Václav Tryhuk

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FUNCTIONAL EQUATIONS AND A THEORETICAL MODEL OF DLTS

VÁCLAV TRYHUK, Brno

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Summary. The paper deals with a theoretical model of the Crowel-Alipanahi correlator. The model describes a new possible effect of the DLTS spectra-exponential and nonexponential transient capacitance, normal or anomalous spectra.

Keywords: deep level transient spectroscopy, nonexponential transient capacitance, functional equations in several variables.

AMS classification: 39B99

INTRODUCTION

We investigate the deep trap in avalanche photodiodes measured by the deep level transient spectroscopy (DLTS). This method is based on the analysis of transient capacitance induced by thermal emission of carriers from traps within the depletion region of the diode structure.

In conventional DLTS measurements, the analysis of the result is based upon the assumptions of an exponential transient capacitance and of negligible errors due to the system of data analysis technique (see [3], [7]).

In this paper we generalize the theoretical model

$$S_0(x,\tau) = (1/x)\left(\int_0^{x/2} C(t/\tau)\,\mathrm{d}t - \int_{x/2}^{x} C(t\tau)\,\mathrm{d}t\right) \tag{1}$$

of the Crowel-Alipanahi correlator (see [4]–[6]) to

$$S(x,\tau) = (A/x)\left(\int_{d(x)}^{x/2+d(x)} C(t/\tau)\,\mathrm{d}t - \int_{x/2+d(x)}^{x+d(x)} C(t/\tau)\,\mathrm{d}t\right) \tag{2}$$

and then to

$$\tilde{S}(x,\tau) = x^p S(x,\tau), \tag{3}$$

where $S(x,\tau)$ is the function (2) and p is a real number. Without loss of generality we can suppose that A is positive real constant.

The model (3) describes a new possible effect of the DLTS spectra given by the process transformation of the capacitance relaxation into DLTS spectra, and also by the physical properties of the samples, especially the high-resistive ones.

Using a theory of functional equations (see [1], [2]) we present a description of the anomalous DLTS spectra observed in experiments (see [7]).

We also discuss a nonexponential transient capacitance and a possibility to use a new theoretical model a modelling of DLTS spectra.

We remark that the results of mathematical modelling are in good agreement with experiments.

Notation, basic definitions

We denote by $\mathbb{N}$ the set of natural numbers, by $\mathbb{R}$ ($\mathbb{R}_+$) the set of real (positive real) numbers, $\mathbb{R}_+^0 = \mathbb{R}_+ \cup \{0\}$. We have the symbol t for time, T for temperature, $\tau = \tau(T)$ for a time constant, C for a transient capacitance, x for a pulse period and $S = S(x,\tau)$ for the DLTS function.

We use the following forms of the function $S(x,\tau)$:

$$S(x,\tau) = \frac{A\tau}{x}\left(\int_{d(x)/\tau}^{(x/2+d(x))/\tau} C(s)\,\mathrm{d}s - \int_{(x/2+d(x))/\tau}^{(x+d(x))/\tau} C(s)\,\mathrm{d}s\right), \tag{4}$$

where $\tau s = t$;

$$S(x,\tau) = \frac{A}{x}\left(\int_0^{x/2} C((s-d(x))/\tau)\,\mathrm{d}s - \int_{x/2}^{x} C((s-d(x))/\tau)\,\mathrm{d}s\right), \tag{5}$$

where $s = t + d(x)$;

$$S(x,\tau) = A\left(\int_0^{1/2} C((xs+d(x))/\tau)\,\mathrm{d}s - \int_{1/2}^{1} C((xs+d(x))/\tau)\,\mathrm{d}s\right), \tag{6}$$

where $xs + d(x) = t, x \in \mathbb{R}_+$ being fixed.

In the paper we will use the following assumptions:

(A1) $d(x)$ is a continuous function satisfying $0 \leqslant d(x) < x$ for every $x \in \mathbb{R}_+$ fixed;

(A2) $C(s)$ is a positive decreasing bounded and continuous function on $\mathbb{R}_+$.

If $d(x) \equiv 0$ and $A = 1$ then the expressions (2), (4)–(6) are equivalent to (1). According to (A2) there exists a finite $\lim C(u) = L$ for $u \to \infty$, $L \in \mathbb{R}_+^0$.

Definition 1. Let p be an arbitrary fixed real number. We say that $g(x,y)$ is a homogeneous function of the order p if the Euler's functional equation

$$g(\alpha x, \alpha y) = \alpha^p g(x,y) \tag{7}$$

is valid for every $\alpha, x, y \in \mathbb{R}_+$.

A general solution $g\colon \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ of (7) is of the form

$$g(x,y) = x^p h(y/x) \tag{8}$$

for an arbitrary function $h\colon \mathbb{R}_+ \to \mathbb{R}$ (see [1], [2]).

Definition 2. For any $x \in \mathbb{R}_+$ fixed, let the DLTS function (2) reach its maximum S_m at a point τ_m, i.e. $S_m = S(x,\tau_m)$. We say that the DLTS function $S(x,\tau)$ forms a normal spectra if for every $\alpha \in \mathbb{R}_+$ the function $S(\alpha x, \alpha\tau)$ reaches its maximum at the point $\alpha\tau_m$ and $S(\alpha x, \alpha\tau_m) = S_m$. Otherwise we call the DLTS spectra anomalous.

The exponential case

Lemma 1. *If $C(t) = \exp\{-t\}$, $t \in \mathbb{R}_+$, then*

$$S(x,\tau) = AC(-d(x)/\tau)S_0(x,\tau) \tag{9}$$

for all $x, \tau \in \mathbb{R}_+$. Conversely, if $F\colon \mathbb{R}^0_+ \to \mathbb{R}_+$ and

$$S(x,\tau) = F(d(x)/\tau)S_0(x,\tau) \tag{10}$$

then there exist solutions $C(t) = \exp\{-kt\}$, $k \in \mathbb{R}_+$ such that $F(d(x)/\tau) = AC(-d(x)/\tau)$. Hence (9) is satisfied.

Proof. Substituting $C(t) = \exp\{-t\}$ into the expression (5) we get $C((s - d(x))/\tau) = C(s/\tau)C(-d(x)/\tau)$ and

$$\begin{aligned} S(x,\tau) &= AC(-d(x)/\tau)(1/x)\Big(\int_0^{x/2} C(s/\tau)\,\mathrm{d}s - \int_{x/2}^{x} C(s/\tau)\,\mathrm{d}s\Big) \\ &= AC(-d(x)/\tau)S_0(x,\tau) \end{aligned}$$

for $x, \tau \in \mathbb{R}_+$.

Suppose that $S(x,\tau) = F(d(x)/\tau)S_0(x,\tau)$ for $x,\tau \in \mathbb{R}_+$. According (1), (2) and using the expression (5) we obtain

$$A\left(\int_0^{x/2} C\big((s-d(x))/\tau\big)\,\mathrm{d}s - \int_{x/2}^{x} C\big((s-d(x))/\tau\big)\,\mathrm{d}s\right)$$
$$= F(d(x)/\tau)\left(\int_0^{x/2} C(s/\tau)\,\mathrm{d}s - \int_{x/2}^{x} C(s/\tau)\,\mathrm{d}s\right).$$

From linearity of integrals and in accordance with the assumption (A2) we obtain for the decreasing function C the condition $C(s/\tau - d(x)/\tau) = F(d(x)/\tau)C(s/\tau)/A$, i.e.

$$C(u-v) = \alpha(v)C(u), \tag{11}$$

where $u = s/\tau$, $v = d(x)/\tau$; $u, v \in \mathbb{R}_+$ and

$$\alpha(v) = F(v)/A > 1. \tag{12}$$

The method of invariants then yields a mapping $u \to u - v$, $C \to \alpha C$ thus $u/v \to u/v - 1$, $\ln C/\ln\alpha \to \ln C/\ln\alpha + 1$. The expression $\ln C/\ln\alpha + u/v$ is an invariant of the given mapping and we have

$$C(u) = \exp\left\{-\frac{\ln\alpha(v)}{v}u\right\}.$$

The function C is a solution of the functional equation (11). But $C(u)$ is independent of v, hence $\ln\alpha(v)/v = k > 0$ and $\alpha(v) = \exp\{kv\}$, $k \in \mathbb{R}_+$ being constant.

We conclude that

$$C(u) = \exp\{-ku\}, \qquad u = s/\tau$$

and

$$F(v) = A\alpha(v) = A\exp\{kv\} = AC(-v), \qquad v = d(x)/\tau$$

according to (12). □

Remark 1. It follows from Lemma 1 that if the function C is exponential, then the theoretical models (1) and (2) are equivalent in the following sense: We can integrate over the interval $\langle 0, x\rangle$ instead of $\langle d(x), x + d(x)\rangle$ for every $x \in \mathbb{R}_+$ fixed.

Remark 2. In the same manner as the function $C(t) = \exp\{-t\}$, any function $C_\tau(t) = C(t/\tau)$ for $\tau \in \mathbb{R}_+$, can be considered. This follows from the fact that C satisfies Cauchy's functional equation $C(u+v) = C(u)C(v)$ on $\mathbb{R}_+$. A continuous decreasing solution of Cauchy's functional equation is of the form $C(u) = \exp\{-ku\}$, $k > 0$, and we can always put $k = 1/\tau$, $\tau > 0$ (see [1]).

R e m a r k 3. In the exponential case we obtain from (2) that

$$S(x,\tau) = A(\tau/x)\exp\{-d(x)/\tau\}\left(1-\exp\left\{-\frac{x}{2\tau}\right\}\right)^2 \tag{13}$$

and

$$S_0(x,\tau) = (\tau/x)\left(1-\exp\left\{-\frac{x}{2\tau}\right\}\right)^2 \tag{14}$$

for $d(x) \equiv 0$, $A = 1$.

The function $h_0(x/\tau) := S_0(x,\tau)$ is homogeneous (of the order $p = 0$) since

$$S_0(\alpha x, \alpha\tau) = h_0\left(\frac{\alpha x}{\alpha\tau}\right) = h_0\left(\frac{x}{\tau}\right) = S_0(x,\tau)$$

for every $\alpha, x, \tau \in \mathbb{R}_+$.

It is clear that (13) is a homogeneous function if and only if $d(x) = kx$ for a fixed $k \in \langle 0,1)$ since $d(x)$ is a continuous solution of the functional equation $d(\alpha x) = \alpha d(x)$ on $\mathbb{R}_+$ and $d(x)$ satisfies the assumption (A1). In this case we have

$$S(x,\tau) = A\exp\{-kx/\tau\}S_0(x,\tau). \tag{15}$$

Let us denote $u = x/\tau$ and consider a function

$$h(u) = A\exp\{-ku\}(1-\exp\{-u/2\})^2/u \tag{16}$$

on $\mathbb{R}_+$. Then

$$h(u) > 0, \quad \lim h(u) = 0 \quad \text{for } u \to 0+,\ u \to \infty \tag{17}$$

and there exists a local maximum of $h(u)$ at some point u_m. This point u_m is a unique solution of the equation $h'(u) = 0$, which is equivalent to

$$(u + 1 + ku)\exp\{-u/2\} = 1 + ku, \qquad u \in \mathbb{R}_+. \tag{18}$$

Thus if $x \in \mathbb{R}_+$ is fixed then $S(x,\tau)$ reaches its maximum at the point $\tau_m = x/u_m$ and we see that the function $S(x,\tau)$ forms normal spectra.

Lemma 2. *Let C be exponential and let $x \in \mathbb{R}_+$, $\alpha \in \mathbb{R}_+ - \{1\}$ be fixed. Then the value*

$$S(\alpha x, \alpha\tau_m) = \lambda S(x,\tau_m), \qquad \lambda \in \mathbb{R}_+ \tag{19}$$

is the maximum of the function $S(\alpha x, \alpha \tau)$ whenever $\tau_m = k_1 x$, $k_1 = 1/u_m$ and

(20) $$d(x) = \left(k - k_1 \frac{\ln x}{\ln \alpha} \ln \lambda\right) x$$

on $\mathbb{R}_+$, $k \in \mathbb{R}_+$ is arbitrary and u_m is a solution of the equation (18).

P r o o f. We consider the normal and anomalous spectra of the function (2) for $\lambda \equiv 1$ and $\lambda \in \mathbb{R}_+ - \{1\}$, respectively. Since $C(u)$ is an exponential function, we obtain from (13) and (14) that

$$S(x, \tau) = A \exp\{-d(x)/\tau\} S_0(x, \tau).$$

Using (13) we then have a condition equivalent to (19) in the form

(21) $$d(\alpha x) = \alpha d(x) - \alpha \tau_m \ln \lambda$$

for every $\alpha, x \in \mathbb{R}_+$, $\alpha \neq 1$ and the given $\lambda \in \mathbb{R}_+$; $\tau_m = 1/u_m$ according to Remark 3.

Hence we need to solve a functional equation

(22) $$d(\alpha x) = \alpha(d(x) - k_2 x)$$

for α, $x \in \mathbb{R}_+$, $\alpha \neq 1$, $\lambda \in \mathbb{R}_+$; $k_2 = \ln \lambda / u_m$.

In the case that $\lambda = 1$ we obtain $k_2 = 0$ and the equation $d(\alpha x) = \alpha d(x)$ has a continuous solution of the form $d(x) = kx$, $k \in \mathbb{R}_+$.

In the latter case $\lambda \in \mathbb{R}_+ - \{1\}$ we can substitute values $x = 1, \alpha, \alpha^2, \dots$ into (22). Thus

$$d(\alpha) = \alpha(d(1) - k_2),$$

$$d(\alpha^2) = \alpha(d(\alpha) - k_2 \alpha) = \alpha^2(d(1) - 2k_2),$$

and by the assumption

(23) $$d(\alpha^n) = \alpha^n(d(1) - nk_2)$$

we obtain

$$d(\alpha^{n+1}) = \alpha(d(\alpha^n) - k_2 \alpha^n) = \alpha^{n+1}(d(1) - (n+1)k_2).$$

Hence (23) is satisfied for every $n \in \mathbb{N}$, $\alpha \in \mathbb{R}_+ - \{1\}$.

If we denote $x = \alpha^n$ then $n = \ln x / \ln \alpha$ and

(24) $$d(x) = \left(d(1) - \frac{\ln x}{\ln \alpha} k_2\right) x.$$

If we suppose that the function $d(x)$ is defined by (24) for all $x \in \mathbb{R}_+$, then we have

$$d(\alpha x) = \alpha\Big(d(1) - \frac{\ln x}{\ln \alpha}k_2 - k_2\Big) = \alpha d(x) - k_2\alpha x$$

for arbitrary $\alpha \in \mathbb{R}_+ - \{1\}$ and (24) is a continuous solution of the equation (22) on $\mathbb{R}_+$. Thus we get (20) for $k = d(1)$, $k_2 = k_1 \ln \lambda$, $k_1 = 1/u_m$.

Using (13) and (20) we prove that (19) is fulfilled and the assertion of Lemma 2 is proved. □

Remark 4. In the case $\lambda = 1$ we have $d(x) = kx$. Hence (13) and also (2) is a homogeneous function described in Remark 3. The condition $o \leqslant d(x) < x$ is then equivalent to $0 \leqslant k < 1$.

The situation is different on the case $\lambda \neq 1$. Here $0 \leqslant d(x) < x$ and (2) imply that $\ln\lambda/\ln\alpha > -u_m/\ln x$ for any fixed $\lambda, \alpha \in \mathbb{R}_+ - \{1\}$ and $x \neq 1$. In the limit case $x \to \infty$ we obtain $\ln\lambda\ln\alpha \geqslant 0$ and $\alpha < 1$ implies $\lambda < 1$ which does not agree with experiments. Hence the theoretical model (2), (24) is false for anomalous DLTS spectra. Thus we will always suppose that

$$d(x) = kx, \qquad x \in \langle 0, 1) \tag{25}$$

The general case

Lemma 3. *Let $d(x) = kx$ for some $k \in \langle 0, 1)$. The function (2) is homogeneous and*

$$S(x,\tau) = h(x/\tau) = h(u) = \frac{A}{u}\Big(\int_{ku}^{(k+1/2)u} C(s)\,\mathrm{d}s - \int_{(k+1/2)u}^{(k+1)u} C(s)\,\mathrm{d}s\Big) \tag{26}$$

for $u = x/\tau$ and $x, \tau \in \mathbb{R}_+$. Moreover, $h(u)$ is nonnegative and $\lim h(u) = 0$ for both $u \to 0_+$ and $u \to \infty$.

Proof. The relation $h(u) \geqslant 0$ follows from the assumption (A2). It is clear that $S(x,\tau)$ is a homogeneous function, i.e. $S(\alpha x, \alpha\tau) = S(x,\tau)$ for $\alpha, x, \tau \in \mathbb{R}_+$. If $u \to 0_+$ then we have

$$\begin{aligned}\lim h(u) &= A \cdot \lim \frac{1}{u}\Big(\int_{ku}^{(k+1/2)u} C(s)\,\mathrm{d}s - \int_{(k+1/2)u}^{(k+1)u} C(s)\,\mathrm{d}s\Big)\\ &= A \lim[(1+2k)C((k+\tfrac{1}{2})u) - kC(ku) - (1+k)C((1+k)u)] = 0,\end{aligned}$$

using l'Hospital's rule.

We have

$$C(s) \leqslant C(ku) \quad \text{on} \quad \langle ku, (k+\tfrac{1}{2})u\rangle ,$$
$$-C(s) \leqslant -C((1+k)u) \quad \text{on} \quad \langle (k+\tfrac{1}{2})u, (k+1)u\rangle$$

for every fixed $u \in \mathbb{R}_+$. Hence if $u \to \infty$ then

$$0 \leqslant h(u) = \frac{A}{u}(\tfrac{1}{2}C(ku)u - \tfrac{1}{2}C((1+k)u)u) = \frac{A}{2}(C(ku) - C((1+k)u)) \to \frac{A}{2}(L-L)$$

and $\lim_{u\to\infty} h(u) = 0$. □

Remark 5. It is a basic fact that for any homogeneous function S there is a function $h\colon \mathbb{R}_+ \to \mathbb{R}$ such that $S(x,\tau) = h(x/\tau)$. Let us consider the expression (26). Due to Lemma 3, it is possible to investigate functions $C(u)$ other than $\exp\{-u\}$. For example, the function

$$C(u) = 1/(u+a)^r \tag{27}$$

for $a, r \in \mathbb{R}_+$ or the function $C(u) = \operatorname{arccotan} u$ can be considered.

The function $h(u)$ corresponding to (27) will be of the form

$$h(u) = \frac{A}{u} \ln \Big(1 + \frac{u^2}{4(ku+a)((k+1)u+a)}\Big) \quad \text{for} \quad r = 1, \tag{28}$$
$$h(u) = \frac{A}{u}\,\frac{((1+k)u+a)^{1-r} - 2((k+\frac{1}{2})u+a)^{1-r} + (ku+a)^{1-r}}{r-1}$$
$$\text{for} \quad r \in \mathbb{R}_+ - \{1\},$$

so that $h(u)$ reaches its maximum value at the same point u_m as in the exponential case. For example, if $k = 1/25$ then $u_m \doteq 2.226$ and the related constants are

r	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$\sqrt{2}$	$\sqrt{3}$	2	3	4	5
a	0.09	0.18	0.4	0.86	1.27	1.55	1.8	2.8	3.8	4.8

where the values of a were computed from the equation $h'(u) = 0$ for $u_m = 2.226$ by means of (28).

Lemma 4. *Consider a function* (2) *in the form* (6) *and* $d(x) = kx$, $k \in \langle 0,1)$, *i.e.*

$$S(x,\tau) = A\Big(\int_0^{1/2} C((s+k)x/\tau)\,\mathrm{d}s - \int_{1/2}^1 C((s+k)x/\tau)\,\mathrm{d}s\Big), \tag{29}$$

x, $\tau \in \mathbb{R}_+$. *If the function C satisfies the relation*

$$C\Big((s+k)\frac{\alpha x}{\beta\tau}\Big) = \gamma C\Big((s+k)\frac{x}{\tau}\Big) \tag{30}$$

for every $\alpha, \beta \in \mathbb{R}_+$ such that $\alpha \neq 1$, $\beta \neq \alpha$, $\gamma \in \mathbb{R}_+$, then

$$S(\alpha x, \beta\tau) = \gamma S(x, \tau) \tag{31}$$

and there exists a solution

$$C(v) = v^{\ln\gamma/\ln(\alpha/\beta)} \tag{32}$$

on $\mathbb{R}_+$.

Proof. The equation (31) is a consequence of (29) and (30). Let us denote $a = \alpha/\beta, v = (s+k)\frac{x}{\tau}$ and consider (30) in the form $C(av) = \gamma C(v)$ for α, v, γ positive numbers. By means of the method of invariants we obtain a mapping $v \to av, C \to \gamma C$, thus $\frac{\ln v}{\ln a} \to \frac{\ln v}{\ln a} + 1$, $\frac{\ln C}{\ln\gamma} \to \frac{\ln C}{\ln\gamma} + 1$ and $\frac{\ln C}{\ln\gamma} - \frac{\ln v}{\ln a}$ is an invariant. Hence we have $C(v) = v^{\ln\gamma/\ln\alpha}$ and $C(av) = a^{\ln\gamma/\ln a} v^{\ln\gamma/\ln a} = \gamma C(v)$. The function (32) is a solution of the equation (30). □

Remark 6. The function $C(v)$ obtained in Lemma 4 cannot describe the transient capacitance because $C(v)$ is decreasing only in the case that $\ln\gamma/\ln(\alpha/\beta) < 0$. But in this case $C(v)$ is unbounded at $v = 0$ which is a contradiction to (A2). Hence the function (2) cannot form anomalous spectra even though (31) would be satisfied for arbitrarily small $\beta - \alpha$, $\beta \neq \alpha$.

Theorem 1. *For a fixed $x \in \mathbb{R}_+$. If the DLTS spectra are normal or anomalous, then the DLTS function is expressible as*

$$\tilde{S}(x, \tau) = x^p S(x, \tau), \tag{33}$$

where $S(x,\tau) = h(x/\tau) = h(u)$ is the homogeneous function given by (26) and p is fixed real number. Moreover, if for a fixed $\alpha \in \mathbb{R}_+ - \{1\}$ and a given $\lambda \in \mathbb{R}_+$

$$\tilde{S}(\alpha x, \alpha\tau) = \lambda \tilde{S}(x, \tau)$$

holds, then $\lambda = \alpha^p$, i.e. $p = \ln\lambda/\ln\alpha$, and the maximal values of the family of functions

$$\tilde{S}(x, \tau_m),\ \tilde{S}(\alpha x, \alpha\tau_m),\ \tilde{S}(\alpha^2 x, \alpha^2\tau_m),\ \ldots$$

satisfy the relation

$$\tilde{S} = \tilde{S}_m \cdot (\tau/\tau_m)^p \tag{34}$$

where $\tilde{S}_m = \tilde{S}(x, \tau_m)$ is the maximal value of $\tilde{S}(x, \tau)$ at the corresponding point $\tau_m = x/u_m$. Here u_m is a solution of the equation $h'(u) = 0$.

P r o o f. Let the function $\tilde{S}(x, \tau)$ satisfy

$$\tilde{S}(\alpha x, \alpha\tau) = \lambda\tilde{S}(x, \tau) \tag{35}$$

for $x, \alpha, \tau, \lambda \in \mathbb{R}_+$. Then there exists a real number $p = \ln\lambda/\ln\alpha$ such that $\lambda = \alpha^p$ and (35) defines a homogeneous function of the order p of the form $\tilde{S}(x, \tau) = x^p h(x/\tau)$, where $h\colon \mathbb{R}_+ \to \mathbb{R}$ is a suitable function. Now, by means of the generalized DLTS function (26) we obtain the function (33). We can choose a fixed $\alpha \in \mathbb{R}_+ - \{1\}$ and investigate a family of functions

$$\tilde{S}(x, \tau_m),\ \tilde{S}(\alpha x, \alpha\tau_m),\ \ldots,\ \tilde{S}(\alpha^q x, \alpha^q\tau_m); \qquad q \in \mathbb{N},$$

for a fixed $x \in \mathbb{R}_+$. For a fixed x, the function (33) reaches its maximal value at the point $\tau_m = x/u_m$, where u_m is a stationary point of the function $h(u) = h(x/\tau) = S(x, \tau)$ given by (26). Since $\tilde{S}(\alpha x, \alpha\tau) = \alpha^p\tilde{S}(x, \tau)$, we obtain $\tilde{S}(\alpha x, \alpha\tau_m) = \alpha^p\tilde{S}(x, \tau_m) = \alpha^p\tilde{S}_m$, where $\tilde{S}_m = \tilde{S}(x, \tau_m)$. Hence $\tilde{S}(\alpha^n x, \alpha^n\tau_m) = \alpha^{np}\tilde{S}_m = \lambda^n\tilde{S}_m$ are maximal values of the functions $\tilde{S}(\alpha^n x, \alpha^n\tau)$ at the points $\alpha^n\tau_m$, $n = 1, 2, \ldots, q$. Thus $\tilde{S}(\alpha^n x, \alpha^n\tau_m)$ satisfy the equation $\tilde{S} = \tilde{S}_m \cdot (\tau/\tau_m)^p$ for all $n \in \{0, 1, \ldots, q\}$ and Theorem 1 is proved. □

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Author's address: *Václav Tryhuk*, Department of Mathematics, Faculty of Civil Engineering, Technical University of Brno, Žižkova 17, 602 00 Brno, Czech Republic (private: Husova 1006, 665 01 Rosice).