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PARTIAL GENERALIZED SYNCHRONIZATION THEOREMS OF DIFFERENTIAL AND DISCRETE SYSTEMS

Jianyi Jing, Lequan Min and Geng Zhao

This paper presents two theorems for designing controllers to achieve directional partial generalized synchronization (PGS) of two independent (chaotic) differential equation systems or two independent (chaotic) discrete systems. Two numerical simulation examples are given to illustrate the effectiveness of the proposed theorems. It can be expected that these theorems provide new tools for understanding and studying PGS phenomena and information encryption.

Keywords: partial generalized synchronization, differential system, discrete system

AMS Subject Classification: 34K23, 34K99

1. INTRODUCTION

Since the last two decades, the research on chaos and chaos synchronization has received increasing attention ([1 – 5, 7, 8, 10, 17, 20, 24, 26]). One of the reasons for this is that synchronization can be found in many physical, biological and engineering systems.

Generalized synchronization (GS) means all state trajectories of a driven system synchronize with that of a driving system via a transformation. Recently, the research on GS has also been gradually developed ([9, 12, 16, 25]), which may provide new tools for constructing better secure communication systems ([6, 13 – 16, 25, 27, 28]).

Partial synchronization (PS) is defined as the situation where part of the state trajectories of synchronized systems mutually asymptotically converge as time goes to infinity. Recently, research on PS in networks has received some attention ([11, 18, 21, 22]).

In this paper, two constructive theorems on PGS for differential equation systems and discrete map systems are presented, respectively. Based on the two theorems, one can design controllers such that two independent systems are in PGS with respect to a prescribed transformation. The first example shows that the simplest quadratic chaotic system can be in PGS with the Rucklidge chaotic system by designing a controller. As a second example, the discrete three-dimensional chaotic Lorenz map is controlled to achieve PGS with the discrete Lozi map.

2. PGS THEOREM OF DIFFERENTIAL EQUATION SYSTEMS

Given two independent chaotic systems, one is used as the driving system and the other as the driven system. The goal of partial generalized synchronization is to design an appropriate controller for the driven system, such that part of the state variables of the controlled driven system can be in GS with the driving system.

Definition 1. (Yang and Chua [25], Kocarev and Parlitz [12]) Consider two systems:

$$\dot{\boldsymbol{X}} = F(\boldsymbol{X}), \quad \dot{\boldsymbol{Y}} = G(\boldsymbol{Y}, \boldsymbol{X}), \tag{1}$$

where

$$\begin{aligned}&\boldsymbol{X} \in \mathbb{R}^n, \boldsymbol{Y} \in \mathbb{R}^m, F(\boldsymbol{X}) = (f_1(\boldsymbol{X}), f_2(\boldsymbol{X}), \ldots, f_n(\boldsymbol{X}))^T \in \mathbb{R}^n,\\ &G(\boldsymbol{Y}, \boldsymbol{X}) = (g_1(\boldsymbol{Y}, \boldsymbol{X}), g_2(\boldsymbol{Y}, \boldsymbol{X}), \ldots, g_m(\boldsymbol{Y}, \boldsymbol{X}))^T \in \mathbb{R}^m.\end{aligned}$$

If there exists a transformation $H : \mathbb{R}^n \to \mathbb{R}^m$, and a subset $B = B_x \times B_y \subset \mathbb{R}^n \times \mathbb{R}^m$, such that all trajectories of (1) with initial conditions in B satisfies

$$\lim_{t\to+\infty} \|H(\boldsymbol{X}) - \boldsymbol{Y}\| = 0,$$

then the systems given in (1) are said to be in GS with respect to the transformation H.

Definition 2. Consider two independent systems:

$$\begin{aligned}\dot{\boldsymbol{X}} &= F(\boldsymbol{X}), &(2)\\ \dot{\boldsymbol{Y}} &= \begin{pmatrix}\dot{\boldsymbol{Y}}_k\\ \dot{\boldsymbol{Y}}_{m-k}\end{pmatrix} = \begin{pmatrix}G_k(\boldsymbol{Y})\\ G_{m-k}(\boldsymbol{Y})\end{pmatrix}, &(3)\end{aligned}$$

where

$$\begin{aligned}&\boldsymbol{X} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n,\\ &\boldsymbol{Y} = (y_1, y_2, \ldots, y_m)^T \in \mathbb{R}^m,\\ &\boldsymbol{Y}_k = (y_1, y_2, \ldots, y_k)^T \in \mathbb{R}^k,\\ &\boldsymbol{Y}_{m-k} = (y_{k+1}, y_{k+2}, \ldots, y_m)^T \in \mathbb{R}^{m-k},\\ &F(\boldsymbol{X}) = (f_1(\boldsymbol{X}), f_2(\boldsymbol{X}), \ldots, f_n(\boldsymbol{X}))^T,\\ &G_k(\boldsymbol{Y}) = (g_1(\boldsymbol{Y}), g_2(\boldsymbol{Y}), \ldots, g_k(\boldsymbol{Y}))^T,\\ &G_{m-k}(\boldsymbol{Y}) = (g_{k+1}(\boldsymbol{Y}), g_{k+2}(\boldsymbol{Y}), \ldots, g_m(\boldsymbol{Y}))^T.\end{aligned}$$

If there exists a controller $U_{m-k}(\boldsymbol{X}, \boldsymbol{Y}) = (u_{k+1}(\boldsymbol{X}, \boldsymbol{Y}), \ldots, u_m(\boldsymbol{X}, \boldsymbol{Y}))^T$ such that $\boldsymbol{Y}_k$ in the controlled system

$$\dot{\boldsymbol{Y}} = \begin{pmatrix}\dot{\boldsymbol{Y}}_k\\ \dot{\boldsymbol{Y}}_{m-k}\end{pmatrix} = \begin{pmatrix}G_k(\boldsymbol{Y})\\ G_{m-k}(\boldsymbol{Y}) + U_{m-k}\end{pmatrix} \tag{4}$$

and $\boldsymbol{X}$ in system (2) are in GS with respect to a transformation $L : \mathbb{R}^n \to \mathbb{R}^k$, that is,

$$\lim_{t\to+\infty} \|L(\boldsymbol{X}) - \boldsymbol{Y}_k\| = 0,$$

then systems (2) and (4) are said to be in PGS with respect to the transformation $\boldsymbol{Y}_k = L(\boldsymbol{X})$.

Theorem 1. Let two independent differential (chaotic) systems be defined by equations (2) and (3), respectively, and $L : \mathbb{R}^n \to \mathbb{R}^k$ be a transformation given by

$$L(\boldsymbol{X}) = (l_1(\boldsymbol{X}), l_2(\boldsymbol{X}), \ldots, l_k(\boldsymbol{X}))^T \triangleq \boldsymbol{Y}_k.$$

Suppose that for any variables $\widetilde{\boldsymbol{X}}$ and $\widetilde{\boldsymbol{Y}}$, the functions $q_i(\widetilde{\boldsymbol{X}}, \widetilde{\boldsymbol{Y}})$, $i = 1, 2$, ensure that the zero solution of the following error equation (5) is asymptotically stable with respect to $\boldsymbol{e} = \widetilde{\boldsymbol{Y}} - \widetilde{\boldsymbol{X}}$, satisfying

$$\dot{e} = q_i(\widetilde{\boldsymbol{X}}, \widetilde{\boldsymbol{Y}}). \tag{5}$$

If vector $\boldsymbol{Y}_{m-k}$ can be solved from the equation

$$G_k(\boldsymbol{Y}_k, \boldsymbol{Y}_{m-k}) - \left(\frac{\partial l_i}{\partial x_j}\right)_{k\times n} F(\boldsymbol{X}) = q_1(L(\boldsymbol{X}), \boldsymbol{Y}_k) \tag{6}$$

via function $\boldsymbol{Y}_{m-k} = M(\boldsymbol{X}, \boldsymbol{Y}_k) = (M_1(\boldsymbol{X}, \boldsymbol{Y}_k), M_2(\boldsymbol{X}, \boldsymbol{Y}_k), \ldots, M_{m-k}(\boldsymbol{X}, \boldsymbol{Y}_k))^T$, then one can design a controller U_{m-k} by

$$U_{m-k}(\boldsymbol{X}, \boldsymbol{Y}) = \frac{\mathrm{d}M(\boldsymbol{X}, \boldsymbol{Y}_k)}{\mathrm{d}t} + q_2(M(\boldsymbol{X}, \boldsymbol{Y}_k), \boldsymbol{Y}_{m-k}) - G_{m-k}(\boldsymbol{Y}), \tag{7}$$

where

$$\frac{\mathrm{d}M(\boldsymbol{X}, \boldsymbol{Y}_k)}{\mathrm{d}t} = \begin{pmatrix} \dfrac{\partial M_1}{\partial x_1} & \cdots & \dfrac{\partial M_1}{\partial x_n} & \dfrac{\partial M_1}{\partial y_1} & \cdots & \dfrac{\partial M_1}{\partial y_k} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial M_{m-k}}{\partial x_1} & \cdots & \dfrac{\partial M_{m-k}}{\partial x_n} & \dfrac{\partial M_{m-k}}{\partial y_1} & \cdots & \dfrac{\partial M_{m-k}}{\partial y_k} \end{pmatrix} \begin{pmatrix} F(\boldsymbol{X}) \\ G_k(\boldsymbol{Y}) \end{pmatrix},$$

such that (2) and (4) are in PGS with respect to $\boldsymbol{Y}_k = L(\boldsymbol{X})$.

P r o o f. Denote the control error by

$$\boldsymbol{e}_u = \boldsymbol{Y}_{m-k} - M(\boldsymbol{X}, \boldsymbol{Y}_k).$$

Then

$$\begin{aligned} \dot{\mathbf{e}}_u &= \dot{\boldsymbol{Y}}_{m-k} - \dot{M}(\boldsymbol{X}, \boldsymbol{Y}_k) \\ &= G_{m-k}(\boldsymbol{Y}) + U_{m-k} - \frac{\mathrm{d}M(\boldsymbol{X}, \boldsymbol{Y}_k)}{\mathrm{d}t} \\ &= G_{m-k}(\boldsymbol{Y}) + \frac{\mathrm{d}M(\boldsymbol{X}, \boldsymbol{Y}_k)}{\mathrm{d}t} \\ &\quad + q_2(M(\boldsymbol{X}, \boldsymbol{Y}_k), \boldsymbol{Y}_{m-k}) - G_{m-k}(\boldsymbol{Y}) - \frac{\mathrm{d}M(\boldsymbol{X}, \boldsymbol{Y}_k)}{\mathrm{d}t} \\ &= q_2(M(\boldsymbol{X}, \boldsymbol{Y}_k), \boldsymbol{Y}_{m-k}). \end{aligned}$$

From the assumption, the zero solution of the error equation is asymptotically stable so that (5) holds by selecting $\boldsymbol{Y}_{m-k}(0) = M(\boldsymbol{X}(0), \boldsymbol{Y}_k(0))$.

Similarly, denote PGS error as

$$\boldsymbol{e} = \boldsymbol{Y}_k - L(\boldsymbol{X}).$$

Then

$$\begin{aligned} \dot{\boldsymbol{e}} &= \dot{\boldsymbol{Y}}_k - L(\dot{\boldsymbol{X}}) \\ &= G_k(\boldsymbol{Y}_k, \boldsymbol{Y}_{m-k}) - \left(\frac{\partial l_i}{\partial x_j}\right)_{k\times n} F(\boldsymbol{X}) \\ &\overset{(6)}{=} q_1(L(\boldsymbol{X}), \boldsymbol{Y}_k). \end{aligned}$$

Therefore, the zero solution of the error equation is asymptotically stable so that equations (2) and (4) are in PGS via transformation L. This completes the proof. □

3. PGS THEOREM OF DISCRETE SYSTEMS

Definition 3. Let

$$\boldsymbol{X}(i+1) = F(\boldsymbol{X}(i)), \tag{8}$$

$$\begin{aligned} \boldsymbol{Y}(i+1) &= \begin{pmatrix} \boldsymbol{Y}_k(i+1) \\ \boldsymbol{Y}_{m-k}(i+1) \end{pmatrix} = \begin{pmatrix} G_k(\boldsymbol{Y}(i)) \\ G_{m-k}(\boldsymbol{Y}(i)) \end{pmatrix}, \\ & i = 1, 2, \ldots \end{aligned} \tag{9}$$

be two discrete systems, where

$$\boldsymbol{X} \in \mathbb{R}^n, \quad \boldsymbol{Y} \in \mathbb{R}^m, \quad \boldsymbol{Y}_k \in \mathbb{R}^k, \boldsymbol{Y}_{m-k} \in \mathbb{R}^{m-k}.$$

If there exists a transform $L : \mathbb{R}^n \to \mathbb{R}^k, k \le m$, such that

$$\lim_{i\to+\infty} \|L(\boldsymbol{X}(i)) - \boldsymbol{Y}_k(i)\| = 0,$$

then systems (8) and (9) are said to be in PGS with respect to the transform L.

Theorem 2. Given two discrete systems defined by (8) and (9). Let $L : \mathbb{R}^n \to \mathbb{R}^k$ be a transform given by

$$L(\boldsymbol{X}) = (l_1(\boldsymbol{X}), l_2(\boldsymbol{X}), \ldots, l_k(\boldsymbol{X}))^T \triangleq \boldsymbol{Y}_k.$$

Suppose that for any variables $\widetilde{\boldsymbol{X}}(i)$ and $\widetilde{\boldsymbol{Y}}(i)$, the functions $q_j(\widetilde{\boldsymbol{X}}(i), \widetilde{\boldsymbol{Y}}(i)), j = 1, 2$, ensure that the zero solution of the following error equation (10) is asymptotically stable with respect to $\mathbf{e}(i+1) = \widetilde{\boldsymbol{Y}}(i+1) - \widetilde{\boldsymbol{X}}(i+1)$:

$$\mathbf{e}(i+1) = q_j(\widetilde{\boldsymbol{X}}(i), \widetilde{\boldsymbol{Y}}(i)), \quad j = 1, 2. \tag{10}$$

i. e.,

$$\lim_{i\to+\infty} \|\boldsymbol{e}(i+1)\| = \lim_{i\to+\infty} \|q_j(\widetilde{\boldsymbol{X}}(i), \widetilde{\boldsymbol{Y}}(i))\| = 0, \quad j = 1, 2.$$

If the vector $\boldsymbol{Y}_{m-k}(i)$ can be solved from the equation

$$G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - L(F(\boldsymbol{X}(i))) = q_1(L(\boldsymbol{X}(i)), \boldsymbol{Y}_k(i)) \tag{11}$$

via function

$$\begin{aligned}\boldsymbol{Y}_{m-k}(i) &= M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)) \\ &= (M_1(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)), M_2(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)), \ldots, M_{m-k}(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)))^T,\end{aligned}$$

and if $G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i))$ is a continuous function in $\boldsymbol{Y}_{m-k}(i)$, that is, if

$$\lim_{i\to+\infty} \|\boldsymbol{Y}_{m-k}(i) - \boldsymbol{S}(i)\| = 0,$$

then

$$\lim_{i\to+\infty} \|G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - G_k(\boldsymbol{Y}_k(i), \boldsymbol{S}(i))\| = 0.$$

Under the above assumptions, one can design a controller $U_{m-k}(i)$ by

$$\begin{aligned}U_{m-k}(i) &= M(F(\boldsymbol{X}(i)), G_k(\boldsymbol{Y}(i))) \\ &\quad + q_2(\boldsymbol{Y}_{m-k}(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i))) - G_{m-k}(\boldsymbol{Y}(i)) \\ &\triangleq (u_{k+1}(i), u_{k+2}(i), \ldots, u_m(i))^T\end{aligned}$$

such that the following two systems

$$\boldsymbol{X}(i+1) = F(\boldsymbol{X}(i)), \tag{12}$$

$$\boldsymbol{Y}(i+1) = \begin{pmatrix} \boldsymbol{Y}_k(i+1) \\ \boldsymbol{Y}_{m-k}(i+1) \end{pmatrix} = \begin{pmatrix} G_k(\boldsymbol{Y}(i)) \\ G_{m-k}(\boldsymbol{Y}(i)) + U_{m-k}(i) \end{pmatrix}, \tag{13}$$

are in PGS with respect to $\boldsymbol{Y}_k(i+1) = L(\boldsymbol{X}(i+1))$.

P r o o f. Denote the control error by

$$\boldsymbol{e}_u(i+1) = \boldsymbol{Y}_{m-k}(i+1) - M(\boldsymbol{X}(i+1), \boldsymbol{Y}_k(i+1)).$$

Then

$$\begin{aligned}\boldsymbol{e}_u(i+1) &= G_{m-k}(\boldsymbol{Y}(i)) + U_{m-k}(i) - M(F(\boldsymbol{X}(i)), G_k(\boldsymbol{Y}(i))) \\ &\overset{(12)}{=} q_2(\boldsymbol{Y}_{m-k}(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i))).\end{aligned}$$

Hence,

$$\lim_{i\to+\infty} \|e_u(i+1)\| = \lim_{i\to+\infty} \|\boldsymbol{Y}_{m-k}(i+1) - M(\boldsymbol{X}(i+1), \boldsymbol{Y}_k(i+1))\| = 0.$$

From the continuity assumption of $G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i))$, one has

$$\lim_{i\to+\infty} \|G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - G_k(\boldsymbol{Y}_k(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)))\| = 0.$$

Denote the PGS error as

$$e(i+1) = \boldsymbol{Y}_k(i+1) - L(\boldsymbol{X}(i+1)).$$

Then

$$\begin{aligned}
\|e(i+1)\| &= \|G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - L(F(\boldsymbol{X}(i)))\| \\
&= \|G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - G_k(\boldsymbol{Y}_k(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i))) \\
&\quad + G_k(\boldsymbol{Y}_k(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i))) - L(F(\boldsymbol{X}(i)))\| \\
&\leq \|G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - G_k(\boldsymbol{Y}_k(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)))\| \\
&\quad + \|G_k(\boldsymbol{Y}_k(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i))) - L(F(\boldsymbol{X}(i)))\| \\
&= \|G_k(\boldsymbol{Y}_k(i), \boldsymbol{Y}_{m-k}(i)) - G_k(\boldsymbol{Y}_k(i), M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)))\| \\
&\quad + \|q_1(L(\boldsymbol{X}(i)), \boldsymbol{Y}_k(i))\| \\
&\to 0.
\end{aligned}$$

Therefore, the zero solution of the error equation is asymptotically stable so that systems (12) and (13) are in PGS via transformation L.

Specially, if one takes

$$\boldsymbol{Y}_{m-k}(0) = M(\boldsymbol{X}(0), \boldsymbol{Y}_k(0)),$$

then

$$\begin{aligned}
\boldsymbol{Y}_{m-k}(i) &\equiv M(\boldsymbol{X}(i), \boldsymbol{Y}_k(i)), \\
\boldsymbol{Y}_k(i) &\equiv L(\boldsymbol{X}(i)),
\end{aligned}$$

so that systems (12) and (13) are precisely in PGS via transformation L. □

Remark. The previous version of this theorem and its proof can be found in [11]. In this new version, a flaw in [11] has been corrected.

4. NUMERICAL SIMULATIONS

4.1. Example for Theorem 1

Let two differential chaotic systems be the simplest quadratic chaotic system [23] (as a driving system)

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = x_3 \\
\dot{x}_3 = -ax_3 + x_2^2 - x_1
\end{cases} \tag{14}$$

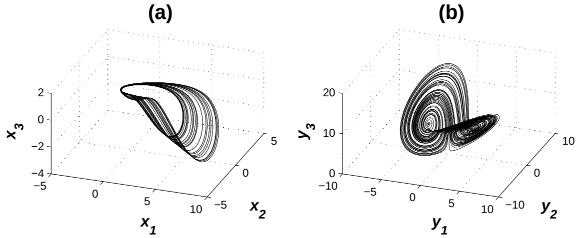

Fig. 1. Chaotic trajectories of (a) the simplest quadratic chaotic system and (b) the Rucklidge system.

where $a = 2.017$, and the Rucklidge system [19] (as driven system)

$$\left\{\begin{array}{lcl}\dot{y}_1 & = & y_2\\ \dot{y}_2 & = & -ky_2 + \lambda y_1 - y_1 y_3\\ \dot{y}_3 & = & -y_3 + x_1^2\end{array}\right. \tag{15}$$

where $k = 2,\ \lambda = 6.7$.

If initial conditions of systems (14) and (15) are selected as $(-0.9,\ \ 0,\ \ 0.5)^T$ and $(-1.8641,\ \ 0,\ \ 1, 4.5)^T$, then the two systems are chaotic. Figure 1 shows their chaotic trajectories.

Choose $q_i(\widetilde{\boldsymbol{X}}, \widetilde{\boldsymbol{Y}}) = \widetilde{\boldsymbol{X}} - \widetilde{\boldsymbol{Y}},\ i = 1, 2$, that is,

$$\begin{array}{rcl} q_1(L(\boldsymbol{X}), \boldsymbol{Y}_k) & = & L(\boldsymbol{X}) - \boldsymbol{Y}_k,\\ q_2(M(\boldsymbol{X},\boldsymbol{Y}_k), \boldsymbol{Y}_{m-k}) & = & M(\boldsymbol{X},\boldsymbol{Y}_k) - \boldsymbol{Y}_{m-k}.\end{array}$$

Now, let systems (14) and (15) be in PGS via the following transformation:

$$L(\boldsymbol{X}) = l_1(\boldsymbol{X}) = \frac{x_1}{x_1^2 + x_2^2}. \tag{16}$$

Then, from Theorem 1, one obtains that

$$U_{3-2}(\boldsymbol{X},\ \boldsymbol{Y}) = (u_2(\boldsymbol{X},\ \boldsymbol{Y}), u_3(\boldsymbol{X},\ \boldsymbol{Y})),$$

where

$$\begin{array}{rcl} u_2 & = & \dfrac{\ddot{x}_1 + 2\dot{x}_1 + x_1}{x_1^2 + x_2^2} - y_1 - 2y_2,\\ u_3 & = & 0,\end{array}$$

where $\dot{x}_i, \ddot{x}_i, i = 1, 2, 3$, can be obtained from system (14). The simulation result for the PGS of variables $\boldsymbol{X}$ and $\boldsymbol{Y}_2$ is shown in Figure 2. The calculated errors converge to zero. The numerical simulation verifies the theoretical expectation.

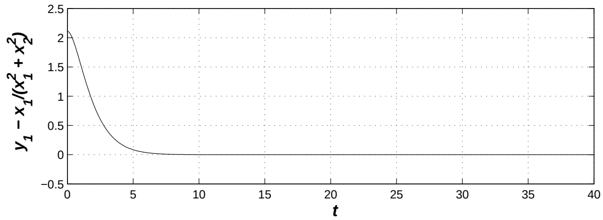

Fig. 2. PGS errors vs. time: $e_1 = y_1 - x_1/(x_1^2 + x_2^2)$.

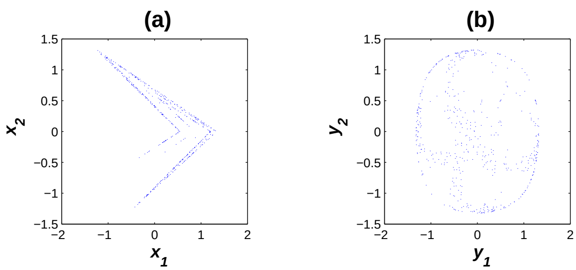

Fig. 3. Chaotic trajectories of (a) the Lozi map system and (b) the three-dimensional chaotic Lorenz map system.

4.2. Example for Theorem 2

Let two discrete chaotic systems be the Lozi map system (as driving system)

$$\begin{cases} x_1(i+1) & = & 1 - a|x_1(i)| + bx_2(i) \\ x_2(i+1) & = & x_1(i), \end{cases} \tag{17}$$

where $a = 1.7,\ b = 0.5$, and the three-dimensional chaotic Lorenz map system (as driving system)

$$\begin{cases} y_1(i+1) & = & y_3(i) \\ y_2(i+1) & = & y_1(i) \\ y_3(i+1) & = & y_1(i)y_3(i) - y_2(i). \end{cases} \tag{18}$$

If initial conditions of systems (17) and (18) are selected as $(-0.1,\quad 0.1)^T$ and $(0.5,\quad -1,\quad 0.5)^T$, then the two systems are chaotic. Figure 3 shows their chaotic trajectories.

Choose $q_j(\widetilde{\boldsymbol{X}}, \widetilde{\boldsymbol{Y}}) = \frac{1}{2}(\widetilde{\boldsymbol{X}} - \widetilde{\boldsymbol{Y}}),\ j = 1, 2$. The object is for systems (17) and

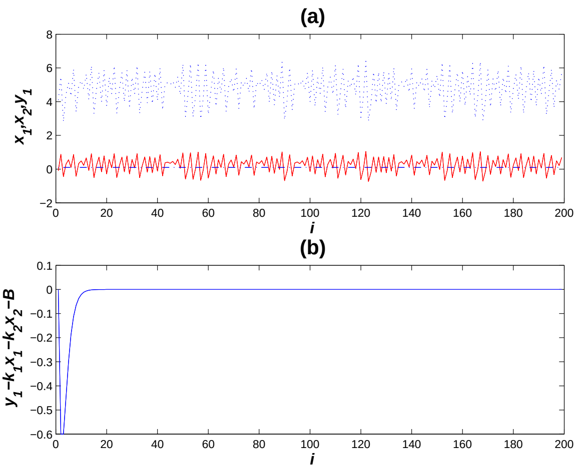

Fig. 4. (a) Trajectories of variables x_1 (solid line), x_2 (dashed line), and y_1 (dotted line). (b) PGS errors vs. time: $e = y_1 - k_1x_1 - k_2x_2 - B$.

(18) to be in PGS via the following transformation:

$$L(\boldsymbol{X}) = l_1(\boldsymbol{X}) = k_1x_1 + k_2x_2 + B,$$

where $k_1 = 2,\ k_2 = 3,\ B = 4$. Then, from Theorem 2, one can get

$$U_{3-1}(i) = (u_2(i), u_3(i))^T,$$

where

$$\begin{aligned} u_2(i) &= 0 \\ u_3(i) &= L\left(F(F(X(i))) - L(F(X(i))) + \frac{1}{4}L(X(i))\right) \\ &\quad -\frac{1}{4}y_1(i) + y_3(i) - y_1(i)y_3(i) + y_2(i). \end{aligned}$$

The simulation results of PGS of variables $\boldsymbol{X}$ and $\boldsymbol{Y}_1$ are shown in Figure 4. The calculated errors converge to zero. The numerical simulation verifies the theoretical anticipation.

5. CONCLUDING REMARKS

Two new theorems on PGS of (chaotic) systems have been established. The theorems provide two general methods for designing controllers to achieve the partial generalized synchronization of a large classes of independent (chaotic) systems. As the first example, it has been shown that the simplest quadratic chaotic system can be in PGS with the chaotic Rucklidge system by designing a controller. The second example shows that the discrete three-dimensional chaotic Lorenz map can be in PGS with the discrete Lozi map. The simulations of these examples illustrate the effectiveness and feasibility of the new theorems. It can be expected that these methods will have a wide spectrum of practical applications.

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