Book Reviews

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BOOK REVIEWS

P. G. LeFloch: HYPERBOLIC SYSTEMS OF CONSERVATION LAWS. THE THEORY OF CLASSICAL AND NONCLASSICAL SHOCK WAVES. Lectures in Mathematics ETH Zürich. Birkhäuser-Verlag, Basel, 2002. ISBN 3-7643-6687-7, 304 pages, softcover, price EUR 27,10.

This set of lecture notes was written for a Nachdiplom-Vorlesungen course given at the Forschungsinstitut für Mathematik, ETH Zürich during the Fall Semester 2000. One of the main objective in this course is to provide a self-contained presentation of the well-posedness theory for nonlinear hyperbolic systems of first order partial differential equations in divergence form, also called hyperbolic systems of conservation laws. A complete account of the existence, uniqueness, and continuous dependence theory for the Cauchy problem associated with stricly hyperbolic systems with genuinely nonlinear chararacteristic fields is given. The developing theory of nonclassical shock waves for strictly hyperbolic systems whose characteristic fields are not genuinely nonlinear is also discussed.

The book is intended to contribute and establish a unified framework encompassing both what is called here classical and nonclassical entropy solutions. No familiarity with hyperbolic conservation laws is a priori assumed in this course.

The well-posedness theory for classical entropy solutions of genuinely nonlinear systems is entirely covered by Chapter I (Sections 1 and 2), Chapter II (Sections 1 and 2), Chapter VI (Sections 1 and 2), and Chapter X. The other sections contain more advanced material and provide an introduction to the theory of nonclassical shock waves.

After the already mentioned Chapter I containing also various singular perturbation limits (as diffusion and dispersive limits), Part 1, entitled Scalar Conservation Laws, starts with the analysis of the Riemann problem (Chapter II) and continues with Diffusive-dispersive travelling waves (Chapter III), Existence theory for the Cauchy problem (Chapter IV), and Continuous dependence of solutions (Chapter V).

Part 2 is devoted to Systems of Conservation Laws. It consists of Chapter VI, the Riemann problem, Chapter VII, Classical entropy solutions of the Cauchy problem, Chapter VIII, Nonclassical entropy solutions of the Cauchy problem, Chapter IX, Continuous dependence of solutions, and finally Chapter X, Uniqueness of entropy solutions. An appendix entitled Functions with bounded variation is added. The book is concluded with Bibliographical notes and 14 pages of References.

The book can serve as an introduction to the present stage of the theory of nonlinear hyperbolic equations. It will be useful for graduates who would like to specialize in this field as well as for specialists. Parts of the book will certainly be useful in arranging undergraduate courses in PDE's.

Ivan Straškraba

M. M. Rao (Ed.): REAL AND STOCHASTIC ANALYSIS. NEW PERSPECTIVES. Trends in Mathematics. Birkhäuser-Verlag, Boston, 2004. ISBN 0-8176-4332-X, ix + 405 pages, price EUR 128,-.

The book under review comprises six papers tied together by a joint focus on results testifying a deep interplay between functional and stochastic analysis. Although most of the papers contain new results and/or proofs, their authors tried to make them reasonably self-contained and provide a wide background to the problems studied, so the papers should be accessible to advanced graduate students and serve as an introduction to some topics which have been paid a considerable attention recently.

Let us list the articles included in the book: D. R. Bell contributed with a paper Stochastic differential equations and hypoelliptic operators, devoted to existence of smooth densities for solutions of stochastic delay equations of the form $\mathrm{d}x(t) = g(x(t-r))\,\mathrm{d}w + b(t, x(t))\,\mathrm{d}t$. B. Driver's article Curved Wiener space analysis aims at developing analysis on a path space of a Riemannian manifold endowed with the Wiener measure (quasi-invariance, Poincaré and logarithmic Sobolev inequalities,...), however, the reader is presupposed to be acquainted only with standard stochastic calculus, so necessary facts from the Riemannian geometry and stochastic analysis on manifolds are carefully explained as well. Hence this article (by far the longest in the book, almost a hundred and fifty pages) may be a good choice for anybody who wants to learn stochastic calculus on manifolds *ab initio*. S. Gudder's paper Noncommutative probability and applications surveys various versions of noncommutative probability theory, with particular emphasis laid upon a discussion of observables, statistical maps and sequential effect algebras. B. Jefferies in his paper Advances and applications of the Feynman integral surveys in a concise way some recent contributions to the Feynman integral theory as treated in his book "Evolution processes and the Feynman-Kac formula" and in the monograph "The Feynman integral and Feynman's operational calculus" by G. W. Johnson and M. L. Lapidus. Hiroshi Kunita's paper is devoted to an in-depth study of properties of stochastic flows associated with stochastic differential equations driven by Lévy processes. Finally, the editor in his paper considers random walks on locally compact nonabelian groups and relates spectral properties of convolution operators in various function spaces to amenability of the group.

M. M. Rao has already edited two books of the same name (published in 1986 and 1997 by John Wiley & Sons and CRC Press, respectively), both being devoted to interactions between probability theory and functional analysis as well. Some of the papers which appeared therein have become rather standard references and one may expect that the present book will have the same fate.

Bohdan Maslowski