((a \backslash x) \vee (a \backslash y)) \leqq$$

$$\leqq J^*(a \backslash x)+J^*(a \backslash y)-J^*(a \backslash (x \vee y)),$$

hence

$$J^*(a \wedge (x \wedge y))+J^*(a \backslash (x \wedge y)) \leqq$$

$$\leqq J^*(a\wedge x)+J^*(a\backslash x)+J^*(a\wedge y)+J^*(a\backslash y)-$$
$$-J^*(a\wedge(x\vee y))-J^*(a\backslash(x\vee y)).$$

But (by 3)

$$a=(a\wedge(x\vee y))+(a\backslash(x\vee y)).$$

hence (by Lemma 1.1)

$$J^*(a)\leqq J^*(a\wedge(x\vee y))+J^*(a\backslash(x\vee y))$$

and therefore

$$J^*(a\wedge(x\wedge y))+J^*(a\backslash(x\wedge y))\leqq$$
$$\leqq J^*(a)+J^*(a)-J^*(a)=J^*(a).$$

The last inequality and Lemma 1.2 imply $x\wedge y\in M$. The relation $x\vee y\in M$ can be proved similarly.

Lemma 1.3. *Let* $x_1, \ldots, x_n \in H$, $x_1\leqq x_2\leqq\ldots\leqq x_n$, $x_i\leqq a_i$, $a_i\in B$, $J^*(x_i)+\frac{\varepsilon}{2^i}>J_0(a_i)$ $(i=1, \ldots, n)$. *Then*

$$J_0\left(\bigvee_{i=1}^{n} a_i\right)\leqq J_0(a_n)+\sum_{i=1}^{n-1}\frac{\varepsilon}{2^i}.$$

Proof. The assertion can be easily proved by induction.

The following lemma is a generalization of a theorem due to Choquet ([3]).

Lemma 1.4. *If* $x_n\nearrow x$, $x_n\in H$ $(n=1, 2, \ldots)$, $x\in H$, *then* $J^*(x_n)\nearrow J^*(x)$.

Proof. Evidently $\lim\limits_{n\to\infty} J^*(x_n)\leqq J^*(x)$ and the equality holds if $\lim\limits_{n\to\infty} J^*(x_n)=\infty$. If $\lim\limits_{n\to\infty} J^*(x_n)<\infty$, then $J^*(x_n)<\infty$ $(n=1,2,\ldots)$. Take $\varepsilon>0$, $b\geqq x$ and a_n such that

$$J^*(x_n)+\frac{\varepsilon}{2^n}>J_0(a_n),\ x_n\leqq a_n\leqq b.$$

By Lemma 1.3 we have

$$J^*(x_n)+\sum_{i=1}^{n}\frac{\varepsilon}{2^i}\geqq J_0\left(\bigvee_{i=1}^{n} a_i\right).$$

Since $\bigvee_{i=1}^{n} a_i\leqq b$ $(n=1, 2, \ldots)$, there exists $\bigvee_{i=1}^{\infty} a_i$, $x\leqq\bigvee_{i=1}^{\infty} a_i$ and by Axiom (iv)

$$J_0\left(\bigvee_{i=1}^{\infty} a_i\right)=\lim_{n\to\infty} J_0\left(\bigvee_{i=1}^{n} a_i\right),$$

hence

$$J^*(x) \leqq J_0\left(\bigvee_{i=1}^{\infty} a_i\right) \leqq \lim_{n\to\infty} J^*(x_n) + \varepsilon$$

and therefore

$$J^*(x) \leqq \lim_{n\to\infty} J^*(x_n).$$

Theorem 1.2. *If $y_n \in M$ $(n=1, 2, \ldots)$, $y_n \nearrow y \in H$, then $y \in M$. (Of course, $J^*(y) = \lim_{n\to\infty} J^*(y_n)$ by Lemma 1.4.)*

Proof. Let $a \in B$. Then

$$J^*(a) = J^*(a \wedge y_n) + J^*(a \backslash y_n) \geqq J^*(a \wedge y_n) + J^*(a \backslash y).$$

Since H is σ-continuous,

$$a \wedge y_n \nearrow a \wedge y$$

hence by Lemma 1.4

$$J^*(a) \geqq \lim_{n\to\infty} J^*(a \wedge y_n) + J^*(a \backslash y) = J^*(a \wedge y) + J^*(a \backslash y).$$

i.e. $y \in M$.

Theorem 1.3. *If $z_n \in M$ $(n=1, 2, \ldots)$, $z_n \searrow z \in H$, then $z \in M$. If $\lim_{n\to\infty} J^*(z_n) < \infty$, then $J^*(z) = \lim_{n\to\infty} J^*(z_n)$.*

Proof. Let $a \in B$. Then

$$J^*(a) = J^*(a \wedge z_n) + J^*(a \backslash z_n) \geqq J^*(a \wedge z) + J^*(a \backslash z_n).$$

By Axiom 6 we have $a \backslash z_n \nearrow a \backslash z$, hence by Lemma 1.4

$$J^*(a \backslash z) = \lim_{n\to\infty} J^*(a \backslash z_n).$$

Therefore

$$J^*(a) \geqq J^*(a \wedge z) + \lim_{n\to\infty} J^*(a \backslash z_n) =$$

$$= J^*(a \wedge z) + J^*(a \backslash z) \geqq J^*(a),$$

i.e. $z \in M$. Simultaneously

$$J^*(a)=\lim_{n\to\infty} J^*(a\wedge z_n)+\lim_{n\to\infty} J^*(a\backslash z_n)=$$

$$=\lim_{n\to\infty} J^*(a\wedge z_n)+J^*(a\backslash z),$$

hence

$$\lim_{n\to\infty} J^*(a\wedge z_n)+J^*(a\backslash z)=J^*(a\wedge z)+J^*(a\backslash z).$$

If $J^*(z_k)<\infty$ for some k, then there is $a_0\in B$, $a_0\geqq z_k$ such that $J^*(a_0)<\infty$. Then also $J^*(a_0\backslash z)<\infty$ and

$$\lim_{n\to\infty} J^*(z_n)=\lim_{n\to\infty} J^*(a_0\wedge z_n)=J^*(a_0\wedge z)=J^*(z).$$

Remark. Does there hold $B\subset M$? Yes, if $J_0(a)=J_0(a\wedge b)+J_0(a\backslash b)$ for all a, $b\in B$. Namely, $b\in B$ and

$$J_0(a)=J_0(a\wedge b)+J_0(a\backslash b)=J^*(a\wedge b)+J^*(a\backslash b)$$

imply $b\in M$.

2

First the new axioms. H is a relatively σ-complete lattice with the least element O. There are two binary operations $+$, $\backslash$ on H satisfying the conditions 1, 2, 3, 4, 5 and

2'. $(x+y)\backslash x\leqq y$, $x+y\geqq y$ for every $x, y\in H$; if $x, y\in H$, $x\leqq y$, then $x\backslash y=O$.

6'. If $y_n\nearrow y$, y_n, $y\in H$ $(n=1, 2, \dots)$, then $x\backslash y_n\searrow x\backslash y$. Further, there is a sublattice C of H closed under the operation $+$, containing O and such that $a_n\in C$ $(n=1, 2, \dots)$ implies $\bigwedge_{n=1}^{\infty} a_n\in C$. ($C$ plays here a similar role as the family of compact sets in the measure theory.)

Finaly, we shall list some properties of an initial mapping $J_0: C\to R$. But first one more notion: Two elements $a, b\in H$ are called disjoint if there are such $x, y\in H$ that $a\leqq x$ and $b\leqq y\backslash x$.

The mapping $J_0: C\to R$ satisfies the following conditions: (i), (ii),

(iii') $J_0(x)+J_0(y)\leqq J_0(x\vee y)+J_0(x\wedge y)$ for every $x, y\in B$.

(iv') If $a_n\searrow a$, $a_n\in C$ $(n=1, 2, \dots)$, then $J_0(a)=\lim_{n\to\infty} J_0(a_n)$.

(v') If a, b are disjoint, $a, b\in C$, then $J_0(a+b)=J_0(a)+J_0(b)$.

Definition 2.1. *For every $x\in H$ we define*

$$J_*(x)=\sup\{J_0(a);\, a\leqq x,\, a\in C\}.$$

Lemma 2.1. *J_* is an extension of J_0, J_* is non decreasing and for every $x, y \in H$ $J_*(x)+J_*(y) \leqq J_*(x \vee y)+J_*(x \wedge y)$ and $J_*(x+y) \geqq J_*(x)+J_*(y)$ for every two disjoint elements x, y.*

Proof. We prove only the last assertion, the three first being trivial. If $J_*(x+y)=\infty$, then the claimed inequality holds. Let $J_*(x+y)<\infty$. Then by Axiom 2′ also $J_*(x)<\infty$ and $J_*(y)<\infty$. Hence to every $\varepsilon>0$ there are $a, b \in C$ such that $a \leqq x$, $b \leqq y$ and

$$J_*(x)-\varepsilon<J_0(a), \quad J_*(y)-\varepsilon<J_0(b).$$

The elements a, b are disjoint, since x, y are disjoint. Therefore

$$J_*(x+y) \geqq J_0(a+b)=J_0(a)+J_0(b)>J_*(x)+J_*(y)-2\varepsilon$$

by Axioms 2 and (v′), hence the assertion follows.

Definition 2.2. *By M we denote the set of elements $x \in H$ with the following property:*

$$J_0(a)=J_*(a \wedge x)+J_*(a \backslash x)$$

for arbitrary $a \in C$.

Lemma 2.2. *If $x \in M$, then for every $y \in H$ there holds*

$$J_*(y)=J_*(x \wedge y)+J_*(y \backslash x).$$

Proof. The elements $y \wedge x$ and $y \backslash x$ are evidently disjoint. Hence by Axiom 3 and Lemma 2.1

$$J_*(y)=J_*((y \wedge x)+(y \backslash x)) \geqq J_*(y \wedge x)+J_*(y \backslash x).$$

On the orther hand, for every $a \in C$, $a \leqq y$ we have

$$J_0(a)=J_*(a \wedge x)+J_*(a \backslash x) \leqq J_*(y \wedge x)+J_*(y \backslash x),$$

hence

$$J_*(y)=\sup\{J_0(a);\, a \in C,\, a \leqq y\} \leqq J_*(y \wedge x)+J_*(y \backslash x).$$

Lemma 2.3. *If $x \in M$, then for every $y \in H$ there holds*

$$J_*(x+y) \leqq J_*(x)+J_*(y).$$

Proof. By Lemma 2.2 we obtain

$$J_*(x+y)=J_*((x+y) \wedge x)+J_*((x+y) \backslash x).$$

But Axiom 2′ implies $x+y \geqq x$, hence $(x+y) \wedge x=x$ and also $(x+y) \backslash x \leqq y$. Therefore $J_*(x+y) \leqq J_*(x)+J_*(y)$.

Lemma 2.4. *An element $x \in H$ belongs to M if and only if for every $a \in C$ we have*

$$J_*(a) \leqq J_*(a \wedge x) + J_*(a \backslash x).$$

Proof. The part "only if" is clear. Let $J_*(a) \leqq J_*(a \wedge x) + J_*(a \backslash x)$ for every $a \in C$. The elements $a \wedge x$, $a \backslash x$ are disjoint, hence by Lemma 2.1 $J_*(a) = J_*((a \wedge x) + (a \backslash x)) \geqq J_*(a \wedge x) + J_*(a \backslash x)$.

Theorem 2.1. *M is a sublattice of the lattice H.*

Proof. Let x, $y \in M$, $a \in C$. By Lemma 2.1, the distributive law and Axioms 4 and 5 we obtain

$$J_*(a \wedge x \wedge y) \geqq J_*(a \wedge x) + J_*(a \wedge y) - J_*(a \wedge (x \vee y)),$$

$$J_*(a \backslash (x \wedge y)) \geqq J_*(a \backslash x) + J_*(a \backslash y) - J_*(a \backslash (x \vee y)).$$

Since the elements $a \wedge (x \vee y)$, $a \backslash (x \vee y)$ are disjoint, we have by Lemma 2.1 and Axiom 3

$$J_*(a) \geqq J_*(a \wedge (x \vee y)) + J_*(a \backslash (x \vee y)),$$

hence

$$J_*(a \wedge x \wedge y) + J_*(a \backslash (x \wedge y)) \geqq J_*(a).$$

Lemma 2.4 then implies that $x \wedge y \in M$. The relation $x \vee y \in M$ can be proved similarly.

The following lemma is analogous to Lemma 1.4 and therefore we omit its proof.

Lemma 2.5. *Let $x_n \searrow x$, $x_n \in H$ $(n = 1, 2, \ldots)$, $x \in H$, $\lim\limits_{n \to \infty} J_*(x_n) < \infty$. Then $J_*(x_n) \searrow J_*(x)$.*

Theorem 2.2. *If $y_n \in M$ $(n = 1, 2, \ldots)$, $y_n \nearrow y \in H$, then $y \in M$ and $J_*(y) = \lim\limits_{n \to \infty} J_*(y_n)$.*

Proof. Let $a \in C$. Then

$$J_*(a) = J_*(a \wedge y_n) + J_*(a \backslash y_n) \leqq J_*(a \wedge y) + J_*(a \backslash y_n).$$

Since $J_*(a) = J_0(a) \in R$, also $J_*(a \backslash y_n) < \infty$. Then by Lemma 2.5 and the relation $a \backslash y_n \searrow a \backslash y$ we obtain

$$\lim_{n \to \infty} J_*(a \backslash y_n) = J_*(a \backslash y).$$

Therefore

$$J_*(a) \leqq J_*(a \wedge y) + \lim_{n \to \infty} J_*(a \backslash y_n) = J_*(a \wedge y) + J_*(a \backslash y),$$

hence $y \in M$ by Lemma 2.4. Evidently $J_*(y) \geqq \lim_{n\to\infty} J_*(y_n)$. Take $a \leqq y$, $a \in C$. Then $a \backslash y_n \searrow a \backslash y = O$ by Axiom 2′. Therefore

$$J_*(a) = \lim_{n\to\infty} J_*(y_n \wedge a) + \lim_{n\to\infty} J_*(a \backslash y_n) =$$

$$= \lim_{n\to\infty} J_*(y_n \wedge a) + 0 \leqq \lim_{n\to\infty} J_*(y_n),$$

hence

$$J_*(y) = \sup\{J_0(a)\,;\, a \leqq y\} = \lim_{n\to\infty} J_*(y_n).$$

Theorem 2.3. *If $z_n \in M$ $(n = 1, 2, \dots)$, $z_n \searrow z$, $z \in H$, then $z \in M$.*
Proof. Take $a \in C$. Then

$$J_*(a) = J_*(a \wedge z_n) + J_*(a \backslash z_n) \leqq J_*(a \wedge z_n) + J_*(a \backslash z).$$

Since $J_*(a \wedge z_n) < \infty$ and $a \wedge z_n \searrow a \wedge z$, we have by Lemma 2.5

$$J_*(a) \leqq \lim_{n\to\infty} J_*(a \wedge z_n) + J_*(a \backslash z) = J_*(a \wedge z) + J_*(a \backslash z).$$

i.e. $z \in M$ by Lemma 2.4.

Remark. We present two sufficient conditions for the inclusion $C \subset M$. The first: J_0 is tight if for every a, $b \in C$, $a \leqq b$ there holds

$$J_0(b) - J_0(a) = J_*(b \backslash a).$$

Hence for every $c \in C$ we have

$$J_0(a) - J_0(a \wedge c) = J_*(a \backslash (a \wedge c))$$

or

$$J_0(a) = J_*(a \wedge c) + J_*(a \backslash c).$$

(Of course, we used moreover the equality $a \backslash (a \wedge c) = a \backslash c$ holding in both special cases, rings of sets as well as cones of real-valued functions.)

The second condition is more simple, but it is not valid in the case where C is the family of all compact sets: the closeness of C with respect to $\backslash$. Under the assumption ($a \wedge c$, $a \backslash c$ are disjoint)

$$J_0(a) = J_0((a \wedge c) + (a \backslash c)) = J_0(a \wedge c) + J_0(a \backslash c).$$

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О МЕТОДЕ КАРАТЭОДОРИ ПРОДОЛЖЕНИЯ МЕР И ИНТЕГРАЛОВ

Белослав Риечан

Резюме

Доказывается теорема о продолжении для вещественных функций определенных на некоторой подструктуре данной структуры. Теорема о продолжении меры и теорема о продолжении интеграла являются частными случаями этого результата.