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ON BARTLETT'S TEST FOR CORRELATION BETWEEN TIME SERIES

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An explicit formula for the correlation coefficient in a two-dimensional AR(1) process is derived. Approximate critical values for the correlation coefficient between two one-dimensional AR(1) processes are tabulated. They are based on Bartlett's approximation and on an asymptotic distribution derived by McGregor. The results are compared with critical values obtained from a simulation study.

1. INTRODUCTION

Let $(X_1, Y_1)', \dots, (X_n, Y_n)'$ be a sample from a bivariate regular normal distribution with independent components. If r' is the sample correlation coefficient then it is known that

$$Er' = 0, \qquad \operatorname{var} r' = \frac{1}{n} + O(n^{-\frac{3}{2}}) \tag{1.1}$$

(see Cramér [4], § 27.8 and § 29.7). If $\{X_t\}$ and $\{Y_t\}$ are independent time series then the variance of the sample correlation coefficient does not obey the formula (1.1). Let $\{\varepsilon_t\}$ and $\{\eta_t\}$ be two independent strict white noises such that $\varepsilon_t \sim N(0, \sigma_1^2)$ and $\eta_t \sim N(0, \sigma_2^2)$. Consider AR(1) processes

$$X_t = \rho_1 X_{t-1} + \varepsilon_t, \qquad Y_t = \rho_2 Y_{t-1} + \eta_t.$$

Their variances are

$$v_1^2 = \operatorname{var} X_t = \frac{\sigma_1^2}{1-\rho_1^2}, \qquad v_2^2 = \operatorname{var} Y_t = \frac{\sigma_2^2}{1-\rho_2^2}.$$

If we define

$$r^* = \frac{\frac{1}{n}\sum_{t=1}^{n} X_t Y_t}{v_1 v_2}$$

then it is easy to prove that under our assumptions $Er^* = 0$ and

$$\operatorname{var} r^* = \frac{1}{n}\frac{1+\rho_1\rho_2}{1-\rho_1\rho_2} - \frac{2\rho_1\rho_2}{n^2}\frac{1-(\rho_1\rho_2)^n}{(1-\rho_1\rho_2)^2} \tag{1.2}$$

(see Anděl [1]). Usually, only the first term on the right-hand side of (1.2) serves as an approximation of the $\operatorname{var} r^*$. This result is due to Bartlett [3]. Of course, in practical applications the variances v_1^2 and v_2^2 are not known. If it is known that $EX_t = 0$ and $EY_t = 0$ then the statistic

$$r = \frac{\sum_{t=1}^{n} X_t Y_t}{\sqrt{\sum_{t=1}^{n} X_t^2 \sum_{t=1}^{n} Y_t^2}}$$

is calculated. However, if $\{X_t\}$ and $\{Y_t\}$ are stationary AR(1) processes with non-vanishing means the usual correlation coefficient

$$r' = \frac{\sum_{t=1}^{n} (X_t - \bar{X})(Y_t - \bar{Y})}{\sqrt{\sum_{t=1}^{n} (X_t - \bar{X})^2 \sum_{t=1}^{n} (Y_t - \bar{Y})^2}} \tag{1.3}$$

is preferred. McGregor [9] showed that

$$\operatorname{var} r \sim V_1 = \frac{1}{n} \frac{1 + \rho_1\rho_2}{1 - \rho_1\rho_2}, \tag{1.4}$$

i.e., that Bartlett's approximation derived for r^* is also valid for r. Let $\alpha = \rho_1\rho_2$ and $N = n + \frac{\alpha(4-3\alpha)}{1-\alpha^2}$. McGregor [9] proved that the density of r is

$$p(r) = f(r)[1 + O(n^{-1})], \qquad -1 < r < 1 \tag{1.5}$$

where the function

$$\begin{aligned} f(r) &= \frac{2^{N-2}\sqrt{1-\alpha}}{B\left(\frac{N-1}{2}, \frac{1}{2}\right)} (1 - r^2)^{\frac{1}{2}(N-3)} \\ &\times \frac{\sqrt{\sqrt{(1+\alpha)^2 - 4\alpha r^2} + 1 + \alpha}}{\left[\sqrt{(1+\alpha)^2 - 4\alpha r^2} + 1 - \alpha\right]^{N-\frac{3}{2}} \sqrt{(1+\alpha)^2 - 4\alpha r^2}} \end{aligned} \tag{1.6}$$

is also a density.

As for the correlation coefficient r' defined in (1.3), McGregor and Bielenstein [10] proved that its density is also given by (1.5) but N must be replaced by $M - 1$ where $M = n + \alpha(6 - 5\alpha)/(1 - \alpha^2)$.

A simple procedure for testing statistical significance of r was suggested by Hannan [7], namely to use r "as an ordinary correlation from $n(1 - \rho_1\rho_2)/(1 + \rho_1\rho_2)$ observations. (Of course, ρ_1 and ρ_2 would need to be estimated from the data and mean corrections would have to be made.)" Hannan notes that this procedure was suggested by Bartlett in 1935. In statistical papers this procedure is called Bartlett's approximation. Let r_1 and r_2 be sample first-lag autocorrelations calculated from $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$, respectively. Nakamura et al [12] published a table of critical values for r given r_1 and r_2 when $n = 30$. Their critical values are based on a simulation study. It is shown that in some cases Bartlett's approximation is not very satisfactory. For example, if $n = 30$ and $\rho_1 = \rho_2 = 0.9$ the five per cent two-sided critical value for r given by Bartlett's approximation is 0.87 but the critical value obtained from simulations is 0.71. Nakamura et al also investigated the approximation

$$\operatorname{var} r^* \sim V_2 = \frac{1}{n}\frac{1+\rho_1\rho_2}{1-\rho_1\rho_2} - \frac{1}{n^2}\frac{2\rho_1\rho_2}{(1-\rho_1\rho_2)^2}$$

and a sample modification of it. Bartlett's approximation based on V_2 was found to be better especially when ρ_1 and ρ_2 have their absolute values near to 1.

It is also possible to calculate critical values for r using the density f introduced in (1.6). McGregor [9] calculated values of $f(r)$ and published some graphs of this density. Although "the corresponding approximate values of the cumulative distribution function $P(r) = \int_{-1}^{r} p(r)\,dr$ were found as a check" they were not published in the paper.

Hannan [6] proposed an exact test for correlation between two autoregressive processes $\{X_t\}$ and $\{Y_t\}$. However, to make the test exact, not all information in the data is used. Haugh [8] introduced a general method for testing the correlation using the residuals. Tests based on comovements between time series are described by Goodman and Grunfeld [5]. Some tests in frequency domain are reviewed in Anděl [1].

In this paper we proceed as follows. In Section 2 we discuss some properties of the theoretical correlation coefficient ρ between the variables X_t and Y_t when $(X_t, Y_t)'$ is a stationary two-dimensional AR(1) process. Critical values based on McGregor's density, critical values based on Bartlett's approximation and critical values obtained from a simulation study are given in Section 3. Some conclusions and recommendations are given in Section 4.

2. CORRELATION COEFFICIENT IN A TWO-DIMENSIONAL AR(1) PROCESS

Consider a stationary two-dimensional AR(1) process $\boldsymbol{Z}_t = (X_t, Y_t)'$ given by $\boldsymbol{Z}_t = \boldsymbol{U}\boldsymbol{Z}_{t-1} + \boldsymbol{\varepsilon}_t$ where $\boldsymbol{\varepsilon}_t$ is a white noise such that $E\boldsymbol{\varepsilon}_t = \boldsymbol{0}$ and $\operatorname{var}\boldsymbol{\varepsilon}_t = \boldsymbol{S}$ where

$$\boldsymbol{U} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}, \qquad \boldsymbol{S} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}.$$

Of course, $s_{12} = s_{21}$. Assume that $\{\boldsymbol{Z}_t\}$ is stationary, i.e., that both the roots of the matrix $\boldsymbol{U}$

$$\lambda_{12} = \frac{1}{2}\left[u_{11} + u_{22} \pm \sqrt{(u_{11} - u_{22})^2 + 4u_{12}u_{21}}\right]$$

are inside the unit circle. Define $u = u_{11}u_{22} - u_{12}u_{21}$. It is known that the variance matrix $\boldsymbol{B} = var\, \boldsymbol{Z}_t$ is given by the formula

$$\begin{aligned}[(1-u_{11}^2)(1-u_{22}^2) &- u_{12}u_{21}(u+u_{11}u_{22}+2)](1-u)\boldsymbol{B} \\ &= (1+u)\boldsymbol{USU}' - u(u_{11}+u_{22})(\boldsymbol{SU}'+\boldsymbol{US}) \\ &+ [(1-u_{11}^2)(1-u_{22}^2) - u_{12}u_{21}(u+u_{11}u_{22}+2) \\ &+ u(u_{11}^2+u_{22}^2+u_{12}u_{21}+u_{11}u_{22}-1)]\boldsymbol{S}\end{aligned} \tag{2.1}$$

(see Anděl [2], p. 242). If we denote

$$\boldsymbol{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

then the correlation coefficient ρ between X_t and Y_t can be written in the form $\rho = b_{12}/\sqrt{b_{11}b_{22}}$. Inserting from (2.1) we get after some computations that $\rho = A/\sqrt{BC}$ where

$$\begin{aligned} A &= s_{12}[(1-u_{11}^2)(1-u_{22}^2) - u_{12}^2u_{21}^2] + s_{11}u_{21}(u_{11}-u_{22}u) + s_{22}u_{12}(u_{22}-u_{11}u), \\ B &= s_{22}[1-u_{11}u_{22}-u_{12}u_{21}-u_{11}^2(1-u)] + 2s_{12}u_{21}(u_{22}-u_{11}u) + s_{11}u_{21}^2(1+u), \\ C &= s_{11}[1-u_{11}u_{22}-u_{12}u_{21}-u_{22}^2(1-u)] + 2s_{12}u_{12}(u_{11}-u_{22}u) + s_{22}u_{12}^2(1+u). \end{aligned}$$

The formula for ρ is quite complicated. It can be simplified in special cases, e. g. when $s_{12} = 0$ or when $u_{12} = u_{21} = 0$. If $s_{12} = 0$ and $u_{12} = u_{21} = 0$ then, of course, $\rho = 0$.

It must be stressed, however, that ρ is not a good measure of dependence between $\{X_t\}$ and $\{Y_t\}$ since there exist two-dimensional AR(1) processes $\boldsymbol{Z}_t = (X_t, Y_t)'$ such that $\rho = 0$ although $\{X_t\}$ and $\{Y_t\}$ are dependent. We introduce some examples.

Example 1. Let $\{\eta_t\}$ be a one-dimensional white noise with $E\eta_t = 0$ and $var\,\eta_t > 0$. If we define $X_t = \eta_t$ and $Y_t = \eta_{t-1}$ then $cov(X_t, Y_t) = 0$ but $cov(X_{t-1}, Y_t) = var\,\eta_{t-1} > 0$. This process can be expressed in the form

$$\boldsymbol{Z}_t = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \boldsymbol{Z}_{t-1} + \begin{pmatrix} \eta_t \\ 0 \end{pmatrix}$$

i.e., $\boldsymbol{Z}_t$ is a two-dimensional stationary AR(1) process.

Example 2. One could object that Example 1 is in some sense degenerated. However, it is possible to construct a "normal" model with correlated components such that $\rho = 0$. Define $\boldsymbol{Z}_t = \boldsymbol{U}\boldsymbol{Z}_{t-1} + \boldsymbol{\varepsilon}_t$ where

$$\boldsymbol{U} = \begin{pmatrix} 0.7 & 0.3 \\ 0.1 & 0.5 \end{pmatrix}, \qquad \boldsymbol{S} = \begin{pmatrix} 1 & -1368/3816 \\ -1368/3816 & 1 \end{pmatrix}.$$

The process $\{\boldsymbol{Z}_t\}$ is stationary since $\lambda_1 = 0.8$, $\lambda_2 = 0.4$ and $\boldsymbol{S}$ is positive definite. Inserting into (2.1) we get

$$\boldsymbol{B} = \begin{pmatrix} 2.20126 & 0 \\ 0 & 1.36268 \end{pmatrix}$$

and thus $\rho = 0$. Since the covariance function $\boldsymbol{R}(s)$ of AR(1) process satisfies

$$\boldsymbol{R}(s) - \boldsymbol{U}\boldsymbol{R}(s-1) = 0 \qquad \text{for } s \geq 0$$

and $\boldsymbol{R}(0) = \boldsymbol{B}$ we get

$$\boldsymbol{R}(1) = \boldsymbol{U}\boldsymbol{B} = \begin{pmatrix} 1.54088 & 0.40880 \\ 0.22013 & 0.68134 \end{pmatrix}.$$

Then

$$\begin{aligned} corr(X_{t+1}, Y_t) &= \frac{0.40880}{\sqrt{2.20126 \times 1.36268}} = 0.23604, \\ corr(X_t, Y_{t+1}) &= \frac{0.22013}{\sqrt{2.20126 \times 1.36268}} = 0.12710. \end{aligned}$$

3. CRITICAL VALUES

In Tables 1 − 9 we summarize selected critical values suitable for the testing of statistical significance of the correlation coefficient between two AR(1) processes. We used following approaches to obtain them:

- simulations,
- Bartlett's approximation,
- numerical integration.

For $n \in \{10, 20, 30, 40, 50, 100, 200, 500\}$ and for each couple (ρ_X, ρ_Y) such that $\rho_X \in \{0.1, 0.4, 0.8\}$ and $\rho_Y \in \{0.2, 0.6, 0.9\}$ we generated 100 000 independent realizations $\{X_1, \ldots, X_n\}$ and $\{Y_1, \ldots, Y_n\}$ where $\{X_t\}$ and $\{Y_t\}$ are independent AR(1) processes with the autocorrelations ρ_X and ρ_Y, respectively. From the each pair $\{X_1, \ldots, X_n\}$ and $\{Y_1, \ldots, Y_n\}$ of realizations the statistics r and r' were calculated. Based on these values we found corresponding 0.95 and 0.99 sample quantiles. Programs for simulations were coded in *Matlab v. 4.2.1c* and run on both Pentium based PC and DEC workstations. In Tables 1 − 9 we denote these sample quantiles R_S if the sample correlation coefficient r was used and R'_S if the usual sample correlation coefficient r' was used.

For the calculation of Bartlett's approximation we applied procedure `Quantile [StudentTDistribution[n],q]` implemented in *Mathematica* v.2.2 for DEC workstations. The results were checked using the function `tinv` implemented in the *Statistical Toolbox v. 2.0* for *Matlab*. In Tables 1 − 9 we denote these critical values by R_B. Principal advantage of mentioned procedures is that one can use them even in the case when the number of degrees of freedom is not an integer.

Numerical integration was calculated using the procedure `NIntegrate` implemented in *Mathematica* v. 2.2 for DEC workstations. In Tables 1 − 9 we denote by R_I the quantiles based on the density f given by (1.6) and by R'_I the quantiles based on the analogical density of r'.

Much more detailed results covering broader range of values of ρ_X and ρ_Y etc. are available from the authors on request.

Table 1. $\rho_X = 0.1, \ \rho_Y = 0.2.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.529	.555	.562	.527	.554	.689	.724	.728	.690	.719
20	.376	.383	.386	.374	.384	.511	.523	.526	.510	.522
30	.306	.311	.312	.306	.311	.420	.427	.431	.422	.429
40	.264	.267	.269	.265	.268	.366	.369	.374	.368	.372
50	.238	.241	.240	.237	.239	.329	.334	.335	.331	.334
100	.169	.169	.169	.168	.169	.236	.237	.237	.236	.237
200	.119	.119	.119	.119	.119	.167	.167	.168	.168	.168
500	.074	.075	.075	.075	.075	.105	.106	.106	.106	.106

Table 2. $\rho_X = 0.1, \ \rho_Y = 0.6.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.538	.564	.587	.539	.563	.704	.729	.755	.701	.726
20	.385	.393	.403	.386	.395	.526	.537	.546	.524	.534
30	.315	.320	.326	.317	.321	.434	.441	.448	.435	.441
40	.272	.276	.281	.275	.278	.380	.385	.389	.380	.384
50	.246	.249	.250	.246	.248	.341	.346	.348	.342	.345
100	.175	.176	.176	.174	.175	.246	.247	.247	.244	.246
200	.124	.124	.124	.123	.124	.173	.173	.175	.174	.174
500	.078	.078	.078	.078	.078	.110	.110	.110	.110	.110

Table 3. $\rho_X = 0.1, \ \rho_Y = 0.9.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.551	.571	.607	.548	.570	.711	.734	.775	.709	.732
20	.396	.401	.416	.395	.403	.535	.540	.563	.534	.544
30	.323	.327	.336	.325	.329	.445	.448	.462	.445	.451
40	.281	.282	.290	.282	.285	.391	.393	.401	.390	.394
50	.251	.255	.258	.253	.255	.347	.352	.359	.351	.354
100	.179	.181	.181	.179	.180	.252	.253	.254	.251	.252
200	.127	.127	.128	.127	.127	.180	.180	.180	.179	.179
500	.081	.080	.081	.080	.081	.113	.113	.114	.114	.114

Table 4. $\rho_X = 0.4,\ \rho_Y = 0.2.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.547	.570	.600	.545	.567	.708	.734	.768	.706	.730
20	.393	.401	.412	.392	.400	.530	.542	.557	.531	.541
30	.332	.328	.333	.322	.327	.441	.448	.457	.442	.448
40	.278	.281	.287	.280	.283	.387	.390	.397	.387	.391
50	.251	.253	.255	.250	.253	.346	.349	.355	.348	.351
100	.178	.179	.179	.178	.178	.248	.249	.252	.249	.250
200	.126	.126	.126	.126	.126	.178	.178	.178	.177	.178
500	.079	.080	.080	.080	.080	.113	.112	.113	.112	.113

Table 5. $\rho_X = 0.4,\ \rho_Y = 0.6.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.603	.613	.721	.597	.613	.760	.772	.876	.753	.768
20	.445	.449	.492	.445	.451	.596	.601	.652	.590	.597
30	.369	.373	.396	.370	.373	.499	.504	.537	.500	.505
40	.321	.324	.340	.323	.326	.442	.445	.467	.442	.445
50	.291	.292	.303	.291	.292	.398	.400	.418	.400	.402
100	.209	.209	.212	.208	.208	.291	.291	.297	.290	.291
200	.148	.148	.149	.148	.148	.207	.206	.210	.207	.208
500	.094	.094	.094	.094	.094	.133	.132	.133	.132	.132

Table 6. $\rho_X = 0.4,\ \rho_Y = 0.9.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.652	.638	.831	.643	.654	.800	.793	.949	.792	.801
20	.494	.485	.568	.491	.496	.644	.634	.735	.640	.646
30	.412	.409	.455	.413	.416	.550	.546	.610	.551	.554
40	.363	.358	.391	.362	.364	.492	.488	.531	.490	.493
50	.325	.325	.347	.327	.328	.442	.444	.476	.446	.448
100	.236	.236	.243	.235	.236	.328	.326	.338	.327	.327
200	.168	.168	.171	.168	.168	.237	.236	.239	.235	.236
500	.107	.107	.107	.107	.107	.151	.150	.152	.151	.151

Table 7. $\rho_X = 0.8,\ \rho_Y = 0.2.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.572	.589	.657	0.570	.589	.736	.753	.822	.729	.747
20	.419	.423	.449	0.417	.425	.562	.572	.602	.558	.568
30	.345	.349	.362	0.345	.349	.474	.474	.495	.470	.475
40	.299	.301	.312	0.300	.303	.411	.414	.430	.413	.417
50	.270	.272	.278	0.270	.272	.373	.377	.385	.373	.375
100	.192	.192	.195	0.192	.193	.269	.270	.273	.268	.269
200	.137	.137	.137	0.136	.136	.192	.192	.193	.192	.192
500	.086	.086	.087	0.086	.086	.122	.122	.122	.122	.122

Table 8. $\rho_X = 0.8,\ \rho_Y = 0.6.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.701	.674	.948	.695	.702	.837	.823	.994	.833	.839
20	.548	.536	.668	.546	.550	.697	.687	.832	.696	.700
30	.465	.459	.534	.464	.466	.608	.601	.699	.609	.611
40	.410	.406	.457	.410	.412	.546	.543	.611	.547	.549
50	.373	.371	.405	.372	.373	.500	.501	.549	.501	.502
100	.269	.269	.282	.270	.270	.372	.372	.391	.372	.373
200	.193	.194	.198	.194	.194	.271	.272	.277	.270	.270
500	.123	.124	.125	.123	.123	.174	.174	.175	.174	.174

Table 9. $\rho_X = 0.8,\ \rho_Y = 0.9.$

	$\alpha = 0.95$					$\alpha = 0.99$				
n	R_S	R'_S	R_B	R_I	R'_I	R_S	R'_S	R_B	R_I	R'_I
10	.825	.736	–	.820	.822	.919	.865	–	.918	.919
20	.695	.636	.971	.696	.698	.829	.781	.998	.830	.831
30	.613	.557	.815	.614	.615	.758	.721	.940	.759	.760
40	.554	.532	.697	.555	.556	.703	.678	.856	.703	.703
50	.507	.494	.616	.510	.511	.649	.636	.783	.657	.657
100	.382	.378	.422	.382	.382	.514	.509	.569	.512	.513
200	.279	.278	.293	.279	.279	.384	.383	.406	.383	.383
500	.180	.180	.184	.180	.180	.252	.252	.257	.251	.251

4. CONCLUSIONS

The difference between R_S and R'_S typically grows either if ρ_X and/or ρ_Y increases or if n decreases. However, this difference is practically negligible for $n \geq 50$ irrespective of the values of ρ_X and/or ρ_Y. For smaller values of n is R_S usually larger than R'_S.

On the contrary, the difference between R_I and R'_I increases both if n decreases and if ρ_X and/or ρ_Y decreases. However, the difference in all considered situations is practically negligible provided $n \geq 50$.

Difference between R_S and R_I is very small already for $n = 10$ and practically negligible for $n \geq 20$. The situation is almost the same in the case of R'_S and R'_I and small values of ρ_X and ρ_Y. On the other hand, the situation is worse in the case of R'_S and R'_I and larger values of ρ_X and ρ_Y. The values of R'_I are typically greater than those of R'_S and the difference start to be negligible only for $n \geq 100$.

As for Bartlett's approximation, it gives in all cases more conservative values (as expected). While this approximation seems to give very well acceptable results for $n \geq 50$ and at least one of ρ's small, the discrepancy is quite big even for $n = 200$ and both ρ_X and ρ_Y large.

The values R_I are closer to R_S than the values R_B. Similarly, R'_I are closer to R'_S than the values R_B. This leads to the recommendation that the approximations R_I and R'_I should be preferred to the approximation R_B.

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