Kybernetika

Book reviews

RADIM BĚLOHLÁVEK

Fuzzy Relational Systems. Foundations and Principles

Kluwer Academic / Plenum Publishers. New York, Boston 2002.
x + 369 pages.
ISBN 0-306-46777-1.

The referred book is devoted to the fuzzy relational modeling, its theoretical foundation, potential application, and to some closely related topics and results.

Its text is, beside a short *"Preface"*, divided into seven chapters. The first one of them, *"Preliminaries"* very briefly informs about some mathematical concepts used in the following explanation, like sets and ordered sets, mappings, lattices, equivalences, and algebras. Chapter 2, titled *"Fuzzy Approach, Graded Truth and Structures of Truth Values"*, offers a general discussion on the notions mentioned in its title, and on their properties. It represents a not-trivial introduction of the reader into the environment of the fuzziness, its conceptual background and tools.

The most extensive Chapter 3 *"Fuzzy Structures"* is important for the understanding further chapters oriented to more specific topics. It summarizes the general concepts and results related to the fuzzy relations, starting by the concept of fuzzy sets, fuzzy relations and fuzzy logic, and continuing with a presentation of such topics like similarity, morphism, substructures, cutlike semantics, fuzzy equalities and algebras using them, and others. Chapter 4 dealing with *"Binary Fuzzy Relations"* is focused on operations and special types of binary fuzzy relations and their properties. They include sections devoted to formal tools representing such relations like similarity and hierarchy. Binary fuzzy relations are interpreted as relations between objects and attributes. Topics being related to those presented in Chapter 4 are investigated in Chapter 5, *"Object-Attribute Fuzzy Relations and Fuzzy Concept Lattices"*. Particular sections are oriented to conceptual granulation, similarity related to logical precision, fuzzy Galois connections, and fuzzy concept lattices.

Chapter 6 *"Composition and Decomposition of Fuzzy Relations"* treats the concepts contained in its title, and their properties. It surveys solutions and approximate solutions of relational equations, systems of equations and their sensitivity, and the structure of solutions. The last Chapter 7 headed *"Miscellanea"* contains some miscellaneous topics like fuzzy closure operators, applications of some selected results to some problems and models and fuzzy logic with truth degrees in residuated lattices.

The book is concluded by an extensive (222 items) and representative list of *"References"* and by the *"Index"*. In addition, each of Chapters 2 – 7 is concluded by "Bibliographical Remarks" containing references to the related bibliographical items and also brief characterization of alternative approaches.

The author in his explanation stresses the issues and result which are new, not so frequently discussed or not generally known, meanwhile the space devoted to the topics which are relatively well known from other sources is shorter, and the existing literature is referred. The book aims to address two types of readers. Theoreticians being interested in the recent state of art in the subject treated in the book, and applied researchers looking for theoretical tools supporting the development of new methods of control, decision reasoning, knowledge extraction, systems analysis, etc. It is to be stressed that the respect to this second group of readers does not influence the mathematical exactness of the explanation.

The book is written in a precise and consequently exact style, based on definitions, theorems and proofs. The ordering of chapters and sections is logical and natural enabling their reading in the order of their presentation.

The referred publication represents a not-trivial, representative and for the readers evidently valuable survey of the related branches of the fuzzy set theory with special stress

on the new and modern methods and results. It can be recommended to every reader of this type as a useful contribution to the existing literature on the fuzzy set theory.

Milan Mareš

A. MUNTEAN
Fixed Point Principles and Applications to Mathematical Economics

Cluj University Press. Cluj–Napoca 2002.
198 pages.
ISBN 973-610-082-0.

The fixed point theorem and related concepts and results belong to the mathematical tools which are frequently and effectively used in many fields of mathematical economics, game theory, and some other applications of mathematics in optimization modeling. The referred booklet offers an overview of the development and some recent results regarding this topic. A significant part of those results which are presented in this book was derived by the author and his colleagues.

The work is, besides a brief "*Introduction*" simplifying the orientation in the text, divided into four chapters. The first one, headed "*Elements of the Multivalued Operators Analysis*", is of rather auxiliary character. It aims to introduce some terms and results necessary for the presentation of the main concepts dealt in the next chapters. It presents the notions of some functionals on space of subsets of metric space, multivalued operators including Lipschitz operators, and techniques of the fixed point structures.

The second chapter, "*Coincidence and Fixed Point Principles for Multivalued Operators*", is oriented to the topic formulated in its heading. It shows some results regarding the topological fixed point theorems with a stress on the coincidence theorems with economical relevance, existence of fixed points for expansive multivalued operators, common fixed point theorems for sequences of single valued operators, and generalization of the multivalued case of intermediate values theorem.

The third chapter, "*Strict Fixed Point*", which is the largest one of the book, deals with the properties of the mentioned concept. Some existence results, but also commutativity condition for the composition, conditions for the exclusive existence of the strict fixed point, validity of the properties of the usual fixed points for the strict ones, and a few new applications in mathematical economics and game theory are presented in this chapter.

The last chapter headed "*Maximal Elements Structures and Applications to Mathematical Economics*" is focused on the applicability of the maximal elements technique to the problems of mathematical modeling of economic phenomena. Some already known results of the maximal elements theory are summarized here, and the maximal elements methodology is applied to the solution of some problems of the economic equilibrium theory.

The explanation is supported by an impressive list of references including 269 items.

The referred book is written with respect to mathematical correctness and logical formalism. The presentation of particular topics is well organized and lucid.

The book can be recommend to any reader who is interested in recent trends in economical modeling and operations research methods in which the fixed point theory and related topics play a significant role.

Milan Mareš