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ON–OFF INTERMITTENCY IN CONTINUUM SYSTEMS DRIVEN BY THE CHEN SYSTEM

Qian Zhou, Zeng-Qiang Chen and Zhu-Zhi Yuan

Previous studies on on-off intermittency in continuum systems are generally based on the synchronization of identical chaotic oscillators or in nonlinear systems driven by the Duffing oscillator. In this paper, one-state on-off intermittency and two-state on-off intermittency are observed in two five-dimensional continuum systems, respectively, where each system has a two-dimensional subsystem driven by the chaotic Chen system. The phenomenon of intermingled basins is observed below the blowout bifurcation. Basic statistical properties of the intermittency are investigated. It is shown that the distribution of the laminar phase follows a $-3/2$ power law, and that of the burst amplitudes follows a -1 power law, consistent with the basic statistical characteristics of on-off intermittency.

Keywords: on-off intermittency, Chen system, Blowout bifurcation, intermingled basin, power law

AMS Subject Classification: 37C70, 93C10

1. INTRODUCTION

Intermittency refers to the random switching of system behaviors between the relatively "regular" laminar phase and "irregular" burst phase during the evolution of time. Since intermittency appears whenever a dynamical system is close to several types of bifurcations, the phenomenon is very common in various fields such as biological behavior [8, 10], fluid dynamics [4, 14], human activities [2, 3], earthquake occurrence [13], etc. Different types of intermittency have been reported, including the Pomeau–Manneville intermittency [18], crisis-induced intermittency [7] and in-out intermittency [1].

On-off intermittency is another type of intermittency, first reported by Platt, Spiegel and Tresser [17]. With its close connection to chaotic synchronization [12, 19, 22], it has attracted a great deal of attention in recent years. In "off" state, the dynamical system stays in the vicinity of an invariant manifold for a long period of time, and in "on" state, it bursts out of the invariant manifold occasionally. Ott and Sommerer [15] found that the occurrences of on-off intermittency and riddled basins are on the two sides of the blowout bifurcation, which is related to the transverse stability of an invariant subspace. The statistical properties of on-off intermittency have been extensively investigated [5, 9, 16]. It is confirmed that on-off intermittency follows some well-known power laws, e. g. the asymptotic scaling of the laminar phase lengths follows a power law with exponent $-3/2$, and the distribution of the burst amplitude follows a power law with exponent -1. These power laws are widely used to characterize on-off intermittency.

In deterministic systems, generation of on-off intermittency in general needs a drive system and a response system. Moreover, the drive system determines the dynamics of the chaotic attractor in the invariant subspace, while the response system determines the positions and the number of invariant subspaces, that is, the number of laminar phases. According to [21], among the two physical situations that could display on-off intermittency, one is the synchronization of identical chaotic oscillators, and the other is some physical systems with a spatial symmetry.

In this paper, we deal with two five-dimensional systems with a skew product structure [17]. In each system, a two-dimensional subsystem is driven by the chaotic Chen system [6]. By calculating the largest transverse Lyapunov exponent, we find the location of blowout bifurcation of each system. Just beyond the bifurcation, the Chen attractor in the subspace loses its transverse stability and becomes weakly unstable, consequently we observe one-state and two-state on-off intermittency in the two systems respectively. And below the blowout bifurcation the phenomenon of intermingled basins is observed. Through investigating the statistical properties of the intermittency in these systems, we verify that the intermittency is on-off intermittency.

2. MODEL OF ONE–STATE ON–OFF INTERMITTENCY AND NUMERICAL RESULTS

We construct the following two-dimensional system as the response system:

$$\begin{aligned} \dot{u} &= v, \\ \dot{v} &= -\gamma u^3 + p_1 S u + p_2 v, \end{aligned} \tag{1}$$

where S is a signal from a drive system, p_1 and p_2 are control parameters, $\gamma > 0$. Let system (1) be driven by the Chen system [6]. Then we get the following five-dimensional continuous system:

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x + cy - xz, \\ \dot{z} &= xy - bz, \\ \dot{u} &= v, \\ \dot{v} &= -\gamma u^3 + p_1 x u + p_2 v. \end{aligned} \tag{2}$$

Obviously, the dynamics of variables x, y and z are independent of those of variables u and v, thus system (2) has a skew product structure. Moreover, the system is symmetrical with respect to $(u, v) \to (-u, -v)$, therefore $u = v = 0$ is an invariant three-dimensional manifold in the phase space. In this invariant subspace, the dynamics is the chaotic Chen attractor. Here, we take $\gamma = 1$, $a = 35$, $b = 3$, $c = 28$.

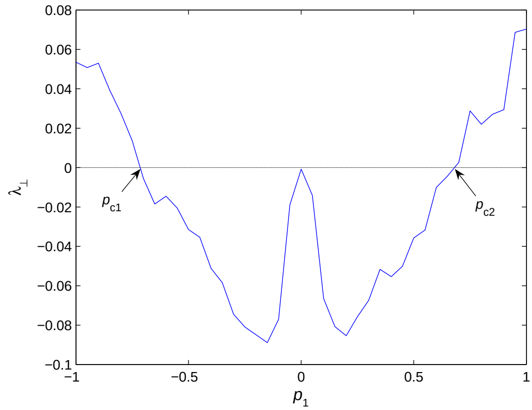


Fig. 1. The transverse Lyapunov exponent $\lambda_\perp$ versus the control parameter $p_1 \in [-1, 1]$ in system (2).

Since on-off intermittency occurs on the loss of transverse stability of an invariant manifold, we calculate the largest transverse Lyapunov exponent $\lambda_\perp$ of system (2). Take $(\delta u, \delta v)$ as infinitesimal perturbation transverse to the invariant subspace $u = v = 0$. Here, we take p_1 as the control parameter and p_2 as a constant $p_2 = -0.6$. According to system (1), the motion equation of the perturbation $(\delta u, \delta v)$ near the invariant subspace $u = v = 0$ is

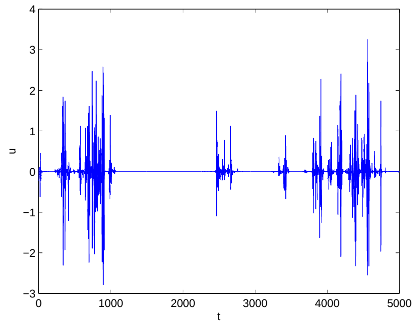
$$\begin{aligned} \dot{\delta u} &= \delta v, \\ \dot{\delta v} &= p_1 x \delta u + p_2 \delta v, \end{aligned} \tag{3}$$

where x is a chaotic trajectory produced by system (2) in the invariant subspace. The largest transverse Lyapunov exponent $\lambda_\perp$ is calculated via

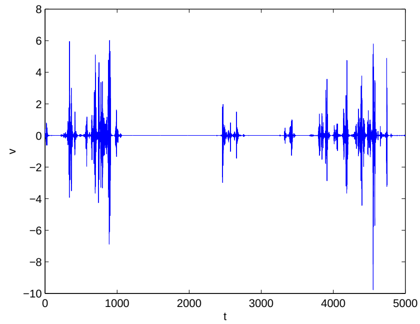
$$\lambda_\perp = \lim_{t \to \infty} (1/t) \ln(\delta(t)/\delta(0)), \tag{4}$$

where $\delta(t) = ((\delta u(t))^2 + (\delta v(t))^2)^{1/2}$. A plot of $\lambda_\perp$ versus p_1 for $p_1 \in [-1, 1]$ is shown in Figure 1. We see that $\lambda_\perp$ passes through zero at $p_{c1} \approx -0.702$ and $p_{c2} \approx 0.713$. Thus, there symmetrically exist two blowout bifurcation points of system (2) in the parameter range. If we take p_2 as the control parameter, and p_1 as a constant $p_1 = 0.15$, with the method above we find another blowout bifurcation point. Thus, any combination of p_1 and p_2 that satisfies $\lambda_\perp \cong 0$ corresponds to a bifurcation point of system (2).

Near each blowout bifurcation point, while the two control parameters p_1 and p_2 take values corresponding to slightly positive $\lambda_\perp$, we observe typical on-off intermittency in the time series of $u(t)$ and $v(t)$ as shown in Figure 2. We present the projections of on-off intermittent attractors of system (2) in Figure 3. In Figure 3 (a) and Figure 3 (b), it can be clearly seen that trajectories repeatedly spend long stretches of time on or near the Chen attractor during which the on-off intermittency is in



(a)



(b)

Fig. 2. (a) On-off intermittent time series of $u(t)$; (b) On-off intermittent time series of $v(t)$, with $p_1 = 0.8, p_2 = -0.6$ from an arbitrary initial condition in system (2).

"off" state, and they occasionally are repelled away from the attractor which corresponds to "on" state, and later are attracted to the attractor again. From these phase portraits, we see that when on-off intermittency occurs, the chaotic attractor in the subspace becomes a bursting attractor.

As the control parameter approaches the critical value of the bifurcation, the frequency of bursting becomes less and less, approaching zero.

Scaling laws are used widely to describe different intermittent phenomena. It is well known that on-off intermittency has the following power laws:

(i) In the range of moderate lengths, the distribution of laminar phase lengths follows a power law with exponent $-3/2$ [9].

(ii) The distribution of burst amplitudes satisfies a power law with exponent -1 when their values are small, but deviates from the power law when their values are large [20].

The intermittent behavior of system (2) is shown to share the same power laws. Let τ denote the length of the laminar phase, which is defined by $|u(t)| \leq 0.001$, and $P(\tau)$ the distribution function of τ. We analyze the time-series data of $u(t)$ for $p_1 = 0.8, p_2 = -0.6$, and collect about 21200 laminar phases. Numerical analysis shows a power law,

$$P(\tau) \propto \tau^{\alpha}. \tag{5}$$

Figure 4 shows the distribution of the laminar phase lengths in log-log coordinates for the on-off intermittency in system (2). In Figure 4 the fitted line is drawn by the least-squares method and the slope is calculated to be -1.463; in other words, $\alpha \approx -1.463$, which is close to -1.5. It is confirmed that the power law (i) holds well for the distribution of the laminar phase lengths of the intermittency in system (2).

Next, we investigate the burst amplitude r, which is defined by $|u(t)|$. With $p_1 = 0.75, p_2 = -0.6$, we take a time series of 4×10^7sampled data points for $|u(t)|$.

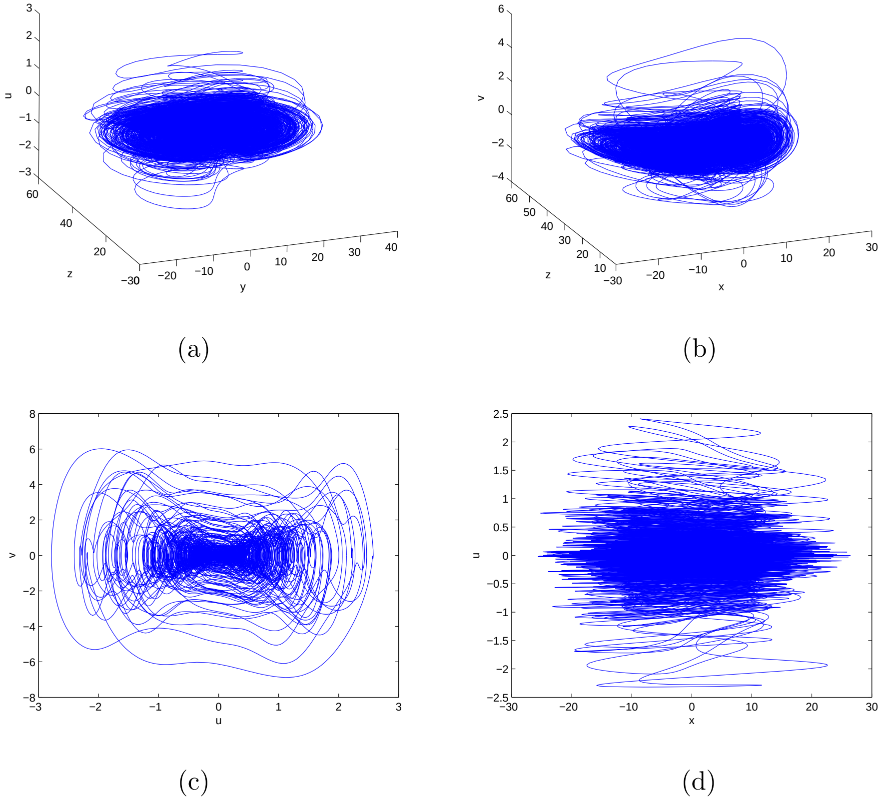


Fig. 3. On-off intermittent attractor of system (2) with $p_1 = 0.8, p_2 = -0.6$.

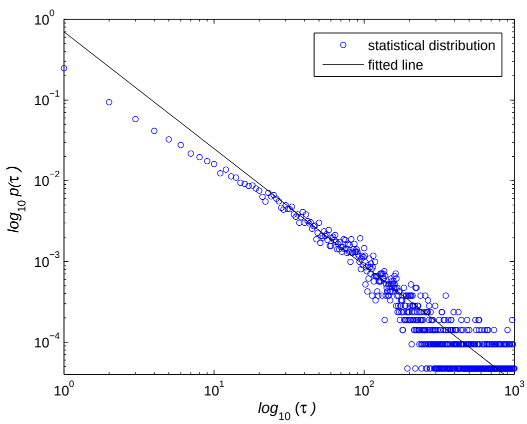


Fig. 4. Distribution of laminar duration time of the intermittency in system (2).

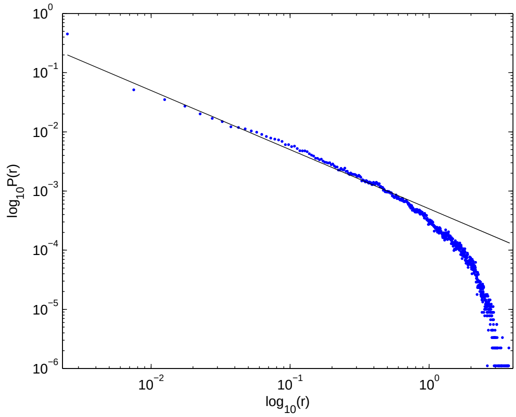


Fig. 5. Distribution of burst amplitudes of the intermittency in system (2). The straight line is fitted to $p(r) = ar + b, a = -0.999$. The slope of the line is very close to -1.

Numerical analysis shows that the distribution of r also follows a power law,

$$P(r) \propto r^{\beta}. \tag{6}$$

Relation (6) is shown in Figure 5, in which the slope of the straight fitted line is -0.999. Therefore, β is very close to -1 and the power law (ii) holds very well for the intermittency in system (2).

3. MODEL OF TWO–STATE ON–OFF INTERMITTENCY AND NUMERICAL RESULTS

If a dynamical system has two low-dimensional symmetric invariant subspaces, the system shows two-state on-off intermittency or intermingled basins near the blowout bifurcation.

Still using the Chen system as the drive system, we modify the response subsystem in Section 2 to make it have two symmetric invariant subspaces. Consequently, we get a model for two-state on-off intermittency as follows:

$$\begin{aligned}
\dot{x} &= a(y - x),\\
\dot{y} &= (c - a)x + cy - xz,\\
\dot{z} &= xy - bz,\\
\dot{u} &= v,\\
\dot{v} &= -\gamma u(u^2 - 1)^3 + p_1 x u(u^2 - 1) + p_2 v.
\end{aligned} \tag{7}$$

Obviously, system (7) has a skew product structure and is symmetrical with respect to $(u, v) \to (-u, -v)$. Moreover, $u = 1,\ v = 0$ and $u = -1,\ v = 0$ are two three-dimensional symmetric invariant subspaces of system (7). In these two

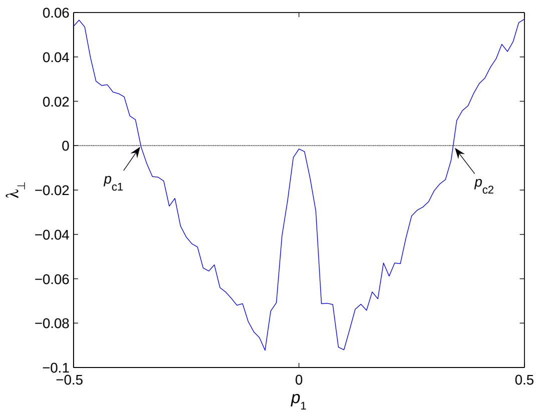


Fig. 6. The largest transverse Lyapunov exponent $\lambda_\perp$ versus the control parameter p_1 for $p_1 \in [-0.5, 0.5]$ in system (7).

invariant subspaces, the dynamics are decided by the chaotic Chen attractor. Like the case of the one-state on-off intermittency model, the control parameter can be either p_1 or p_2. Here, we only consider p_1 as the control parameter and take $\gamma = 1,\ a = 35,\ b = 3,\ c = 28$, and $p_1 = -0.6$.

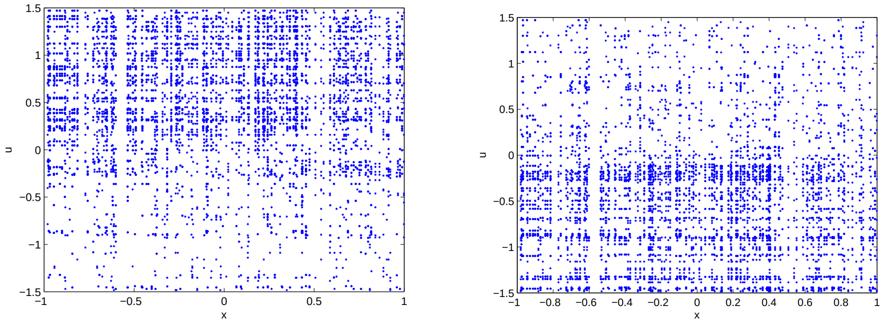
Since the infinitesimal transverse perturbation $(\delta u, \delta v)$ of both symmetric invariant subspaces evolves according to the following equation of motion:

$$\begin{aligned} \dot{\delta u} &= \delta v, \\ \dot{\delta v} &= 2p_1 x \delta u + p_2 \delta v, \end{aligned} \tag{8}$$

the two invariant manifolds have the same transverse stability. The exponent $\lambda_\perp$ is computed via Eq. 4). Figure 6 shows a plot of $\lambda_\perp$ versus the parameter p_1 for $p_1 \in [-0.5, 0.5]$, in which $\lambda_\perp$ passes through zero at $p_{c1} \approx -0.356$, $p_{c2} \approx 0.356$. Thus, system (7) has two blowout bifurcations at p_{c1}, p_{c2}.

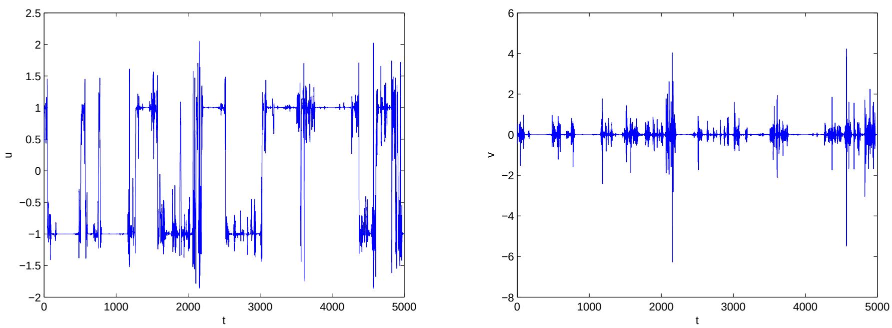
When p_1 takes values corresponding to negative $\lambda_\perp$, the chaotic Chen attractors in the subspaces $u = 1,\ v = 0$ and $u = -1,\ v = 0$ are global attractors. We observe the phenomenon of intermingled basins [11]. Figure 7 shows the attraction basins of the Chen attractors at $u = 1, v = 0$ and at $u = -1,\ v = 0$, for initial conditions taken from the two-dimensional region of $-1 \leq x \leq 1$, $-1.5 \leq u \leq 1.5$ with $y = z = v = 0$. In Figure 7, 49000 randomly chosen initial conditions were integrated to determine their destinations. We see that arbitrarily near any initial point in the attraction basin of one attractor, there exist points which belong to the basin of the other attractor. Therefore, the two attraction basins are intermingled.

When $\lambda_\perp$ is slightly positive, the two attractors in the subspaces lose their transverse stability simultaneously, consequently two-state on-off intermittency appears. The time series of $u(t)$ and $v(t)$ are shown in Figure 8 with $p_1 = 0.43$, $p_2 = -0.6$. We see that the on-off intermittent behavior of $u(t)$ has two laminar states at $u = 1$ and $u = -1$, while $v(t)$ has only one laminar state at $v = 0$.



(a) (b)

Fig. 7. Intermingled basins of attraction for initial conditions taken from the two-dimensional region of $-1 \leq x \leq 1$, $-1.5 \leq u \leq 1.5$ with $\gamma = 0.3$ in system (7). (a) The attraction basin of the Chen attractor at $u = 1,\ v = 0$; (b) The attraction basin of the Chen attractor at $u = -1,\ v = 0$.



(a) (b)

Fig. 8. (a) Two-state on-off intermittent time series of $u(t)$; (b) One-state on-off intermittent time series of $v(t)$ in system (7) when $p_1 = 0.43$, $p_2 = -0.6$.

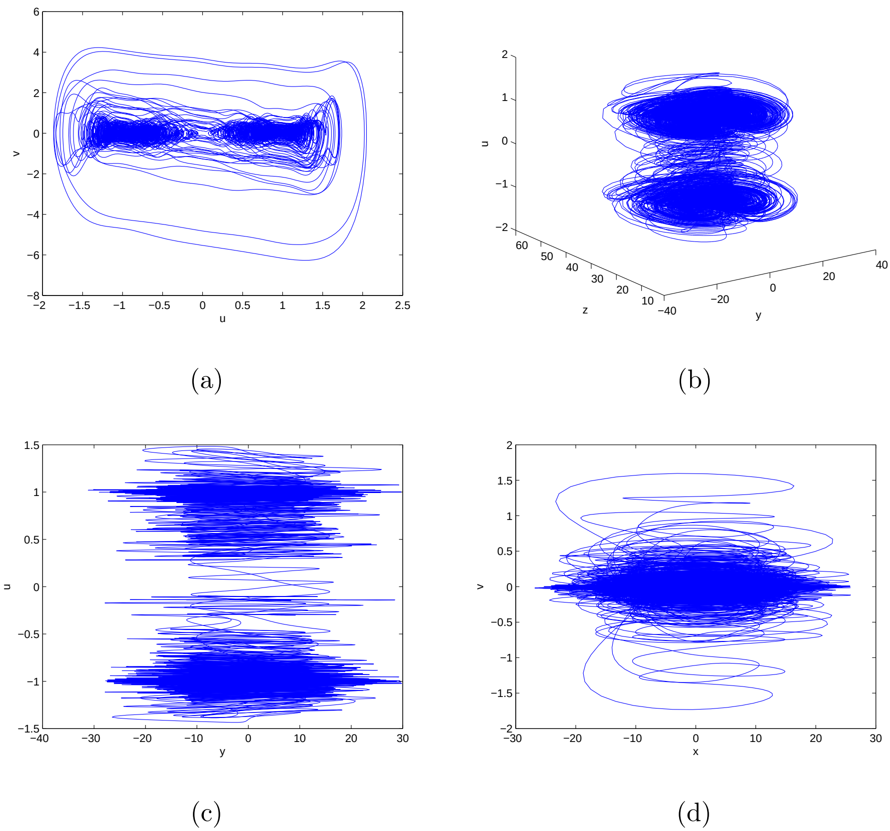


Fig. 9. Projections of on-off intermittent attractor of system (7) with $p_1 = 0.4, p_2 = -0.6$.

The phase portraits of the two-state on-off intermittent attractor with $p_1 = 0.4$, $p_2 = -0.6$ are shown in Figure 9, which enable us to have a better understanding of the nature of two-state on-off intermittency. We can see that on-off intermittency in system (7) has two laminar phases near the two invariant manifolds at $u = \pm 1,\ v = 0$ (dense area corresponds to laminar state) and the trajectories shuttle between the two invariant subspaces and they occasionally burst away from the two attractors in the subspaces (corresponds to burst state). This is confirmed by the time series of $u(t)$ and $v(t)$ shown in Figure 8. Thus, the symmetric existence of two invariant subspaces is a precondition for the onset of two-state on-off intermittency. Figure 9 (a) shows that the projection of the chaotic attractor of system (7) in the $u - v$ plane has four rolls which joint at the invariant manifold $u = 1,\ v = 0$ and $u = -1,\ v = 0$.

The statistical properties of the two-state on-off intermittency in system (7) are analyzed here. First, we show the distribution of the laminar phase duration time.

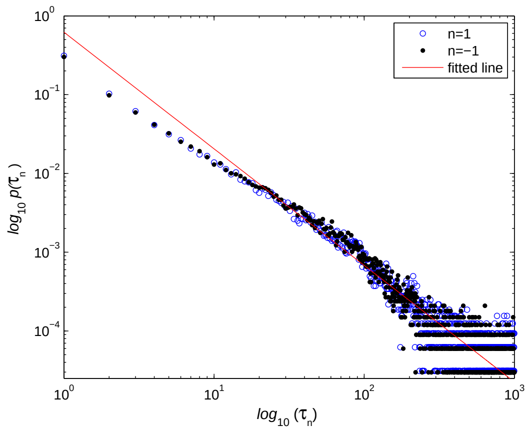


Fig. 10. Distribution of the laminar phase duration $\tau_n(n = 1, -1)$ in system (7). τ_1 is associated with attractor at $u = 1,\ v = 0$, and τ_{-1} associated with attractor at $u = -1,\ v = 0$. The straight line is drawn from a lease-squares fit to the model $p(\tau) = a\tau + b$ and $a = -1.492$. The slope of the line is very close to $-3/2$.

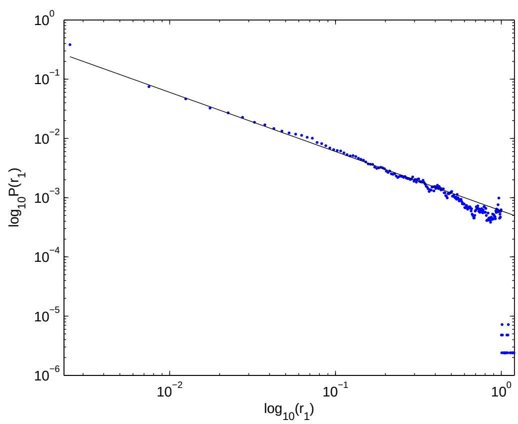


Fig. 11. Distribution of the burst amplitude r_1 associated with the laminar phase at $u = 1$ in system (7). The straight line is drawn from a lease-squares fit to the model $p(r_1) = ar_1 + b$ and $a = -0.998$.

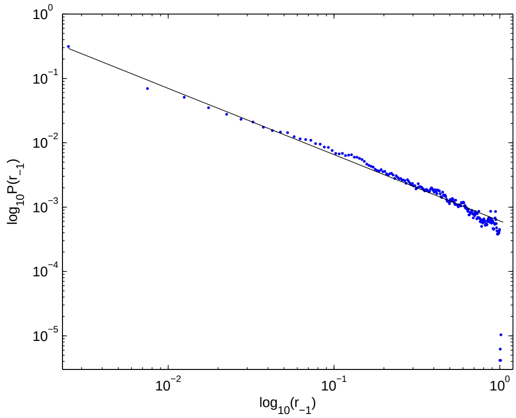


Fig. 12. Distribution of the burst amplitude r_{-1} associated with the laminar phase at $u = -1$ in system (7). The straight line is drawn from a lease-squares fit to the model $p(r_{-1}) = ar_{-1} + b$ and $a = -1.031$.

For each laminar phase level, we collect about 32500 laminar phases defined by $|u(t)| \leq 0.001$. We denote the duration of the laminar phase by τ_n $(n = 1, -1)$. τ_1 is associated with laminar phase near attractor at $u = 1$, $v = 0$, and τ_{-1} associated with laminar phase near attractor at $u = -1$, $v = 0$. Our numerical analysis demonstrates that the universal $-3/2$ power-law distribution of laminar phases still holds true for the two-state on-off intermittency. In Figure 10, we see that the distributions of τ_1, τ_{-1} share the same power law

$$P(\tau_n) \propto \tau_n^{\alpha}. \tag{9}$$

This can be explained by the same transverse stability of the two symmetric invariant subspaces. The straight fitted line in Figure 10 has a slope of -1.492; in other words, $\alpha \approx -1.492$. It is confirmed that the distribution of laminar phase lengths of the two-state on-off intermittency follows the power law (i) with exponent $-3/2$.

Next, we investigate the distributions of the burst amplitudes of the two laminar levels, which are denoted by r_n $(n = 1, -1)$. $r_1$is associated with laminar phase near attractor at $u = 1, v = 0$ and defined by $|u(t) - 1|$, and r_{-1} is associated with laminar phase near attractor at $u = -1$, $v = 0$ and defined by $|u(t) - (-1)|$. Relation of r_n and their distribution are shown in Figure 11 and Figure 12, respectively. In the two figures, the slopes of the fitted solid lines are -0.998 and -1.031, respectively, which are both close to -1. It is confirmed that the power law

$$P(r_n) \propto r_n^{-1} \tag{10}$$

holds very well in the distributions of burst amplitudes for the two laminar levels of two-state on-off intermittency.

4. CONCLUSIONS

Previous studies on on-off intermittency in continuum systems are generally based on the synchronization of identical chaotic oscillators or in nonlinear systems driven by the Duffing oscillator. In this paper, we observe one-state on-off intermittency and two-state on-off intermittency in two five-dimensional continuum systems driven by the chaotic Chen system, respectively. Moreover, we observe the phenomenon of intermingled basins below the blowout bifurcation of the systems. We have investigated the statistical properties of the intermittency in the systems and found that for both one-state intermittency and two-state intermittency observed, the distribution of the laminar phase duration time follows a power law, and that of the burst amplitudes follows a -1 power law.

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