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GEOMETRICAL CHARACTERIZATION OF OBSERVABILITY IN INTERPRETED PETRI NETS

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This work is concerned with observability in Discrete Event Systems (*DES*) modeled by Interpreted Petri Nets (*IPN*). Three major contributions are presented. First, a novel geometric characterization of observability based on input-output equivalence relations on the marking sequences sets is presented. Later, to show that this characterization is well posed, it is applied to linear continuous systems, leading to classical characterizations of observability for continuous systems. Finally, this paper translates the geometric characterization of observability into structural properties of the *IPN*. Thus, polynomial algorithms can be derived to check the observability in a broad class of *IPN*.

Keywords: discrete event systems, observability, Petri nets
AMS Subject Classification: 93C65, 93B07

1. INTRODUCTION

Observability [3] is an important property of structural representations of dynamic systems that allows, through an entity called observer, the computation of state variable values of dynamic systems that cannot be directly measured. Observers are used to estimate the system state in some structures of state feedback controllers as well as to introduce some redundancy in fault tolerant systems, among other applications.

In the Discrete Event Systems (*DES*) area, the observability problem was first addressed using Finite Automata (*FA*) [14, 20]. Based on this representation, [18] reported a technique to build observers allowing state ambiguities. This approach allows determining the system state within some event sequences intervals (resilient observer). The same representation tool is used in [4, 11, 12, 13, 23] to propose some techniques to control *FA* when only partial observation of the language accepted by the *FA* is available. Although *FA* is suitable for describing *DES*, its application is limited to small size systems, since this kind of models explicitly take into account all possible system states, resulting in quite large models when the size of the system grows, and its behavior is complex.

In order to cope with the state explosion problem, research groups throughout the world are increasingly adopting Petri Nets (*PN*) as a modeling formalism for *DES*.

Petri nets provide clearer graphical descriptions and simple and sound mathematical support, allowing to represent causal relationships, process synchronization, resource allocation, and concurrence, inherent to *DES* behavior [6, 17].

The observability problem also has been studied from the *PN* point of view. One of the earliest reported works on observability using *PN* is [10]. In that paper, Ichikawa and Hiraishi divided the observability problem into two subproblems: the computation of the event sequence, and, using that sequence, the computation of the set of possible initial states. Giua and Seatzu [8] addressed the problem of computing the initial marking assuming that the firing transition sequence is fully known, and provided several notions of observability. Rivera et al. [22] introduced a definition for observability that exploits the input and output information of the live and cyclic Interpreted Petri Nets (IPN) and does not assume that the firing transition sequence is fully known. The authors also presented a new methodology for the design of observers using *IPN*.

More recently, [21] addressed the observability in *IPN*. That work exploited the input/output information of the *IPN* to design an *IPN* observer for a *DES*. The observability definition proposed in this paper, however, does not characterize all observable *IPN*. Using the same model, Aguirre et al. [1] derived a characterization of observability based on *IPN* invariant marking sequences. The computation of marking invariants, however, is also a complex problem, limiting the application of the results presented in that paper. A drawback of the observability definition used in the last two works is that, when an observable *IPN* executes a T-semiflow infinitely often, then the computation of the actual marking cannot be carried out since the required information from the *IPN* evolution may be not obtained.

The approach herein presented is based on an observability definition that considers the structure, finite input and finite output words of the *IPN*, avoiding the drawback presented in the definition introduced in [1]. Based on this new definition, a novel characterization of observability in *IPN* is presented. This characterization is based on input-output equivalence relations on the marking sequences sets, denoted S_ω, and the input-output trajectories set, denoted $\Lambda(Q, M_0)$. Since these sets play a similar role that those played by invariant subspaces in linear systems, this characterization is said to be a geometric one. In order to prove that this characterization is well posed, it is applied to linear continuous systems, leading to classical characterizations of observability for continuous systems. Moreover, it is a general characterization of observability and can be applied to other dynamic systems. Finally, since the geometric characterization leads to computational complex algorithms when it is applied to *IPN*, this paper goes deeper and translates the geometric characterization into a structural one. The advantage of this new characterization is that polynomial algorithms can be derived to check the observability in a broad class of *IPN*. Furthermore, the observer scheme presented in [21] can be applied together with the observability concepts and characterizations herein presented. This observer scheme can be used in many applications, for instance in a state feedback controller [2], where the estimated state is needed.

This paper is organized as follows. In Section 2, a formal definition of *IPN* and necessary notation are presented. Section 3 studies the observability problem for

IPN models, where a formal definition and some characterizations of observable *IPN* are presented. Section 4 presents the sequence and marking-detectability properties which are used to derive a structural characterization of observable *IPN*. Section 5 shows that the observer scheme presented in [21] can be used to find out the actual state of an observable *IPN* and introduces an example using the concepts and results herein introduced. Finally, conclusions and some directions for future work are presented in Section 6.

2. INTERPRETED PETRI NETS

This section presents the basic concepts and notation of *PN* and introduces the definition of *IPN* used in this paper. A review of *PN* is presented in [17]. For details about marked graphs, state machines and free choice *PN*, an interested reader can consult [5].

Definition 2.1. A Petri net structure G is a bipartite digraph represented by the 4-tuple $G = (P, T, I, O)$, where:

- $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of vertices called places,
- $T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of vertices called transitions,
- $I : P \times T \longrightarrow \mathbb{Z}^+$ is a function representing the weighted arcs going from places to transitions, where $\mathbb{Z}^+$ is the set of nonnegative integers, and
- $O : P \times T \longrightarrow \mathbb{Z}^+$ is a function representing the weighted arcs going from transitions to places.

The symbol ${}^\bullet t_j$ denotes the set of all places p_i such that $I(p_i, t_j) \neq 0$ and $t_j^\bullet$ the set of all places p_i such that $O(p_i, t_j) \neq 0$. Analogously, ${}^\bullet p_i$ denotes the set of all transitions t_j such that $O(p_i, t_j) \neq 0$ and $p_i^\bullet$ the set of all transitions t_j such that $I(p_i, t_j) \neq 0$. Pictorially, places are represented by circles, transitions are represented by rectangles, and arcs are depicted as arrows.

The pre-incidence matrix of G is $C^- = [c_{ij}^-]$, where $c_{ij}^- = I(p_i, t_j)$, the post-incidence matrix of G is $C^+ = [c_{ij}^+]$, where $c_{ij}^+ = O(p_i, t_j)$, and the incidence matrix of G is $C = C^+ - C^-$. The marking function $M : P \longrightarrow \mathbb{Z}^+$ is a mapping from each place to the nonnegative integers representing the number of tokens (depicted as dots) residing inside each place. The marking of a *PN* is usually expressed as an $n-$entry vector.

Definition 2.2. A Petri Net system or Petri Net (*PN*) is the pair $N = (G, M_0)$, where G is a *PN* structure and M_0 is an initial token distribution.

In a *PN* system, a transition t_j is enabled at marking M_k if $\forall p_i \in P$, $M_k(p_i) \geq I(p_i, t_j)$. An enabled transition t_j can be fired reaching a new marking M_{k+1} which can be computed as $M_{k+1} = M_k + Cv_k$, where $v_k(i) = 0$, $i \neq j$, $v_k(j) = 1$, this equation is called the *PN* state equation. The reachability set of a *PN*, denoted as $R(G, M_0)$, is the set of all possible reachable marking from M_0 firing only enabled transitions.

This work uses Interpreted Petri Nets (*IPN*) [16], an extension to *PN*, since they allow to associate input and output signals to *PN* models. Formally *IPN* are defined as follows.

Definition 2.3. An Interpreted Petri Net (*IPN*) is the 4-tuple $Q = (N, \Sigma, \lambda, \varphi)$ where:

- $N = (G, M_0)$ is a *PN* system,
- $\Sigma = \{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ is the input alphabet of the net, where α_i is an input symbol,
- $\lambda : T \to \Sigma \cup \{\varepsilon\}$ is a labelling function of transitions with the following constraint: $\forall t_j, t_k \in T$, $j \neq k$, if $\forall p_i$, $I(p_i, t_j) = I(p_i, t_k) \neq 0$ and both $\lambda(t_j) \neq \varepsilon$, $\lambda(t_k) \neq \varepsilon$, then $\lambda(t_j) \neq \lambda(t_k)$. In this case ε represents an internal system event.
- $\varphi : R(Q, M_0) \to (\mathbb{Z}^+)^q$ is an output function that associates to each marking in $R(Q, M_0)$ an output vector. Here q is the number of outputs.

Remarks:

2.1. In this work (Q, M_0) will be used instead of $Q = (N, \Sigma, \lambda, \varphi)$ to emphasize the fact that there is an initial marking in an *IPN*.

2.2. Attention is focused on the case when function φ is a $q \times n$ matrix, where q is the number of places representing measurable states in the *DES* and n is the number of places in the model (G, M_0). Each column of this matrix is an elementary or null vector. If the output symbol i is present (turned on) every time that $M(p_j) \geq 1$, then $\varphi(i, j) = 1$, otherwise $\varphi(i, j) = 0$.

2.3. Equivalent transitions are not allowed, i. e. it is assumed that $\forall t_i, t_j$ such that $t_i \neq t_j$, $\lambda(t_i) = \lambda(t_j)$, it holds that $C(\cdot, i) \neq C(\cdot, j)$. This is not a major constraint because those transitions are redundant.

2.4. Notice that by definition of λ, *IPN* are deterministic [9] over labeled transitions, i. e. two transitions with the same associated input symbol (different from symbol ε) cannot have the same input places. However, they can be non deterministic [9] over unlabeled transitions (those t_j such that $\lambda(t_j) = \varepsilon$).

A transition $t_j \in T$ of an *IPN* is enabled at marking M_k if $\forall p_i \in P$, $M_k(p_i) \geq I(p_i, t_j)$. If $\lambda(t_j) = a_i \neq \varepsilon$ is present and t_j is enabled, then t_j must fire. If $\lambda(t_j) = \varepsilon$ and t_j is enabled then t_j can be fired. When an enabled transition t_j is fired in a marking M_k, then a new marking M_{k+1} is reached. This fact is represented as $M_k \xrightarrow{t_j} M_{k+1}$ and M_{k+1} can be computed using the dynamic part of the state equation

$$\begin{aligned} M_{k+1} &= M_k + C v_k \\ y_k &= \varphi(M_k) \end{aligned} \tag{1}$$

where C and v_k are defined as in *PN* and $y_k \in (\mathbb{Z}^+)^{\,q}$ is the kth output vector of the *IPN*.

Definition 2.4. A firing transition sequence of an *IPN* (Q, M_0) is a transition sequence $\sigma = t_i t_j \ldots t_k \ldots$ such that $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \ldots M_w \xrightarrow{t_k} \ldots$.

Definition 2.5. The set $\mathcal{L}(Q, M_0)$ of all firing transition sequences is called the firing language, i. e. $\mathcal{L}(Q, M_0) = \{\sigma | \sigma = t_i t_j \dots t_k \dots \wedge\ M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots M_w \xrightarrow{t_k} \dots\}$.

Definition 2.6. Let $\sigma = t_i t_j t_k \dots$ be a firing transition sequence. The Parikh vector $\vec{\sigma} : T \longrightarrow (\mathbb{Z}^+)^m$ of σ maps every transition $t \in T$ to the number of occurrences of t in σ.

According to functions λ and φ, transitions and places of an *IPN* (Q, M_0) can be classified as follows.

Definition 2.7. If $\lambda(t_i) \neq \varepsilon$, the transition t_i is said to be manipulated. Otherwise it is non manipulated. A place $p_i \in P$ is said to be measurable if the ith column vector of φ is not null, i. e. $\varphi(\cdot, i) \neq 0$. Otherwise it is non measurable. A place p_i is said to be computable if it is measurable and $\forall j,\ i \neq j,\ \varphi(\cdot, i) \neq \varphi(\cdot, j)$. Otherwise it is non computable.

Notice that computable places are measurable and the marking of these places can be computed from the output (no other place, when it is marked, generates the same output value of function φ).

Now, the following definitions relate the input and output symbol sequences with the firing transition sequences and the generated marking sequences. These concepts are useful in the study of the observability property since they are relating the input and output information of the *IPN* with the firing transition and marking sequences.

Definition 2.8. A sequence of input-output symbols of (Q, M_0) is a sequence $\omega = (\alpha_0, y_0)(\alpha_1, y_1) \cdots (\alpha_n, y_n)$, where $\alpha_j \in \Sigma \cup \{\varepsilon\}$ and α_{i+1} is the current input of the *IPN* when the output changes from y_i to y_{i+1}. It is assumed that $\alpha_0 = \varepsilon$, $y_0 = \varphi(M_0)$ and (α_{i+1}, y_{i+1}) belongs to the sequence when:

- (α_i, y_i) belongs to the sequence,
- $y_{i+1} \neq y_i$, and
- there exists no $y_j \neq y_i$, $y_j \neq y_{i+1}$ occurring after the occurrence of y_i and before the occurrence of y_{i+1}.

Definition 2.9. Let (Q, M_0) be an *IPN*. The set

$$\Lambda(Q, M_0) = \{\omega \,|\, \omega \text{ is a sequence of input-output symbols}\}$$

denotes the set of all sequences of input-output symbols of (Q, M_0). The set of all input-output sequences of length greater or equal than k will be denoted by $\Lambda^k(Q, M_0)$, i. e. $\Lambda^k(Q, M_0) = \{\omega \in \Lambda(Q, M_0) \mid |\omega| \geq k\}$.

Definition 2.10. If $\omega = (\alpha_0, y_0)(\alpha_1, y_1) \cdots (\alpha_n, y_n)$ is a sequence of input-output symbols, then the firing transition sequence $\sigma \in \mathcal{L}(Q, M_0)$ whose firing actually generates ω is denoted by σ_ω. The set of all possible firing transition sequences that could generate the word ω is defined as

$$\Omega(\omega) = \{\sigma | \sigma \in \mathcal{L}(Q, M_0) \wedge \text{the firing of } \sigma \text{ produces } \omega\}.$$

Definition 2.11. The set of all input-output sequences leading to an ending marking in the *IPN* (markings enabling no transition or only self-loop transitions) is denoted by $\Lambda_B(Q, M_0)$, i.e.,

$$\Lambda_B(Q,M_0) = \{\omega \in \Lambda(Q,M_0) \mid \exists \sigma \in \Omega(\omega) \text{ such that } M_0 \xrightarrow{\sigma} M_j \wedge \text{if } M_j \xrightarrow{t_i} \text{ then } C(\cdot,t_i) = \vec{0}\}.$$

The prefix of a sequence s is another sequence s' such that there exists a sequence s'' fulfilling that $s = s's''$. The set of all prefixes of s is denoted by $\bar{s}$.

Definition 2.12. Let $\omega = (\alpha_0, y_0)(\alpha_1, y_1)\cdots(\alpha_n, y_n) \in \Lambda(Q, M_0)$ be a sequence of input-output symbols. The marking sequences set corresponding to ω is defined as

$$S_\omega = \{M_0 M_1 \cdots M_k | M_i \in R(Q,M_0) \ \wedge \ M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \cdots \xrightarrow{t_m} M_k \wedge \sigma_\omega = t_i t_j \cdots t_m \in \Omega(\omega)\}.$$

For instance, suppose that in the example of Figure 1 there exist two sensors, sensor one is turned on when place p_1 is marked and sensor two is turned on when places p_4 or p_5 are marked. Then, the output function of the *IPN* is the matrix

$$\varphi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Notice that this matrix has two rows, because there exist two sensors. The element $\varphi(i,j) = 1$ when sensor i is associated to place p_j.

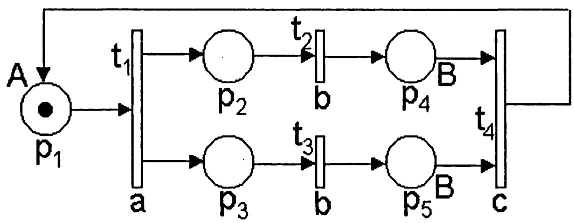

Fig. 1. A non deterministic *IPN*.

Thus, places p_1, p_4 and p_5 are measurable, however, place p_1 is the only computable place. Since all transitions are labeled, then all of them are manipulated. We obtain the following languages:

$$\mathcal{L}(Q, M_0) = \{t_1,\ t_1t_2,\ t_1t_3,\ t_1t_2t_3,\ t_1t_3t_2,\ t_1t_2t_3t_4,\ t_1t_3t_2t_4,\ t_1t_2t_3t_4t_1, ..\}$$

$$\Lambda(Q,M_0) = \left\{\left(\varepsilon,\begin{bmatrix}1\\0\end{bmatrix}\right),\left(\varepsilon,\begin{bmatrix}1\\0\end{bmatrix}\right)\left(a,\begin{bmatrix}0\\0\end{bmatrix}\right),\left(\varepsilon,\begin{bmatrix}1\\0\end{bmatrix}\right)\left(a,\begin{bmatrix}0\\0\end{bmatrix}\right)\left(b,\begin{bmatrix}0\\1\end{bmatrix}\right),\ldots\right\}$$

$$\Lambda^2(Q,M_0) = \left\{\left(\varepsilon,\begin{bmatrix}1\\0\end{bmatrix}\right)\left(a,\begin{bmatrix}0\\0\end{bmatrix}\right),\left(\varepsilon,\begin{bmatrix}1\\0\end{bmatrix}\right)\left(a,\begin{bmatrix}0\\0\end{bmatrix}\right)\left(b,\begin{bmatrix}0\\1\end{bmatrix}\right),\ldots\right\}$$

$$\Lambda_B(Q, M_0) = \{\}.$$

If $\omega=\left(\varepsilon,\begin{bmatrix}1\\0\end{bmatrix}\right)\left(a,\begin{bmatrix}0\\0\end{bmatrix}\right)\left(b,\begin{bmatrix}0\\1\end{bmatrix}\right)$, then $\Omega(\omega) = \{t_1t_2,\ t_1t_3\}$ and

$$S_\omega = \left\{\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}\right\}.$$

Now, the following concept will be needed to characterize observable *IPNs* and establishes that, even when the precise marking of a place is unknown, it can belong to a conservative marking law. In other words, the location or state of the entities (resources, machines, buffer capacities, etc.) that constitute the *DES* may be unknown; however the amount of those entities is known. Suppose, for instance that we do not know the marking in the example of Figure 1. However, from the P-semiflows of the net we have the conservative marking law $1M(p_1) + 1M(p_2) + 1M(p_4) = 1$. This work assumes that these conservative marking laws can be given arbitrarily (not only by the P-semiflows). This concept is analogous to that of "macro-markings" used in [7].

Definition 2.13. Let (Q, M_0) be an *IPN* structure and $M(p_j)$ be any marking of a place p_j in (Q, M_0). The set of s equations

$$CML = \left\{\sum_{j=1}^{n} \gamma_j^i M(p_j) = k_i \,|\, i \in [1,\ldots,s] \wedge \gamma_j^i \in \mathbb{Z}^+\right\}$$

form a set of conservative marking laws (*CML*) if $\forall \gamma_k^i \neq 0$ it holds that k_i/γ_k^i is an integer value and all non computable places p_n are contained in at least one equation of the *CML* set. A *CML* is said to be binary (*BCML*) if it holds that $\forall i, j$, $\gamma_j^i \in \{0, 1\}$ and $k_i = 1$. In addition, the *CML* can be rewritten as

$$\Gamma M = K \tag{2}$$

where M is the marking vector, Γ is the matrix $\Gamma[i, j] = \gamma_j^i$ and K is the vector $K(i) = k_i$.

Remarks:

2.5. Hereafter $\mathcal{M}_0$ will denote the set of all possible initial markings fulfilling the stated *CML*, i. e.

$$\mathcal{M}_0 = \{M_0 |\text{ such that any } M \in R(Q, M_0) \text{ fulfills the } CML \text{ constraints}\}. \tag{3}$$

2.6. The notation $p_l \in e_i$, where $e_i \in CML$ ($e_i \in BCML$) means that there exists an equation $\sum_{j=1}^{n} \gamma_j^i \cdot M(p_j) = k_i$, named e_i in the *CML* (in the *BCML*), such that $\gamma_l^i \neq 0$.

Also, $(Q, \mathcal{M}_0)$ will denote an *IPN* where $M_0 \in \mathcal{M}_0$ and it could be unknown. Notation $\mathcal{M}_0^{\mathrm{B}}$ will be used for a *BCML*.

3. OBSERVABILITY IN *IPN*

Loosely speaking, a state representation of a dynamic system is said to be observable if the knowledge of its inputs, outputs, and structure suffices to uniquely determine its state.

A deterministic and continuous time dynamic model, for instance

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t),$$

is said to be observable at t_0, if there exists a finite time t_1 such that the knowledge of the model structure (A, B, C), the input signal $u(t)$ and the output signal $y(t)$ over the interval $t_0 \leq t \leq t_1$ suffices to uniquely determine the initial state $x(t_0)$. Moreover, since the system is a deterministic one, then $x(t)$ for all $t \geq t_0$, can also be uniquely determined using the knowledge of $x(t_0)$ and $u(t)$, over the interval $t_0 \leq t \leq t_1$.

When the dynamic model is non deterministic (i.e. when the solution of the model is not unique [19]), however, the knowledge of $x(t_0)$ and $u(t)$ over the interval $t_0 \leq t \leq t_1$ does not guarantee the computation of $x(t)$ for all $t \geq t_0$. For instance, even if it is known that the initial marking of the *IPN* depicted in Figure 1 is $M_0 = [1\ 0\ 0\ 0\ 0]^T$ and that the input sequence $\sigma = abbcabbcabbc \ldots ab$ is fired, it is not possible to determine if the reached marking is $[0\ 1\ 0\ 0\ 1]^T$ or $[0\ 0\ 1\ 1\ 0]^T$. In this case, there exists no finite transition sequence σ allowing to know the reached marking and all future reached markings of the *IPN*.

This fact leads to that, in the general case, the observability definition must be changed to ensure that the initial state $x(t_0)$ and all states $x(t)$, $t > t_0$ can be computed in a finite time or length of input firing words.

Because of that, in the general case we can derive the following intuitive definition: A non deterministic dynamic model, for instance an *IPN*, is observable at k_0, if there exists a finite integer k_1 such that the knowledge of the model structure (C, λ, φ) and the sequence of input-output symbols ω_k for any $k \geq k_1$ suffices to uniquely determine the state sequence over $k_0 \leq l \leq k$ $(M_{k_0} \ldots M_k)$. The observability definition in *IPN* can be formally proposed as follows.

Definition 3.1. An *IPN* given by (Q, M_0), where M_0 may be unknown, is observable if there exists an integer $k < \infty$ such that $\forall \omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ it holds that the information provided by ω and (Q, M_0) suffices to uniquely determine the initial marking M_0 and the marking M_i reached by the firing of the underlying firing transition sequence σ_ω.

Therefore an *IPN* is observable if for any sequence of input-output signals of length equal or greater than k and for any blocking sequence, the marking sequence reached by the system can be uniquely determined.

Since the set S_ω contains the marking sequences generated by the same input-output sequence $\omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$, then when $|S_w| = 1$ there exists only one marking sequence for the word ω. Thus the initial and the actual marking can be computed from these marking sequence, leading to an observable *IPN*. This fact is formalized in the following result.

Theorem 3.1. An *IPN* given by (Q, M_0) is observable if and only if there exists an integer $k < \infty$ such that $\forall\omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ it holds that $|S_\omega| = 1$, where S_ω is the marking sequences set corresponding to ω.

Proof. (Sufficiency) Assume that there exists an integer $k < \infty$ such that $\forall\omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ it holds that $|S_\omega| = 1$, then a function $\Psi : \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0) \longrightarrow R(Q, M_0) \times R(Q, M_0)$ can be computed, where Ψ fulfills the following: $\forall\omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ it holds that $\Psi(\omega, (Q, M_0)) = (M_0, M_i)$ where M_0 is the initial marking and M_i is the marking reached by the firing of the underlaying firing transition sequence σ_ω.

(Necessity) Suppose that there is no integer $k < \infty$ such that $\forall\omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ it holds that $|S_\omega| = 1$, then for any k there is at least one $\omega \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ such that $|S_\omega| \neq 1$, therefore $|S_\omega| > 1$. Assume further without lost of generality that $S_\omega = \{\gamma_1 = M_i M_j \cdots M_k \cdots M_n,\ \gamma_2 = M'_i M'_j \cdots M'_k \cdots M'_n\}$.

Since these sequences are different, then there must exist markings M_k, M'_k such that $M_k \neq M'_k$ in γ_1, γ_2 respectively. Notice that when the initial marking of (Q, M_0) is M_k or M'_k then there exist two different values to assign to M_0, or the function Ψ cannot be obtained, a contradiction. □

It is worth noting here that the previous theorem uses the input and output sequences, but it does not use the structure of the model. Thus, this result can be applied to several dynamic system models.

For instance, when the model is the linear differential state equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(t_0) = x_0 \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

then, the system is observable if and only if there exists a finite t_1 such that any pair of different state trajectories $x_1(t)$, $x_2(t)$ can be distinguished using the input signals $u_1(t)$, $u_2(t)$ and the output signals $y_1(t)$, $y_2(t)$, for $t_0 \leq t \leq t_1$. To prove this fact, we use the solution to equation (4) given by

$$x(t) = e^{At}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,d\tau. \tag{5}$$

If the same input is given for two different initial conditions $x_1(t_0)$, $x_2(t_0)$, then the following two possible solutions are found, respectively:

$$\begin{aligned} x_1(t) &= e^{At}x_1(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,d\tau \\ x_2(t) &= e^{At}x_2(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,d\tau. \end{aligned} \tag{6}$$

Now, if the output signals $y_1(t)$ and $y_2(t)$, generated respectively by $x_1(t)$ and $x_2(t)$, are the same for every $t \geq t_0$, then

$$Ce^{At}x_1(t_0) + C\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,\mathrm{d}\tau = Ce^{At}x_2(t_0) + C\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,\mathrm{d}\tau \quad (7)$$

or

$$Ce^{At}x_1(t_0) - Ce^{At}x_2(t_0) = 0. \quad (8)$$

Therefore

$$Ce^{At}(x_1(t_0) - x_2(t_0)) = 0. \quad (9)$$

Using Taylor series, equation (9) can be rewritten as

$$C\left(I + At + \frac{1}{2}A^2t^2 + \frac{1}{3!}A^3t^3 + \ldots + \frac{1}{k!}A^kt^k + \ldots\right)(x_1(t_0) - x_2(t_0)) = 0. \quad (10)$$

Since $e^{At} = \left(I + At + \frac{1}{2}A^2t^2 + \frac{1}{3!}A^3t^3 + \ldots + \frac{1}{k!}A^kt^k + \ldots\right)$ is non singular [3], then, according to Theorem 3.1, the system is non observable if there exist at least two vectors $x_1(t_0)$, $x_2(t_0)$ belonging to $\ker(C) \cap \ker(CA) \cap \cdots \cap \ker(CA^{n-1})$, or equivalently, the system is non observable if the rank of the matrix

$$\mathcal{O} = \begin{bmatrix} C \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (11)$$

is less than n. Thus, the continuous system is non observable if $\ker(C) \cap \ker(CA) \cap \cdots \cap \ker(CA^{n-1}) \neq \{0\}$ or the rank of $\mathcal{O}$ is less than n. The converse can also be proved [24].

Notice that using Theorem 3.1, a structural characterization of the observability for those systems described by equation (4) was derived. Now, using a similar reasoning, a structural characterization for observability in *IPN* models will be derived.

Assume that an *IPN* is non observable, then there exist two initial markings M_0^1, M_0^2 (not necessarily different because *IPN* could be, in the general case, non deterministic) such that when the same input γ is given in both cases, two different sequences of markings $s_1^{n_1} = M_0^1 M_1^1 \cdots M_{n_1}^1$ and $s_2^{n_2} = M_0^2 M_1^2 \cdots M_{n_2}^2$ (reached by the firing transition sequences $\sigma_1 = t_1^1 t_2^1 \cdots t_{n_1}^1$ and $\sigma_2 = t_1^2 t_2^2 \cdots t_{n_2}^2$, respectively) are generated. Both of them generate the same output word ϕ for every $n_1, n_2 \geq 0$. Now, using the *IPN* state equation, it is equivalent to that for all $\sigma_i' \in \overline{\sigma_1}$ there exists a $\sigma_j' \in \overline{\sigma_2}$, $i \neq j$, such that

$$\varphi(M_0^1 + C\vec{\sigma_1'}) = \varphi(M_0^2 + C\vec{\sigma_2'}) \quad (12)$$

or equivalently,

$$\varphi(M_0^1 + C\vec{\sigma_1'}) - \varphi(M_0^2 + C\vec{\sigma_2'}) = 0. \quad (13)$$

In the case when φ and C are linear, the previous condition is equivalent to the following conditions:

$$\begin{aligned} &\varphi(M_0^1 - M_0^2) = 0, \text{ and} \\ &\varphi C(\vec{\sigma_1'} - \vec{\sigma_2'}) = 0, \end{aligned} \quad (14)$$

and other combinations are not possible.

Rewriting previous equations, we obtain

1) $M_0^1 - M_0^2 \in \ker \varphi$, and
2) $(\overrightarrow{\sigma_1} - \overrightarrow{\sigma_2}) \in \ker \varphi C$.

Notice that condition 1) is equivalent to the following conditions:

a) $M_0^1 = M_0^2$, or
b) $\varphi M_0^1 = \varphi M_0^2$ with $M_0^1 \neq M_0^2$,

and condition 2) is equivalent to the following conditions:

c) $\sigma_1' = \sigma_2'$, or
d) $C\overrightarrow{\sigma_1} = C\overrightarrow{\sigma_2}$ with $\sigma_1' \neq \sigma_2'$, or
e) $\varphi C\overrightarrow{\sigma_1} = \varphi C\overrightarrow{\sigma_2}$ with $C\overrightarrow{\sigma_1} \neq C\overrightarrow{\sigma_2}$.

Thus the combination of conditions a) or b) with the conditions c), d) or e) leads to a system of conditions equivalent to the system (14). However, not all combinations are valid under the assumptions of this work. For instance, it has been assumed that $s_1^{n_1}$ and $s_2^{n_2}$ are different, therefore, it is not valid that $M_0^1 = M_0^2$ and $\sigma_1' = \sigma_2'$ since it indicates that $s_1^{n_1} = s_2^{n_2}$. The condition $C\overrightarrow{\sigma_1} = C\overrightarrow{\sigma_2}$ with $\sigma_1' \neq \sigma_2'$ is also not valid since it implies that there exist redundant transitions in σ_1' and σ_2', violating Remark 2.3. Thus the following combinations are the valid ones:

R_1) $M_0^1 = M_0^2$ and $\varphi C\overrightarrow{\sigma_1} = \varphi C\overrightarrow{\sigma_2}$ with $C\overrightarrow{\sigma_1} \neq C\overrightarrow{\sigma_2}$.
R_2) $\varphi M_0^1 = \varphi M_0^2$ with $M_0^1 \neq M_0^2$ and $\sigma_1' = \sigma_2'$.
R_3) $\varphi M_0^1 = \varphi M_0^2$ with $M_0^1 \neq M_0^2$ and $\varphi C\overrightarrow{\sigma_1} = \varphi C\overrightarrow{\sigma_2}$ with $C\overrightarrow{\sigma_1} \neq C\overrightarrow{\sigma_2}$.

The first case characterizes when an initial marking M_0^i enables different firing transition sequences σ_1, σ_2. Thus, two marking sequences generating the same sequence ω of input-output symbols are obtained.

The second case characterizes when two different markings enable the same firing transition sequence, thus two marking sequences generating the same sequence of input-output symbols are obtained.

Finally, the third case characterizes when two different markings enable two different firing transition sequences σ_1, σ_2, such that two marking sequences generating the same sequence of input-output symbols are obtained.

According to the previous argumentation, if the *IPN* is non observable, then condition R_1), or R_2), or R_3) holds. Thus, if these cases are avoided the *IPN* becomes observable.

For instance, the first case is avoided when any pair of firing transition sequences σ_1, σ_2, such that $\sigma_1 \neq \sigma_2$, can be distinguished from each other. Because in this case for each input-output symbol word generated ω, we have $|\Omega(\omega)| = 1$. Thus according to Definition 2.12, $|S_\omega| = 1$, and by Theorem 3.1 the net is observable.

The second case is avoided when there exists a marking M_k that can be computed after the firing of $k < \infty$ transitions. In this case, we claim that either $M_0^1 \xrightarrow{\sigma} M_k$

or $M_0^2 \xrightarrow{\sigma} M_k$, but not both M_0^1, M_0^2 could reach M_k firing the same input word. In order to prove it, assume that this is not true, i. e. $M_0^1 \xrightarrow{\sigma} M_k$ and $M_0^2 \xrightarrow{\sigma} M_k$. Since the firing transition sequence is the same in both cases, using the *IPN* state equation it follows that M_{k-1} is the same in both marking sequences and then, by recursively applying this idea, we obtain that $M_0^1 = M_0^2$, a contradiction. Then, when σ is known, the knowledge of M_k allows to detect the marking sequence, i. e. $|S_\omega| = 1$.

The third case is a combination of previous two cases. Thus, the knowledge of the fired transition sequence and the reached marking M_k are enough to avoid this case.

In order to formalize these concepts, the following definitions are introduced.

Definition 3.2. An *IPN* given by $(Q, \mathcal{M}_0)$ is sequence-detectable if there exists an integer $k < \infty$ such that for any $\omega \in \Lambda^k(Q, \mathcal{M}_0) \cup \Lambda_B(Q, \mathcal{M}_0)$ the information of ω and the structure of the *IPN* suffices to determine the fired transition sequences. In other words there exists a function

$$\Psi_S : (\Lambda^k(Q, \mathcal{M}_0) \cup \Lambda_B(Q, \mathcal{M}_0)) \times (Q, \mathcal{M}_0) \longrightarrow \pounds(Q, \mathcal{M}_0)$$

such that $\forall \omega \in \Lambda^k(Q, \mathcal{M}_0) \cup \Lambda_B(Q, \mathcal{M}_0)$ it holds that $\Psi_S(\omega, (Q, \mathcal{M}_0)) = \sigma_\omega$.

Definition 3.3. An *IPN* given by $(Q, \mathcal{M}_0)$ is marking-detectable if there exists an integer $k < \infty$ such that for any $\omega \in \Lambda^k(Q, \mathcal{M}_0) \cup \Lambda_B(Q, \mathcal{M}_0)$ the *IPN* suffices to determine the actual marking of the *IPN*. In other words there exists a function

$$\Psi_M : (\Lambda^k(Q, \mathcal{M}_0) \cup \Lambda_B(Q, \mathcal{M}_0)) \times (Q, \mathcal{M}_0) \longrightarrow R(Q, \mathcal{M}_0)$$

such that $\forall \omega \in \Lambda^k(Q, \mathcal{M}_0) \cup \Lambda_B(Q, \mathcal{M}_0)$ it holds that $\Psi_M(\omega, (Q, \mathcal{M}_0)) = M_i$, where M_i is the marking reached by the firing of the underlying firing transition sequence σ_ω.

According to these definitions and the previous discussion on the conditions that lead to observable *IPN* it follows that, under conditions of Remark 2.3, sequence and marking-detectability are necessary for observability. Moreover, it can be proved in this case that sequence and marking-detectability are necessary and sufficient for observability. This is formally posed in the following result.

Lemma 3.1. An *IPN* given by $(Q, \mathcal{M}_0)$ is observable if and only if it is both sequence and marking-detectable.

Proof. (Sufficiency) Let $(Q, \mathcal{M}_0)$ be an *IPN* and assume that there exists an integer $k < \infty$ and functions $\Psi_M : \Lambda^k(Q, \mathcal{M}_0) \times (Q, \mathcal{M}_0) \longrightarrow R(Q, \mathcal{M}_0)$ and $\Psi_S : \Lambda^k(Q, \mathcal{M}_0) \times (Q, \mathcal{M}_0) \longrightarrow \pounds(Q, \mathcal{M}_0)$ such that $\forall \omega \in \Lambda^k(Q, \mathcal{M}_0)$ it holds that $\Psi_M(\omega, (Q, \mathcal{M}_0)) = M_i$, and $\Psi_S(\omega, (Q, \mathcal{M}_0)) = \sigma_\omega$, where M_i is the marking reached by the firing of the underlying firing transition sequence σ_ω. Then, a function $\Psi(\omega, (Q, \mathcal{M}_0)) = (M_0, M_i)$ can be built as $\Psi(\omega, (Q, \mathcal{M}_0)) = (M_0, M_M)$ where

$M_M = \Psi_M(\omega, (Q, \mathcal{M}_0))$ and $M_0 = M_M - C\vec{\sigma_\omega}$ is computed using the *IPN* state equation.

(Necessity) It follows from previous discussion on the conditions that lead to observable *IPN*. □

In the following section, a characterization based on the structure of the *IPN* exhibiting sequence and marking-detectability properties (and hence, observability), is addressed.

4. SEQUENCE AND MARKING–DETECTABILITY CHARACTERIZATIONS

Sequence-detectability implies the knowledge of all fireable sequences in the *IPN*, thus the problem of determining when an *IPN* is sequence-detectable is a computational complex task. Fortunately, using event-detectability [22], a stronger property, this complexity can be overcome because it leads to polynomial algorithms. This property is defined as follows.

Definition 4.1. An *IPN* given by $(Q, \mathcal{M}_0)$ is event-detectable if any transition firing can be uniquely determined by the knowledge of the input given to $(Q, \mathcal{M}_0)$ and output signals that it produces.

The following lemma provides a structural characterization of the *IPN* exhibiting event-detectability.

Lemma 4.1. A live *IPN* given by $(Q, \mathcal{M}_0)$ is event-detectable if and only if:

1. $\forall t_i, t_j \in T$ such that $\lambda(t_i) = \lambda(t_j)$ or $\lambda(t_i) = \varepsilon$, it holds that $\varphi C(\cdot, t_i) \neq \varphi C(\cdot, t_j)$, and
2. $\forall t_k \in T$ it holds that $\varphi C(\cdot, t_k) \neq 0$.

Proof. (Sufficiency) Assume that $(Q, \mathcal{M}_0)$ is an *IPN* where $\forall t_i, t_j \in T$ such that $\lambda(t_i) = \lambda(t_j)$ or $\lambda(t_i) = \varepsilon$, it holds that $\varphi C(\cdot, t_i) \neq \varphi C(\cdot, t_j)$, and $\forall t_k \in T$ it holds that $\varphi C(\cdot, t_k) \neq 0$. Let $M_m, M_n \in \mathcal{M}_0$ and a transition $t_p \in T$ such that $M_m \xrightarrow{t_p} M_n$ fire while the input symbol α is given to $(Q, \mathcal{M}_0)$. From the state equation (1), $y_n - y_m$ can be computed as

$$y_n - y_m = \varphi(M_n) - \varphi(M_m) = \varphi(M_m + C(\cdot, t_p)) - \varphi(M_m) = \varphi C(\cdot, t_p).$$

Since $\forall t_k \in T$ it holds that $\varphi C(\cdot, t_k) \neq 0$, the change in the output produced by the firing of t_p is not null, that is $y_n - y_m \neq 0$. Now, there are two possibilities:

a) Suppose that the input symbol is ε. Since $\forall t_i, t_j \in T$ such that $\lambda(t_i) = \lambda(t_j) = \varepsilon$, it holds that $\varphi C(\cdot, t_i) \neq \varphi C(\cdot, t_j)$, then there is no transition $t_q \in T$ with $t_q \neq t_p$ such that $\lambda(t_q) = \varepsilon$ and $\varphi C(\cdot, t_q) = \varphi C(\cdot, t_p)$. Thus, the firing of transition t_p is the only one that could produce the change $y_n - y_m = \varphi C(\cdot, t_p)$ while the null input word $\alpha = \varepsilon$ was given to the system.

b) Suppose now that the input symbol is $\alpha \neq \varepsilon$. Since $\forall t_i, t_j \in T$ such that $\lambda(t_i) = \lambda(t_j) = \alpha$ (or $\lambda(t_i) = \varepsilon$, $\lambda(t_j) = \alpha$) it holds that $\varphi C(\cdot, t_i) \neq \varphi C(\cdot, t_j)$, then there is no transition $t_q \in T$ with $t_q \neq t_p$ such that $\lambda(t_q) = \alpha$ (or $\lambda(t_q) = \varepsilon$) and $\varphi C(\cdot, t_q) = \varphi C(\cdot, t_p)$. Thus, the firing of transition t_p is the only one that could produce the change $y_n - y_m = \varphi C(\cdot, t_p)$ while the non-null input word α was given to the system.

Then, in both cases the firing of transition t_p can be uniquely determined and $(Q, \mathcal{M}_0)$ is event-detectable.

(Necessity) Suppose first that there exist two transitions $t_i, t_j \in T$ such that $\lambda(t_i) = \lambda(t_j) = \alpha$ and $\varphi C(\cdot, t_i) = \varphi C(\cdot, t_j)$. Then for an input word α there are two transitions t_i, t_j that may fire, therefore the input symbol given to $(Q, \mathcal{M}_0)$ does not provide information to distinguish the firings of t_i and t_j. In this case, since $\varphi C(\cdot, t_i) = \varphi C(\cdot, t_j)$, the changes in the output that those firings produce are equal and no further information is provided. Therefore, there is no way to distinguish the firings of t_i and t_j.

Now suppose that there exist two transitions $t_i, t_j \in T$ such that $\lambda(t_i) \neq \varepsilon$, $\lambda(t_j) = \varepsilon$ and $\varphi C(\cdot, t_i) = \varphi C(\cdot, t_j)$. Assume that $\lambda(t_i) = \alpha$. Then for an input word α, both transitions t_i and t_j may fire, again the input symbol does not help to distinguish the firings of t_i and t_j. If also $\varphi C(\cdot, t_i) = \varphi C(\cdot, t_j)$, then both firings produce the same change in the output and once more the firings of those transitions cannot be distinguished.

Finally, assume that $\exists t_k \in T$ such that $\varphi C(\cdot, t_k) \neq 0$. Then the firing of t_k has no effect in the output and for any input symbol α given to $(Q, \mathcal{M}_0)$, there is no way to determine if transition t_k fires.

It follows that in all those cases the firing of the transitions cannot be uniquely determined and if any of those conditions holds, $(Q, \mathcal{M}_0)$ is not event-detectable. □

Now, as we mention in Definition 3.3, marking-detectability deals with the possibility of finding out the current marking of an *IPN*. This property is strongly related with the structural properties of the *IPN* and can be analyzed using the following place classification.

The following set contains the places whose current marking can be computed from the *IPN* output $\varphi(M_k)$.

Definition 4.2. Let $(Q, \mathcal{M}_0^{\mathrm{B}})$ be an *IPN*. The set of output-computable places is defined by $S_m = \{p_i \in P \,|\, p_i \text{ is a computable place}\}$.

Notice that the set S_m can be computed from the knowledge of φC since $p_i \in S_m$ when the column of $\varphi C(\cdot, i)$ is not null and different from each other.

Now, the following set contains the places whose marking can be computed when any T-semiflow is fired.

Definition 4.3. Let $(Q, \mathcal{M}_0^{\mathrm{B}})$ be an *IPN*, $X = \{x_1, \ldots, x_r\}$ be the set of elemental T-components [5, 6] of the *IPN*, P_i be the set of places belonging to x_i, and $E = \{e_1, \ldots, e_s\}$ be the set of *BCML* defined in the *IPN*. The set of *BCML*-computable places is defined by:

$$S_e = \{p_i \in P \mid \forall x_j \in X \text{ there exists an } e_k \in E \text{ with } p_i \in e_k$$

and

$$\left(\bigcup_{p_n \in e_k} \{p_n\} \right) \cap P_j \neq \emptyset\}.$$

In other words, a place p_i belongs to S_e when some places of any T-component x_i are contained in a *BCML*, e_j, also containing place p_i. Thus, when transitions in x_i are fired, the marking of places in e_j is computed; hence the marking of p_i is computed. Notice that $p_i \in S_e$ if the following linear programming problem (*LPP*) has no solution:

$$\begin{aligned} &\nexists Z \text{ such that} \\ &CX = 0;\ X \ngeq 0 \\ &X^T C^{-T} = Z \\ &\Gamma|_{p_i} Z = 0, \end{aligned} \tag{15}$$

where C is the incidence matrix of the *IPN*, C^- is the pre-incidence matrix of the *IPN*, X is a T-semiflow [5], Z are the places contained in the T-component generated by X (these places are computed by the equation $X^T C^{-T}$), and $\Gamma|_{p_i}$ is the matrix of the *CML* (see Definition 2.13) restricted to rows q such that $\Gamma(q,i) = 1$. Thus, $\Gamma|_{p_i} Z = 0$ means that there exist T-components whose places are disjoint from *CML* equations containing place p_i, i. e. $p_i \notin S_e$. Hence, if the previous *LPP* has no solution, then $p_i \in S_e$.

Definition 4.4. Let $(Q, \mathcal{M}_0^{\mathrm{B}})$ be an *IPN*, $E = \{e_1, \ldots, e_s\}$ be the set of *BCML* defined in the *IPN*, and S_m, S_e be the sets of output-computable places and *BCML*-computable places, respectively. The set of transitive-computable places is defined by $S_c = \bigcup_{i=0}^{z} S_c^i$ where:

$S_c^0 = S_m \cup S_e$

$S_c^i = S_c^{i-1} \cup \{p_i \in P | p_i \in e_k, \text{ and every } p_j \in e_k,\ j \neq i, \text{ fulfills that } p_j \in S_c^{i-1}\}$

and z could be at most equal to $|P|$.

Notice that S_c can be computed with a polynomial algorithm.

As we mention in Definition 2.7, the marking of output-computable places can be computed from the output function. This fact is summarized in the following lemma.

Lemma 4.2. Let $(Q, \mathcal{M}_0^{\mathrm{B}})$ be a binary, live and event-detectable *IPN*. Then, there exists an integer $k < \infty$ such that $\forall \omega \in \Lambda(Q, \mathcal{M}_0^{\mathrm{B}})$, $k \leq |\omega| < \infty$, the marking $M_k(p_i)$ for a place p_i can be computed if $p_i \in S_m$.

Proof. Let $\omega \in \Lambda(Q, \mathcal{M}_0^{\mathrm{B}})$ be a word of a finite length, $k \geq 0$, and $\omega = (\alpha_0, y_0) \cdots (\alpha_k, y_k)$. Then the current output of the *IPN* is the vector $y_k = \varphi M_k$. Since p_i is computable, then $\varphi(\cdot, i)$ is different from other columns. Moreover, the columns of φ are elemental vectors, thus $M_k(p_i) = y_k(q)$, where q is the number such that $\varphi(q, i) = 1$. □

Now, the marking of a place belonging to the *BCML*-computable places set can be computed as the following lemma states.

Lemma 4.3. Let $(Q, \mathcal{M}_0^{\mathrm{B}})$ be a binary, live and event-detectable *IPN*, $X = \{x_1, \ldots, x_n\}$ be the set of elemental T-components of the IPN, and $E = \{e_1, \ldots, e_s\}$ be the set of *BCML* defined in the *IPN*. Then, there exists an integer $k_i < \infty$ such that $\forall \omega \in \Lambda(Q, \mathcal{M}_0^{\mathrm{B}})$, $k_i \leq |\omega| < \infty$, the marking $M_0(p_i)$ for a place p_i can be computed if $p_i \in S_e$.

Proof. If p_i is computable, then its marking can be computed using Lemma 4.2. If p_i is a non computable place, then it belongs to an equation $e_k \in E$.

Moreover, since $(Q, \mathcal{M}_0^{\mathrm{B}})$ is a live *IPN*, then there exists a finite integer k' such that $\forall \sigma \in \mathcal{L}(Q, \mathcal{M}_0^{\mathrm{B}})$ fulfilling $|\sigma| \geq k'$, σ contains the firing of every transition in a T-component $x_i \in X$.

By definition, there exists a place p_z such that $p_i, p_z \in e_k$ and $p_z \in x_i$, thus the firing of a prefix of σ will add a token in p_z. Let M_k be the first marking adding tokens to p_z. This marking is detected because the *IPN* is event-detectable, and the firing of a transition in $^\bullet p_z$, adding a token to p_z, can be detected. Since the *IPN* is binary, then $M_k(p_z) = 1$ and using the equation e_k we conclude that $M_k(p_i) = 0$. Moreover, since the net is event detectable, then the marking of p_i will be known for any marking M_l such that $l \geq k$. □

Finally, we prove that the marking into the places belonging to S_c is computable.

Lemma 4.4. Let $(Q, \mathcal{M}_0^{\mathrm{B}})$ be a binary, live and event-detectable *IPN*. Then, there exists an integer $k_i < \infty$ such that $\forall \omega \in \Lambda(Q, \mathcal{M}_0^{\mathrm{B}})$, $k_i \leq |\omega| < \infty$, the marking $M_0(p_i)$ for a place p_i can be computed if $p_i \in S_c$.

Proof. According to previous lemmas, the marking of p_i can be computed if $p_i \in S_m \cup S_e$. Thus, we will focus on places $\{p_a, p_b, \ldots, p_g\} = S_c - (S_m \cup S_e)$. Assume that places $\{p_a, p_b, \ldots, p_g\}$ are given in the following order: the leading places in this set are those belonging to S_c^1, after them are those belonging to S_c^2, and so on.

By definition of S_c^i, for each $p_x^i \in S_c^i$ there exists a set of places $S_{p_x^i} = \{p_w, \ldots, p_s\}$ such that places $\{p_x^i\} \cup S_{p_x^i}$ are the only places belonging to the same *BCML*. Name $e_{p_x^i}$ this *BCML*.

Thus, the following *BCML* holds for any marking M,

$$M(p_x^i) + \sum_{p_r \in S_{p_x^i}} M(p_r) = 1. \tag{16}$$

Hence, the marking of place $p_x^i \in S_c^i$ can be computed as

$$M(p_x^i) = 1 - \sum_{p_r \in S_{p_x^i}} M(p_r). \tag{17}$$

Moreover, by definition of S_c^i, we have that $S_{p_x^i} \subseteq S_c^{i-1}$, therefore the marking of place p_x^i can be computed using the places in S_c^{i-1}. To accomplish this task, we define the vector $\vartheta_{p_x^i}$ in the following way: $\vartheta_{p_x^i}(r) = 1$ if $p_r \in S_c^{i-1} \cap e_{p_x^i}$, otherwise $\vartheta_{p_x^i}(r) = 0$. Thus, equation (17) can be rewritten as

$$M(p_x^i) = 1 - \sum_{p_r \in S_c^{i-1}} \vartheta_{p_x^i}(r) M(p_r). \tag{18}$$

In other words, the marking of places in S_c^i can be computed using the marking of places in S_c^{i-1}. Thus, the marking of places in S_c^1 can be computed using the marking of places in $S_c^0 = S_m \cup S_e$ (whose markings are known). In general, the marking of places in S_c^i can be recursively computed using the marking of places in S_c^{i-1}. □

Finally, previous results lead to the following theorem, that provides a structural characterization of observable *IPN*.

Theorem 4.1. An *IPN* given by $(Q, \mathcal{M}_0^{\mathrm{B}})$ is observable if $P = S_c$ and $(Q, \mathcal{M}_0^{\mathrm{B}})$ is event-detectable.

Proof. If $(Q, \mathcal{M}_0^{\mathrm{B}})$ is event-detectable, it is sequence-detectable. According to Lemma 4.4, if $(Q, \mathcal{M}_0^{\mathrm{B}})$ is event-detectable and $P = S_c$, then $(Q, \mathcal{M}_0^{\mathrm{B}})$ is marking detectable. Thus, because of Lemma 3.1, $(Q, \mathcal{M}_0^{\mathrm{B}})$ is observable. □

5. OBSERVER DESIGN

The adopted observer scheme is the widely known architecture depicted in Figure 2, and used in [21, 22]. In this figure, matrices C^D and C^ε represent, respectively, the incidence matrix columns of the manipulated and non manipulated transitions.

Now, an *IPN* observer structure and the gain terms are defined as in references [21, 22].

Definition 5.1. Let $N_S = (P_S, T_S, I_S, O_S, \Sigma, \lambda, \varphi)$ be a binary and event-detectable *IPN* where $P_S = S_c$. The net $N_O = (P_O, T_O, I_O, O_O, \Sigma, I, I)$ is an observer for N_S, i.e. the marking of N_O will tend to the marking of N_S as transitions are fired, when:

1. Its state equation is

$$\begin{aligned} S_{k+1} &= S_k + \begin{bmatrix} C & -I & I \end{bmatrix} \begin{bmatrix} \gamma_k \\ \beta_k \\ \delta_k \end{bmatrix} \\ \hat{y}_k &= S_k \end{aligned} \tag{19}$$

   where I is the identity function or the identity matrix, depending of the context.

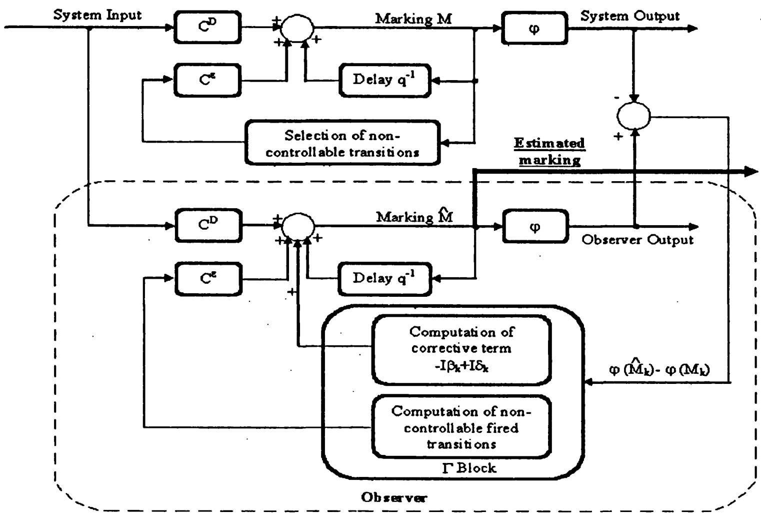

Fig. 2. The system and observer architecture.

2. The admissible initial marking S_0 of N_O is $S_0(p_i) = M_0(p_i)$ if p_i is computable, and $S_0(p_i)$ is any value fulfilling that $0 \leq S_0(p_i) \leq 1$ if p_i is not computable.
3. When a transition t_j fires in N_S then:

- $\gamma_k = \begin{cases} \overrightarrow{t_j} & \text{if } t_j \text{ is enabled in } N_O \\ 0 & \text{any other case} \end{cases}$

- $\beta_k = \begin{bmatrix} \varpi_1 \\ \vdots \\ \varpi_n \end{bmatrix}, \ \delta_k = \begin{bmatrix} \upsilon_1 \\ \vdots \\ \upsilon_n \end{bmatrix}$, where

$$\varpi_i = \begin{cases} 1 & \text{if } \gamma_k = \overrightarrow{t_j}, \text{ and } S_k(p_i) + C(p_i,\cdot)\gamma_k > 1, \text{ or} \\ & \quad \gamma_k \neq \overrightarrow{t_j},\ p_i \in^\bullet (t_j) \text{ and } S_k(p_i) > 0 \\ 0 & \text{any other case} \end{cases}$$

$$\upsilon_i = \begin{cases} 1 & \text{if } \gamma_k = \overrightarrow{t_j}, \text{ and } S_k(p_i) + C(p_i,\cdot)\gamma_k < 0, \text{ or} \\ & \quad \gamma_k \neq \overrightarrow{t_j},\ p_i \in (t_j)^\bullet \text{ and } S_k(p_i) < 1 \\ 0 & \text{any other case.} \end{cases}$$

We present next an example of an observable *IPN* and the use of an observer to compute the current *IPN* marking.

Example 5.1. Consider the producer-consumer scheme depicted in Figure 3. The model consists of a producer unit (*PU*), a consumer unit (*CU*) and a buffer of $k-$slots.

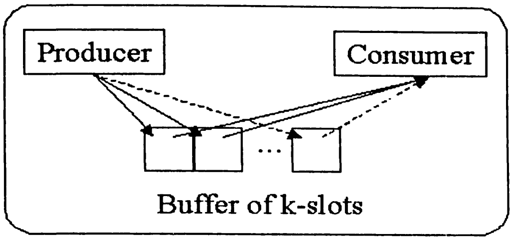

Fig. 3. Producer-consumer with buffer of k-slots scheme.

The behavior of this system is the following: The producer unit *PU* creates and delivers products into free buffer positions. The consumer unit *CU* retrieves products from the buffer when there exists a product stored into a slot buffer. *PU* emits a signal P when is creating a product. Similarly, *CU* emits C when is consuming a product. Each buffer slot s_n emits a signal o_n while it is occupied.

An *IPN* model of this system is shown in Figure 4, where the meaning of each place is depicted in the same figure.

In this model, the input alphabet is $\Sigma = \{\}$, because there are no input signals in the system. The function λ for this model is $\forall t_i,\ \lambda(t_i) = \varepsilon$. Since places $p_3, p_6, p_8, p_9, \ldots, p_{6+2k}$ represent states that have different output signals associated and all other places have no output signal, φ is the $(2+k) \times (6+2k)$ matrix

$$\varphi = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & 0 \\ \vdots & & & & & & & & & \ddots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 1 \end{bmatrix}.$$

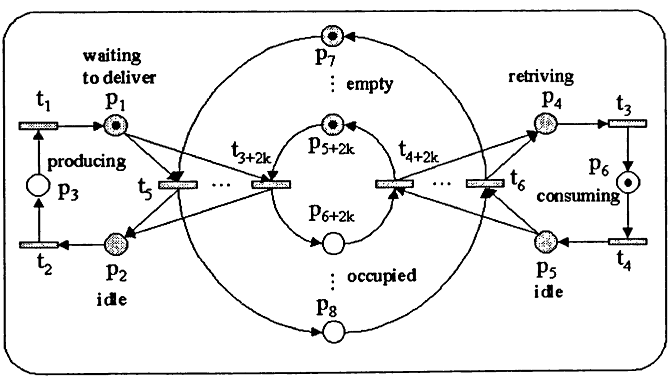

Fig. 4. *IPN* model of the producer-consumer system.

According to the system description and the meaning given to each place, the following *BCML* can be derived:

$$\begin{aligned} M(p_1) + M(p_2) + M(p_3) &= 1 \\ M(p_4) + M(p_5) + M(p_6) &= 1 \\ M(p_7) + M(p_8) &= 1 \\ &\vdots \\ M(p_{5+2k}) + M(p_{6+2k}) &= 1. \end{aligned}$$

The initial marking for the system is $M_0 = [1\ 0\ 0\ 0\ 0\ 1\ 1\ 0 \cdots 1\ 0]^T$ and represents that the buffer is completely empty, the *PU* is waiting to deliver, and the *CU* is consuming a product.

The matrix φC is shown below. Notice that none of the columns of the matrix is null and they are different from each other. Therefore, the *IPN* is event-detectable.

$$\varphi C = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & & & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & & & 0 \\ \vdots & & & & & & & & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

Indeed, the signals associated to places p_{6+2i} allow to uniquely determine the firing of transitions t_{3+2i}, t_{4+2i}. Analogously, the signals associated to p_3 and p_6 allow to uniquely determine, respectively, the firing of t_1, t_2 and t_3, t_4.

Notice that places $p_3, p_6, p_8, \ldots, p_{6+2k}$ belong to the set S_m, because they are computable. Since all T-components contain places p_1, p_2, p_3, besides of p_4, p_5, p_6, then $p_1, p_2, p_3, p_4, p_5, p_6 \in S_e$. Moreover, places $p_7, \ldots, p_{5+2k}$ are included in equations of *BCML* with the form $M(p_{5+2i}) + M(p_{6+2i}) = 1$ and, since $p_{6+2i} \in S_m$ it follows that $p_7, \ldots, p_{5+2k} \in S_c$. Thus, it holds that $P = S_c$ and according to Lemma 4.4, the marking of all places can be computed and therefore the *IPN* is marking detectable. Moreover, being event and marking-detectable, the *IPN* is observable.

An *IPN* observer for this system is depicted in Figure 5, where the initial marking S_0 is chosen as $S_0 = [0\ 0\ 0\ 0\ 0\ 1\ 1\ 0 \cdots 1\ 0]^T$.

The initial estimation error is $S_0 - M_0 = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \cdots 0\ 0]^T$, and according to the definition of firing vectors β_k and δ_k, this error becomes null as soon as any of the transitions t_{3+2i} is fired.

6. CONCLUSIONS

This work introduced a novel definition of observability that considers the structure and finite input and output words of the *IPN*. Based on all the possible marking sequences generated by the *IPN*, a geometric characterization of observability in *IPN* was presented. It is a very general characterization and can be used in any state

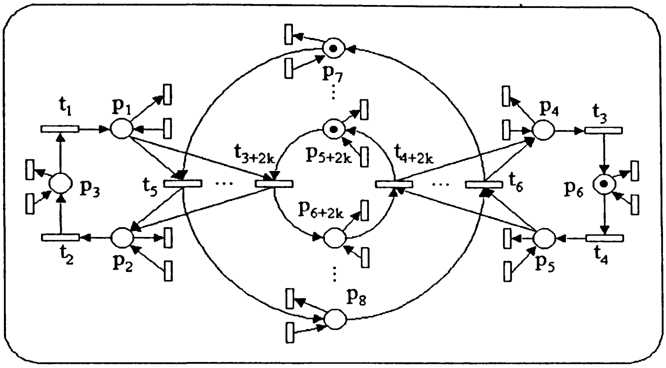

Fig. 5. *IPN* observer for the producer-consumer system.

dynamic model. For instance, when this characterization was applied to the linear continuous systems case, the classical conditions of observability for continuous systems were derived. This fact suggests that the definition and characterization of observability are well posed. This characterization was also applied to the *IPN* model, and structural characterizations for a broad subclass of *IPN* were derived. The main advantage of this last approach lies in the fact that the marking sequences are no longer needed, thus polynomial algorithms can be used to check the observability in those cases.

Current research is being conducted to derive necessary and sufficient conditions for observability based on the *IPN* structure.

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