

Applications of Mathematics

Book Reviews

Applications of Mathematics, Vol. 52 (2007), No. 1, 95–96

Persistent URL: <http://dml.cz/dmlcz/134664>

Terms of use:

© Institute of Mathematics AS CR, 2007

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://dml.cz>

BOOK REVIEWS

R. Zaharopol: INVARIANT PROBABILITIES OF MARKOV-FELLER OPERATORS AND THEIR SUPPORTS. *Frontiers in Mathematics*. Birkhäuser-Verlag, Basel, 2005. ISBN 3-7643-7134-X, xiii+108 pages, price EUR 29,96.

To explain what is Radu Zaharopol's book under review about, we need to introduce several notions. Let X be a separable locally compact metric space, \mathcal{B} the Borel σ -algebra of X , and P a Markovian kernel (a transition probability) on (X, \mathcal{B}) . In a standard way, P defines an operator S on the space of all bounded real Borel functions on X . Its dual operator S^* maps the space $M(X)$ of all finite signed Borel measures on X into itself; let us denote by T the restriction of S^* to $M(X)$. The kernel P is called Feller if the image of the space $C_b(X)$ of all bounded continuous functions under S is contained in $C_b(X)$; only Feller transition probabilities are considered in the book. (In fact, the author starts with a pair of operators (S, T) obeying a duality relation $\langle Sf, \mu \rangle = \langle f, T\mu \rangle$ for any $f \in C_b(X)$ and $\mu \in M(X)$, and he finds a corresponding Feller Markovian kernel by improving on M. Rosenblatt's result known in compact state spaces.) A probability measure μ is called invariant for the kernel P (or for T , or for the pair (S, T)) if $T\mu = \mu$. The author's main goal is to describe explicitly supports of invariant measures for Feller Markov kernels. We quote a particular case of one of the main theorems of the book to give an idea how the results obtained look like. Denote by D the dissipative part of X , that is, the set of all $x \in X$ such that the Cesàro means $n^{-1} \sum_{k=0}^{n-1} S^k f(x)$ converge to zero whenever f is a continuous function vanishing at infinity, and by Γ_0 the complement of D . Let $\mathcal{O}(x)$ be the orbit of a point x , i.e. the union over $n \geq 0$ of supports of the measures $T^n \delta_x$, δ_x being the Dirac measure sitting at x , and let $\text{cl } \mathcal{O}(x)$ be the closure of the orbit. Suppose that μ is the unique invariant measure for the operator T , then the support of μ is equal to $\gamma_0 = \bigcap \{\text{cl } \mathcal{O}(x); x \in \Gamma_0\}$. The whole Chapter 3 of the book is devoted to results of this kind, while converse theorems are presented in the next chapter: for example, if there exists an invariant measure for the Feller Markov operator T and the set γ_0 defined above is non-void, then it is shown that the invariant measure is unique, provided that $\{S^n f; n \geq 0\}$ is an equicontinuous sequence for every continuous function f vanishing at infinity.

The first two chapters are preparatory. In the former, Feller Markov operators are introduced and basic results concerning their ergodic theory are recalled (most of them with proofs). In Chapter 2, generalizations of the Krylov-Bogolyubov decomposition of invariant measures in terms of regular points are treated; the techniques introduced here play an important role in the rest of the book.

To summarize, Zaharopol's book is a research monograph, devoted to results obtained by the author very recently. The rather advanced character of these results notwithstanding, the author tried to make the book fairly self-contained and to avoid non-elementary tools. All proofs are detailed, many illuminating examples are included, and familiarity with only basics of measure theory, general topology of metric spaces and functional analysis is sufficient to follow the exposition. Therefore, potential readers of this monograph are not only those looking for information about supports of invariant measures, but everybody interested in Markov operators who may find the author's approach inspiring in many respects.

Jan Seidler

A. Mallios: MODERN DIFFERENTIAL GEOMETRY IN GAUGE THEORY: MAXWELL FIELDS, Vol. 1. Birkhäuser-Verlag, Basel-Boston-Berlin, 2006. ISBN 0-8176-4378, 293 pages, price EUR 98,-.

The terminology “Modern Differential Geometry” in the title of the book under review is another name for the Abstract Differential Geometry developed by A. Mallios. The essential idea of this abstract theory consists in replacing the classical theory of differential geometry which deals with vector bundles over smooth manifolds, their connections, curvature and related cohomology theories etc. by a theory based on differential triads $(\mathcal{A}, \partial, \Omega)$ as well as \mathcal{A} -modules \mathcal{E} . Here \mathcal{A} is a \mathbb{C} -algebra sheaf over a topological space X , Ω is an \mathcal{A} -module, and $\partial: \mathcal{A} \rightarrow \Omega$ plays the role of the differentiation. A basic example of \mathcal{A} is the algebra of \mathbb{C} -valued smooth functions over X , and a basic example of an \mathcal{A} -module \mathcal{E} is the space of sections of a complex vector bundle over X . A. Mallios calls a pair (X, \mathcal{A}) a \mathbb{C} -algebraized space. (This notion has been introduced under the name of a ringed space by Grothendieck.) A. Mallios developed the theory of connections, etc. based on these concepts.

This volume consists of five chapters. In the first chapter A. Mallios introduces main notions and results of his Abstract Differential Geometry. The main reference for this chapter is the author’s early book “Geometry of Vector Sheafs”. In other chapters, “Elementary Particle: Sheaf-Theoretical Classification, by Spin Structure, According to Selesnick’s Correspondence Principle”, “Electromagnetism”, “Cohomological Classifications of Maxwell and Hermitian Maxwell Fields”, “Geometric Prequantization”, A. Mallios translates some notions and results in physics, e.g. spin-numbers of elementary particles, or Maxwell fields (electromagnetic field), in terms of his theory.

The author promises that an application of his theory which contains a lot of singularities shall be given in the second volume of this book.

It is not easy to read this research monograph. The text is too wordy, and the author refers too often to other sources without quoting those results/definitions explicitly.

Hong Van Le