Book reviews

Mathematica Bohemica, Vol. 126 (2001), No. 4, 787–796

Persistent URL: http://dml.cz/dmlcz/134119

BOOK REVIEWS

S. C. Coutinho: THE MATHEMATICS OF CIPHERS. Number Theory and RSA Cryptography. A. K. Peters, Natick, MA, 1999.

This book is an elementary introduction to number theory. It can easily be read by undergraduate and high school students interested in Mathematics. It presents essentially the usual basic material with emphasis on prime numbers and algorithms for primality testing. The title is rather misleading, as the book contains very little about cryptography: ten chapters on number theory and only the last, the eleventh chapter, is about the RSA cryptosystem. Apparently the aim was to get reader interested in number theory by promising him to learn some cryptography. This was probably also the reason for placing the chapter on RSA at the end of the book, although the necessary mathematics could have been presented easily in two or three chapters. I find the term 'finite induction' denoting mathematical induction rather unusual. This may have been caused by translating it from the Portuguese original.

The book is written in a very readable style, with remarks on history and with pictures of famous mathematicians, so it can be an entertaining reading for people who do not know anything about number theory and want to learn the basics in a leisurely way.

Pavel Pudlák, Praha

Peter J. Cameron: PERMUTATION GROUPS. London Mathematical Society Student Texts 45, Cambridge University Press, 1999.

The book under review offers highly interesting overview of modern theory of permutation groups. Various quite recent developments are included, as well as applications to and relations with other areas (combinatorics, logic, computability in particular). The book is divided into seven chapters: General theory, Representation theory, Coherent configurations, The O'Nan-Scott theorem, Oligomorphic groups, Miscellanea and Tables (with tables on simple group and 2-transitive groups).

The book grew from a set of lecture notes and it shows. The style of the exposition is freshly non-encyclopaedic with the size kept admiringly low (220 pages). The text is accompanied by numerous exercises (some are worked using the computer algebra program GAP) and some historic remarks. The book contains extensive bibliography and useful index. Throughout the book there are also references to web pages with related material that is not readily available in other way (e.g. on the classification of finite simple groups). The reader will appreciate that there is a web page associated with the book itself containing, among other links, the list of misprints and errata. The URL is `http://www.maths.qmw.ac.uk/~pjc/permgps`.

Jan Krajíček, Praha

Elwyn Berlekamp, Tom Rodgers (eds.): THE MATHEMAGICIAN AND PIED PUZZLER. A Collection in Tribute to Martin Gardner. A. K. Peters, Natick, Massachusetts, USA, 1999, x+266 pages, numerous graphs and figures.

The name of Martin Gardner is well known to everybody interested in mathematical recreations, games and puzzles. His columns in Scientific American have appealed to wide intelectual public since 1957. The present collection draws on the program of "Gatherings for Gardner", meetings "of the world's foremost magicians, puzzlers and mathematicians", as the editors mention in the Foreword. (Three of them were held till now, in 1993, 1995 and 1998.)

The book has three parts titled Personal Magic (5 papers), Puzzlers (21) and Mathemagics (12). Among the names of individual papers you find terms well known not only to specialists in mathematical recreations—puzzles, mazes, pentominos, tangrams, but also some curiously sounding (at least to me) exotic names: self-designing tetraflexagons, block-packing jambalaya, metagrobolizers of wire.

The book certainly offers numerous opportunities of exercise in creative thinking even to an advanced mathematician, as well as much inspiration for those looking for new sources of interesting and amusing problems for their students.

Jiří Jarník, Praha

Robert C. Dalang, Marco Dozzi, Francesco Russo (eds.): SEMINAR ON STOCHASTIC ANALYSIS, RANDOM FIELDS AND APPLICATIONS. Progress in Probability, vol. 45, Birkhäuser, Basel, 1999, x+300 pages, ISBN 3-7643-6106-9, DM 188,–.

This volume comprises twenty papers based on lectures presented at the Second seminar on stochastic analysis, random fields and applications, held in Ascona, Switzerland, in September 1996. All contributions have been refereed; more than half of them are full-length papers with proofs, the others being overviews of their authors' recent results. A wide spectrum of problems is addressed in the papers, from infinite dimensional diffusions to statistical manifolds. A specialized minisymposium on stochastic methods in financial modelling took place during the seminar, so several papers are devoted to or aim at applications to mathematical finance.

The reviewer may only regret the high price of the book which is likely to prevent many interested readers from having access to it.

Jan Seidler, Praha

H. Amann, J. Escher: ANALYSIS II. Birkhäuser, Basel, 1999, 424 pages, sFr. 42.–

The second volume of a proposed three volume introduction to Calculus written in German. It is based on the university lectures of the authors (Bochum, Kiel, Zürich and Basel, Kassel).

This volume consists of three chapters numbered consecutively with regard to the first volume.

The sixth chapter concerns integration in one variable. The theory presented there is based on the Riemann concept of integral, the properties of the integral are described together with the classical integration techniques. The connections to series are presented with an exposition of Bernoulli numbers and polynomials and other interesting classical topics. Fourier series are dealt with, improper integrals are presented and the chapter is concluded by a part devoted to all classical aspects of the Gamma-function.

The seventh chapter is devoted to differential calculus in more variables. Besides the classical calculus concepts some more general topics as continuous linear mappings in abstract spaces, multilinear mappings, Nemitskii operators, manifolds, tangents and normals are presented in this part.

The eighth chapter covers curvilinear integration. Curves and their length is the starting point for this chapter. Curvilinear integration is presented with the development of Cauchy integral theorems and many aspects of the theory of holomorphic functions.

The theoretical background is presented in full extent and at many places general concepts are presented which will be surely of great influence for students in the study of advanced parts of mathematics. As in the first volume there is a big amount of exercises. In the field of mathematical analysis this book together with its first volume represents an exact exposition presented in modern language as a nice basis for lectures and also with many topics suitable for more advanced seminars.

Štefan Schwabik, Praha

K. Hrbáček, T. Jech: INTRODUCTION TO SET THEORY. Third edition, revised and expanded. Marcel Dekker, New York, 1999, 312 pages, \$ 69,75.

The book is a very good middle-level set theory text and contains carefully selected material.

The first version of this textbook was written in Czech in spring 1968 and accepted for publication, however both authors left Czechoslovakia in that year and the prepared text never appeared. It was mathematically rewritten in English and published in 1978. In the present third edition the older text is revised and reorganized and especially extended. New text (Chapters 11–14) contains e.g. ultrafilters, basic properties of stationary sets (this part culminates with the proof of Silver's theorem), bases of combinatorial set theory (e.g. Ramsey's theorem, Suslin's problem, Jensen's Principle Diamond and Martin's Axiom) and large cardinals (in particular measurable cardinals).

The authors' approach is axiomatic, however they pay serious attention to an explanation of set theoretical concepts and give many explaining examples. A detailed discussion is provided in "controversial" cases, such as Axiom of Choice. The treatment is not formal, logical apparatus is kept to a minimum. There are many exercises in the book.

The authors consider Chapters 1–9 to be a solid course in set theory. Supplementary chapters are (almost) independent.

The first chapter is devoted to an introduction (including Russell's Paradox), axioms of Zermelo-Fraenkel set theory (without Axiom of Foundation) and elementary set-theoretical operations; the second deals with relations, functions, equivalences and orderings. The set of natural numbers and arithmetical operations are dealt with in the third chapter. The next chapter deals with finite and countable sets.

A substantial part of the book (Chapters 5–7 and 9) is devoted to the study of ordinal and cardinal numbers (which are treated even in the case that the Axiom of Choice is not available) and their arithmetics.

Chapter 8 deals with Axiom of Choice, its equivalents and some of its consequences. The tenth chapter is devoted to the construction and properties of rational and real numbers.

In the last chapter of the book, Gödel's and Cohen's results concerning consistency and independence of Continuum Hypothesis are indicated.

At the end one information for mathematicians reading in Czech. In 2001 a book concerning the set theory written in Czech by B. Balcar and P. Štěpánek (second edition) will appear. Balcar-Štěpánek's text contains more material, however it is more formal than the reviewed book.

Antonín Sochor, Praha

J. Kąkol, N. De Grande-De Kimpe, C. Perez-Garcia (eds.): p-ADIC FUNCTIONAL ANALYSIS. Lectures in Pure and Applied Mathematics, vol. 207, Marcel Dekker, New York, 1999, 331 pages, paperback, ISBN 0-8247-8254-2, \$ 165,–.

This is a proceedings volume from the Fifth International Conference on p-adic Functional Analysis (Poznań, Poland, 1998). It contains 21 papers on various subjects from p-adic functional analysis, function theory and harmonic analysis. The range of topics covered is too diverse to be described here completely, so we just mention some highlights: p-adic Banach spaces and Banach algebras, non-archimedean results about locally convex spaces with Schauder bases, Banach spaces over fields with an infinite rank valuation, ultrametric Hopf algebras, fractional differentiation over local fields, p-adic Nevanlinna theory, p-adic hypergeometric series, Fourier analysis and Gamma function, isometries and orthonormal bases in spaces of continuous functions, the Banach-Dieudonné theorem, and Mahler's expansion in several variables. The book contains many new results and up-to-date information about current developments in the field, and will be of interest to anyone actively working in p-adic analysis.

Miroslav Engliš, Praha

Ronald C. Read, Robin J. Wilson: ATLAS OF GRAPHS. Oxford Science Publications, Oxford University Press, 1998, x+454 pages, GBP 75,–.

The atlas is divided into ten numbered sections: 1. Graphs, 2. Trees, 3. Regular Graphs, 4. Types of Graphs, 5. Planar Graphs, 6. Special Graphs, 7. Digraphs, 8. Signed Graphs, 9. Ramsey Numbers, 10. Polynomials. Each section starts with an introductory text. The first nine sections contain mainly diagrams; in Sections 1, 2, 3, 4, 5, 7 and 8 supplemented by tables of graph (digraph) numbers (in Sections 5 and 8 these tables form part of the introduction) and in Sections 1, 2, 3, 7 and 8 also by tables of parameters; diagrams of graphs presented in Section 9 are equipped with their diagonal Ramsey numbers. Section 10 differs from all the preceding ones: it consists of 20 pages of tables of chromatic polynomials (for graphs, cubic graphs and quartic graphs) and 45 pages of tables of spectral polynomials (for graphs, trees, cubic graphs and quartic graphs). The book is concluded by Notes and References and an Index of Definitions.

Section 1 presents diagrams of graphs with 1–7 vertices. Each of these 1252 graphs has its serial number under which it is referred to in Tables of parameters for graphs. Among parameters given in the table we find e.g. the girth, the independence number, the vertex-connectivity, the number of automorphisms, the chromatic number, but also the complement, the code number of the chromatic polynomial and the code number of the spectral polynomial (under these code numbers the polynomials are presented in Section 10). Section 1 also includes the degree sequences of graphs with at most 8 vertices.

Altogether the Atlas offers more than 40 lists of diagrams of different length. For instance, a list of trees with 1–12 vertices (but also of homeomorphically irreducible trees with 1–16 vertices and of identity trees with 7–14 vertices), diagrams of connected cubic graphs (4–14 vertices), connected quartic graphs (5–11 vertices), connected quintic graphs (6–10 vertices), connected sextic graphs (7–10 vertices), diagrams of some special subclasses of regular graphs, diagrams of Eulerian graphs (1–8 vertices), diagrams of connected line graphs (1–8 vertices), diagrams of 2-connected plane graphs (3–7 vertices), etc. As concerns digraphs

(Section 7), the Atlas offers diagrams of digraphs with 1–4 vertices but also e.g. diagrams of tournaments (1–7 vertices); the diagrams of tournaments are economical and thus very illustrative.

I would like to point out the aesthetic features of the diagrams. Especially some of the diagrams of regular graphs in Section 3 (above all the diagrams of connected cubic and quartic transitive graphs and the diagrams of symmetric cubic graphs) and in Section 6 (here e.g. diagrams of cages and diagrams of snarks) will please both the eye and the soul.

I welcome this Atlas as a very valuable publication in the field of the Graph Theory. I am convinced that it can serve as useful tool for anybody who gets into contact with the Graph Theory in his research or his study.

Ladislav Nebeský, Praha

Jean-François Le Gall: SPATIAL BRANCHING PROCESSES, RANDOM SNAKES AND PARTIAL DIFFERENTIAL EQUATIONS. Lectures in Mathematics: ETH Zürich, Birkhäuser, Basel, 1999, x+163 pages, ISBN 3-7643-6126-3, DM 44,–.

Simple branching processes, like e.g. the Galton-Watson process, were studied already in the nineteenth century. However, if one wants to describe branching processes with a continuous state space and with particles undergoing a spatial motion, a highly nontrivial theory is required, which has been developed only recently. Such branching processes (often called superprocesses) turn out to be measure-valued Markov processes, interesting not only by themselves, but also for their applications to partial differential equations. The well known relation between diffusion processes in $\mathbb{R}^d$ and linear elliptic and parabolic problems has its counterpart in a relation between superprocesses and *nonlinear* partial differential equations. Let us recall the concept of a superprocess: Let $(\xi, \mathit{\Pi}_x)$ be a càdlàg Markov process in a Polish space E, which describes the spatial motion, and ψ a function determining the branching mechanism,

$$\psi(u) = \alpha u + \beta u^2 + \int_{]0,\infty[} (\mathrm{e}^{-ru} - 1 + ru)\, \mathrm{d}\pi(u),$$

where $\alpha, \beta \geqslant 0$ and π is a σ-finite measure on $]0,\infty[$ such that $r \wedge r^2 \in L^1(\pi)$. A Markov process Z on the space of finite measures on E is called a (ξ, ψ)-superprocess if its transition function Q satisfies

$$\int \mathrm{e}^{-\langle \nu, g\rangle} Q_t(\mu,\, \mathrm{d}\nu) = \mathrm{e}^{-\langle \mu, u_t\rangle} \quad \text{for all bounded Borel function } g,$$

where u is the unique nonnegative solution to

$$u_t(x) + \mathit{\Pi}_x \int_0^t \psi\big(u_{t-s}(\xi_s)\big)\, \mathrm{d}s = \mathit{\Pi}_x g(\xi_t).$$

If $\psi(u) = \beta u^2$ (a quadratic branching mechanism) and ξ is an n-dimensional Wiener process then Z is called a super-Brownian motion, a process which arises in other fields of stochastic analysis (e.g. in the theory of stochastic partial differential equations) as well.

Several monographs and survey on measure-valued processes having appeared recently notwithstanding, Le Gall's text under review has its indisputable merits: it is almost self-contained, only moderate preliminary knowledge is presupposed, it is written in a very clear way, and it makes easily accessible many important results available hitherto only in journal articles. There are two main topics covered by the book. The first of them is a construction (due to the author) of the superprocess using a snake—a path-valued process which provides a "particle picture" of the superprocess. The second is the connection between the super-Brownian motion and the elliptic equation $\Delta u = 4u^2$.

Bohdan Maslowski, Praha

P. G. Goerss, J. F. Jardine: SIMPLICIAL HOMOTOPY THEORY. Progress in Mathematics, vol. 174, Birkhäuser, Basel, 1999, ISBN 3-7643-6064-X, ISBN 0-8176-6064-X, sFr 98,–.

As the authors point in the introduction, the book is intended to fill an expanding gap in the expository literature on simplicial sets. The previous major pieces in this area are more than twenty-five years old and do not take into account the vast developments and huge influence of the concept of Quillen's closed model category structures. In the present book, the axioms of a closed model category can be found on page one.

The first two chapters are devoted to basic definitions and properties of (simplicial) model structures. The third chapter is a flashback of some classical results and constructions, such as the fundamental groupoid and Hurewicz map. Bisimplicial sets and various related spectral sequences are discussed in the fourth chapter.

The next two chapters are dedicated to simplicial groups, fibrations and classifying spaces. In Chapter VII, Reedy model structure on the category of simplicial objects in a closed model category is discussed.

Chapter VIII is devoted to cosimplicial sets, with an emphasis on homotopy inverse limits. The following chapter studies the phenomena of coherence, and the last chapter of the book the localization with respect to a map.

The book does not assume much preliminary knowledge beyond standard courses of algebra, topology and homotopy theory. Yet some experience and knowledge of motivating examples is recommended, because the level of abstraction of the exposition is very high.

Martin Markl, Praha

George Grätzer: FIRST STEPS IN LaTeX. Birkhäuser, Boston, 1999, xx+131 pages, ISBN 0-8176-4132-7, DM 44,–.

The experience of every TeX novice depends on many initial conditions. Usually one encounters a sort of more or less friendly user menu program, which enables to run all programs from one place. This is something to be settled down on a local implementation level and it is usually no serious problem. The basic problem then is how to start and process a particular document and this is the moment of more and more intensive feeling of the immense TeX universality, diversity, and flexibility, which together with sometimes very laconic TeX error messages makes the things difficult. Whereas the TeX universality is exactly the unbeatable advantage for an experienced user, there are many traps and blind alleys for a beginner here and at this stage it is apparently not the fastest way to make it out to start studying command syntax with all the options available, to try to understand all the fine possibilities that TeX offers, and the like, in books which become useful and indispensable later. The book under review is exactly the kind vade-mecum that a beginner needs. Reflecting the fact that LaTeX has become a certain math community standard it describes the basic features of LaTeX 2_ε, modelled upon a preparation of a research article.

As to the contents this book is actually a "little brother" of author's Math into LaTeX, which has appeared already in two editions. The book covers the following areas: The first chapter describes how to type a simple text. Chapters 2 and 3 are a basic manual of how to type mathematical formulas, including user-defined commands. This is of course more or less similar to all books on TeX. In addition, some special LaTeX constructions are discussed here, too, in particular, aligned formulas with help of the `align` environment and automatic numbering. Chapter 4 uses a sample article prepared on the basis of the LaTeX article class file and explains the basic building parts of a paper. Chapter 5 is devoted to the AMS packages and, again, with use of a model paper, it is explained, step by step, how to go on. An extremely useful feature of this part is showing the source text and the corresponding output on the same page. The concluding Chapter 6 contains above all some information about LaTeX warning messages. Every TeX user could tell stories about chasing persistent syntax errors; an experience is needed here and the reader is encouraged to make experiments with deliberate mistakes, to analyze links between the cause and the consequences, and to learn the right strategy for spotting troublesome places. There are also some general comments on LaTeX and its development. Three appendices contain text and math symbol tables, information about the TeX community, and about TeX on the web. The book has one index oriented towards word-processing in general and another one related to specific LaTeX concepts.

The book is written by an experienced author in a very carefully balanced way. It can be very much recommended to beginners in LaTeX who want to proceed fast to the first results and also as a solid base for an effective further reading of more advanced books. At the same time it will be very useful for those who have some TeX experience outside LaTeX and have to prepare a LaTeX document.

Miroslav Krbec, Praha

M. Väth: VOLTERRA AND INTEGRAL EQUATIONS OF VECTOR FUNCTIONS. Marcel Dekker, New York, 1999, vi+349 pages, \$ 150,–.

The present research monograph of Martin Väth is devoted to abstract integral equations mostly of Volterra type for functions with values in a general Banach space and to the study of general Volterra operators and equations involving them.

The introductory chapter presents basic concepts concerning compactness and fixed point theory, measurable functions and ideal spaces. To the last topic the author published a Springer Lecture Note in 1997.

In the second chapter the abstract definition of a Volterra operator is given and local existence results are presented. The local existence results are used for extending the solutions and applications are given.

Integral operators in Banach spaces are studied in the third chapter of the book. This involves Carathéodory functions, Urysohn operators, measures of noncompactness, etc.

A nice part of the theory is contained in the last chapter in which continuous dependence results are presented together with the closely related averaging principle.

This monograph will be surely a valuable reference book for specialists in operator equations.

Štefan Schwabik, Praha

Christina Birkenhake, Herbert Lange: COMPLEX TORI. Birkhäuser, Progress in Mathematics, vol. 177, Boston, 1999, 251 pages, DM 118,–.

A complex torus is a connected compact complex Lie group. Any complex torus is of the form $X = \mathbb{C}^g/\Lambda$, where $\mathbb{C}^g$ is the complex vector space of dimension g and Λ is a lattice in $\mathbb{C}^g$. The roots of the subject reach to the 20s of the 19th century, when Jacobi (1) posed his celebrated inversion problem for hyperelliptic integrals as a generalization of the inversion problem for elliptic integrals and (2) proved that the one dimensional complex torus can be isomorphically embedded into $\mathbb{P}^2$ ($\mathbb{P}$ is the complex projective line, equivalently the Riemannian sphere) via the elliptic θ-functions. The generalizations of (1) and (2) led to the theory of Abelian varieties, i.e. algebraic complex tori, which is now very well investigated. Unfortunately, not much is known about arbitrary non algebraic complex tori, although they are among the simplest complex manifolds and occur even if one starts within the category of Abelian varieties (e.g. algebraic cycles via intermediate Jacobians). The aim of the book is the systematic presentation of the theory of general complex tori. It lies on the crossroads of algebraic geometry, several complex variables, differential geometry and representation theory. The reading of the book is not easy for non specialist, because it in fact presupposes the knowledge of the theory of Abelian varieties. All the necessary facts needed are covered by the book Complex Abelian Varieties (Springer, Grundlehren 302) by the same authors.

Jaroslav Fuka, Praha

Josè Ferreirós: LABYRINTH OF THOUGHT. A History of Set Theory and its Role in Modern Mathematics. Birkhäuser, Basel, 1999, 464 pages, DM 198,–.

The topic of the book is the development of the set theory and it covers fifty years before and after 1900. The author does not confine himself to the development of the technical notions and results only, but discusses extensively the attitudes and views of leading German mathematicians of the time, and how the set theory influenced their research (and vice versa). This is illustrated by detailed descriptions of their communications.

I believe that the book is of interest to mathematicians as well as to other scientists with interest in the evolution of main mathematical ideas.

The book consists of three parts: I. The Emergence of Sets within Mathematics (in particular, the influence of B. Riemann and J. W. R. Dedekind on G. Cantor), II. Entering the Labyrinth—Toward an Abstract Set Theory (cardinality, continuum hypothesis, ordinals), and III. In Search of an Axiom System (paradoxes, axiom of choice and on to formal systems and Gödel's work).

Jan Krajíček, Praha

V. Dragan, A. Halanay: STABILIZATION OF LINEAR SYSTEMS. Birkhäuser, Basel, 1999, 328 pages, sFr. 128,–.

This book is a research monograph based on the work of the known Roumanian mathematicians Vasile Dragan and the late Aristide Halanay. It is devoted to the stabilization of linear control systems which is a topic still in focus for engineering.

The mathematical ideas forming the background of the engineering problems are presented. The contents of the work can be described by the titles of the chapters of the book: Stabilization of linear systems (controllability, observability, Liapunov equations, ...), Stabilization of linear systems with two time scales, High-gain feedback stabilization of linear systems, Adaptive stabilization and identification, Discrete implementation of stabilization procedures.

Problems of optimal stabilization are studied in connection with the well known Popov-type frequency domain conditions.

The book is mathematical; rigorous proofs are presented and the book can be a helpful tool for applied scientists and engineers.

Štefan Schwabik, Praha

Gabriel P. Paternain: GEODESIC FLOWS. Progress in Mathematics 180, Birkhäuser, Boston, MA, 1999, ISBN 0-8176-4144-0, DM 108,–.

The book is devoted to the theory of geodesic flows on compete Riemannian manifolds, with an emphasis on dynamical systems and ergodic theory.

The first chapter contains an introductory material and basic definitions. The second chapter deals with the action of geodesic flows on Lagrangian submanifolds. Among other things, the Maslov index of a pseudo-geodesic is introduced here. In the third chapter, the topological entropy and various counting functions are presented, and relations between these quantities are studied (Yomdin's theorem, Manning's inequality). The fourth chapter is devoted to Mañé's formula for the topological entropy of geodesic flows on Riemannian manifolds and to consequences of this formula, e.g. Mañé's formula for convex billiards. In the last chapter, various topological conditions that ensure positive entropy, mostly based on the rational-homotopic dichotomy theorem for manifolds by Félix and Halperin, are discussed. A typical example of a theorem proved in this part of the book is that a rationally hyperbolic manifold has positive topological entropy for any choice of a smooth Riemannian metric.

The book is basically self-contained, though some theorems are stated without proof. We can recommend it to graduate students and to anyone interested in dynamical aspects of Riemannian geometry.

Martin Markl, Praha

Sanford S. Miller, Petru T. Mocanu: DIFFERENTIAL SUBORDINATIONS. THEORY AND APPLICATIONS. Pure and applied mathematics, Marcel Dekker, New York, 2000, ISBN 0-8247-0029-5, $ 175,–.

Obtaining information about properties of a function from inequalities for its derivatives plays an important role in the theory of real functions. During last two decades, such *differential inequalities* on the real line have been intensively generalized to the main subject of the book, namely *differential subordinations* in the complex plane. Before explaining this notion, let us introduce some notation and definitions. Let $\mathbf{U}$ and $\mathcal{H}$ denote the open unit disk in $\mathbb{C}$ and the class of all functions analytic in $\mathbf{U}$, respectively. A function $f \in \mathcal{H}$ is said to be *subordinate* to a function $F \in \mathcal{H}$ (written $f \prec F$) if there exists a function $w \in \mathcal{H}$ with $w(0) = 0$ and $|w(z)| < 1$, and such that $f(z) = F(w(z))$. It is known that if F is *univalent* (i.e. one-to-one in $\mathbf{U}$), then $f \prec F$ if and only if $f(0) = F(0)$ and $f(\mathbf{U}) \subset F(\mathbf{U})$.

Many problems in the field of differential inequalities are of the following type: given a differential operator D and intervals $\mathbf{I}, \mathbf{J} \subset \mathbb{R}$, find the smallest interval $\mathbf{K} \subset \mathbb{R}$ such that the implication $\mathrm{D}[f](\mathbf{I}) \subset \mathbf{J} \implies f(\mathbf{I}) \subset \mathbf{K}$ holds for any smooth function $f\colon \mathbf{I} \to \mathbb{R}$. Such implications have a direct complex analogue, namely the implications $\mathrm{D}[f](\mathbf{U}) \subset \Omega \implies f(\mathbf{U}) \subset \Delta$, where D is (as before) a differential operator, Ω and Δ are sets in $\mathbb{C}$ and f is any function from $\mathcal{H}$. Moreover, if Ω and Δ are simply connected domains, then the above implication can be rewritten as $\mathrm{D}[f] \prec h \implies f \prec q$, with some (given) univalent functions $h, q \in \mathcal{H}$. We have come to the very heart of the book under consideration: a relation $\mathrm{D}[f] \prec h$, with a given univalent function $h \in \mathcal{H}$, is called a *differential subordination*, while any function $f \in \mathcal{H}$ that satisfies this subordination is called a *solution*. If $q \in \mathcal{H}$ is univalent and if $f \prec q$ for any solution f of $\mathrm{D}[f] \prec h$, then h is called a *dominant* (or *the best dominant* if $q \prec q'$ for all dominants q'). Typical problems considered in the book are: (i) given h and q, find a class of operators D such that q is a dominant of $\mathrm{D}[f] \prec h$; (ii) given D and h, find a dominant (the best dominant) of $\mathrm{D}[f] \prec h$; (iii) given D and q, find the largest class of h for which q is a dominant of $\mathrm{D}[f] \prec h$. Thus Chapter 2 deals with such problems in the case when D is the second order operator of the form $\mathrm{D}[f](z) = \Psi(f(z), zf'(z), z^2f''(z); z)$, with a given four-place function $\Psi\colon\ \mathbb{C}^3 \times \mathbf{U} \to \mathbb{C}$.

Chapter 3 is devoted to first-order differential subordinations (abbreviated below as DS's), both the linear ones, namely $A(z)f'(z) + B(z)f(z) \prec h(z)$, and nonlinear ones, such as the Briot-Bouquet DS, which is of the form $f(z) + zf'(z)/(bf(z) + c) \prec h(z)$, with some constants $b, c \in \mathbb{C}$, and its generalization $\tau(f(z)) + zf'(z)\varphi(f(z)) \prec h(z)$, with some analytic functions τ and φ. The results are then applied to the theory of univalent functions as well as to the theory of analytic integral operators and integral operators I that *preserve subordination*: $f \prec g \implies I[f] \prec I[g]$. The key role in Chapter 4 is played by linear second order DS's and their applications to linear integral operators that preserve *positivity of the real part* or the *boundedness* of any function $f \in \mathcal{H}$; the other topics of this chapter are averaging integral operators, hypergeometric functions and conditions relating the Schwarzian derivative to the univalency of the function under derivating. Some special DS's are considered in Chapter 5 to solve problems related to particular subclasses of univalent functions (starlike and convex functions, functions with bounded turning). Chapter 6 deals with higher order DS's, a field in which there is still a great deal of research to be done. Some results about the third order DS's are presented, followed by results about the Euler N-th order DS, being connected with a famous problem which is known as the *Problem of S. Miller*: Prove that $|f(z)| \leqslant 1/2$ whenever $f \in \mathcal{H}$ satisfies $f(0) = 0$ and $|f(z) + zf'(z) + \ldots + z^n f^{(n)}(z)| \leqslant 1$. In Chapter 7, DS's in several complex variables are treated. The final Chapter 8 deals not only with applications of DS's which yield some properties of harmonic and meromorphic functions but also with extensions of DS's to vector-valued functions.

The well-written monograph which summarizes (in a unified fashion) the results of the last two decades' intensive research of differential subordinations, includes over 400 bibliographic items.

Jaromír Šimša, Brno

Giuseppe Peano: GEOMETRIC CALCULUS. According to the Ausdehnungslehre of H. Grassmann. Translated by L. C. Kannenberg. Birkhäuser, Boston, 2000, xvi+276 pages, DM 138,–.

The present book is an English translation of the whole of Peano's Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann, proceduto delle operazioni della logica deduttiva, which appeared 1888 in Torino.

Giuseppe Peano (1858–1932) has been known for his mathematics and is probably the first mathematician who appreciated Grassmann's algebraic ideas. Peano's book is a treatise on mathematical logic and a presentation of Grassmann's work.

It is interesting to see the formalized logical symbolics of G. Peano.

The ninth chapter of the book contains a general concept of the vector space and of the linear mapping in fact in the form of the contemporary abstract notion of the vector space which was commonly recognized many years after Peano's book.

The book will be undoubtedly welcomed by historians of mathematics as a source of Peano's ideas and mathematical methods.

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