BOOK REVIEWS

R. Bañuelos, C. N. Moore: PROBABILISTIC BEHAVIOR OF HARMONIC FUNCTIONS. Birkhäuser, Basel, 1999, ISBN 3-7643-6062-3, hardcover, 224 pages, DM 118,–.

The authors of this monograph have collected their works (some of them in collaboration with Ivo Klemeš) published over the past years on the subject of probabilistic techniques in specific fields of harmonic analysis and in the present book they offer a unified presentation of their results in view of historical precedents, related directions and approaches of other authors together with some new improvements and refinements.

The main interest concerns the classical *nontangential maximal function* of u, $N_\alpha u(x) = \sup\{|u(s,t)|\colon (s,t) \in \Gamma_\alpha(x)\}$ and *the Lusin area function* of u, $A_\alpha u(x) = (\int_{\Gamma_\alpha(x)} |\nabla u(s,t)|^2 t^{1-n} \,\mathrm{d}s\,\mathrm{d}t)^{1/2}$ where u is a harmonic function defined on the upper half space $\mathbb{R}^{n+1}_+$ and $\Gamma_\alpha(x)$ is the cone with vertex at $(x,0)$ and vertical axis $\{(x,t)\colon t>0\}$. The behavior of N_α and A_α resembles the respective behavior of the maximal function X^* of a continuous martingale X and the square root of the supremum of its quadratic variation $\langle X\rangle^{1/2}_\infty$ respectively, and the book opens at this point. The authors prove several so-called *good-lambda inequalities* which are, in a certain sense, the sharpest possible attainable and several *laws of the iterated logarithm* for harmonic functions which measure the relative growth of truncated versions of $N_\alpha u$ and $A_\alpha u$ in a very precise manner. In this way, the law of the iterated logarithm can be thought of as a refinement of the classical theorem of Calderón and Stein which says that, up to sets of Lebesgue measure zero, the area function and the nontangential maximal function are finite or infinite on the same sets.

Looking into the proofs we see, apart from the tools used in the classical harmonic analysis, a utilization of the martingale theory as well as reductions of purely analytical problems to typically probabilistic ones (e.g. the law of the iterated logarithm for harmonic functions is converted into the well known law of the iterated logarithm for Brownian motion).

Another issue is the analogy between the density $D_\alpha u(x;r)$ and the maximal density $D_\alpha u(x)$ of the area integral of a harmonic function on $\mathbb{R}^{n+1}_+$ and their probabilistic counterparts, the local time and the maximal function of the local time of a continuous martingale X respectively. Again, the use of probabilistic ideas and techniques allows to obtain inequalities involving N_α, A_α and D_α that are, in some sense, the sharpest attainable.

The book is divided into six chapters. Chapter 1 is devoted to the requisite background material from harmonic analysis and known results concerning the nontangential maximal function and the area function, as well as to their role in the development of this field. The remaining chapters deal, among others, with approximations of harmonic functions by dyadic martingales and their applications to the problems outlined above, Chung-type and Kesten-type laws of the iterated logarithm for harmonic functions, applications to Brossard and Chevalier's characterization of $L\log L$ within H^1 and applications to lacunary series and Bloch functions.

Several open problems and questions of interest can be found in the text as well as parallels with classical results of this nature.

This monograph is primarily aimed at researchers and students who wish to learn more on how some probabilistic techniques penetrate effectually into harmonic analysis. Yet a minimal knowledge of classical theory of probability, martingales and harmonic functions would facilitate the reading.

Martin Ondreját, Praha

Aleksey I. Prilepko, Dmitry G. Orlowski, Igor A. Vasin: METHODS FOR SOLVING INVERSE PROBLEMS IN MATHEMATICAL PHYSICS. Marcel Dekker, New York, 2000, ISBN 0-8247-1987-5, 744 pages, \$ 195,–.

This extensive monograph concerns problems where not only the solution but rather the equation is the unknown. Usually some information about a hypothetic solution is known and a certain form of equation is sought so that it fits with the information about the solution. For example, the problem of discovering forces making planets move in accordance with Kepler's laws was one of the first inverse problems in the dynamics of mechanical systems.

This book provides a systematic theory of inverse problems and reflects its rapid growth and development over recent years. It covers the basic types of equations as elliptic, parabolic and hyperbolic equations. Special attention is paid to Navier-Stokes equations and kinetic equations as the Boltzman and the neutron transport equation.

The body of the book is formed by ten chapters. First three chapters are devoted respectively to inverse problems for parabolic, hyperbolic and elliptic equations.

In the first chapter auxiliary material is thoroughly presented and the notion of the inverse problem rigorously introduced. Basic methods for its solution are then developed.

In the second chapter first order hyperbolic systems such as those arising in hydrodynamics, aerodynamics or the Maxwell equations are treated. Also the inverse problems for the wave equation with lower order terms are investigated.

The third chapter is almost exclusively devoted to inverse problems in potential theory with strong relation to elliptic equations.

Next, the fourth chapter concerns inverse problems arising in dynamics of viscous incompressible fluids described by the classical Navier-Stokes system for newtonian fluids.

The fifth chapter serves as a preparatory one for the sixth chapter, where an abstract approach to inverse problems, using the theory of differential equations in Banach spaces, is applied.

Chapter seven continues to apply the abstract approach to the so-called two-point inverse problem for first order equations, where not only the initial but also the final state is prescribed.

An analogous problem is solved in Chapter eight for the second order abstract hyperbolic equation. Semilinear equations are also considered in this chapter.

In the ninth chapter the abstract theory built up in previous chapters is applied to concrete equations of mathematical physics. Here, first, linear symmetric hyperbolic systems are addressed, then second-order hyperbolic equations are studied, and finally the system from the elasticity theory, equations of heat transfer, neutron transport equations, linearized Boltzman equation, Navier-Stokes equations and Maxwell equations, are thoroughly treated.

Finally, in Chapter 10 the results presented in the book are extensively commented and detailed references not only to inverse but also to related direct problems are given.

The book would serve specialists in inverse problems, scientists working in the theory of partial differential equations, equations of mathematical physics and applications of mathematics. It mostly assumes the graduate level and as a representative monograph in the field should be included in the libraries of mathematical departments of universities as well as in related research institutions.

Ivan Straškraba, Praha

J. Faraut, S. Kaneyuki, A. Korányi, Q.-K. Lu, G. Roos: ANALYSIS AND GEOMETRY ON COMPLEX HOMOGENEOUS DOMAINS. Progress in Mathematics, vol. 185. Birkhäuser, Basel, 2000, ISBN 3-7643-4138-6, hardcover, 560 pages, DM 138,–.

Analysis on homogeneous spaces has become a very large, fairly complex and intensively developing area. Although several good books on the subject already exist, it is extremely important to have a treatise which would give a general overview of the whole field and put various results, approaches and ideas into perspective. The book under review tries to do precisely that. It consists of five papers (one by each author) of more or less introductory nature which concentrate on different aspects of the field. Though ordered alphabetically, the best ones for the reader to start with are probably *Function spaces on bounded symmetric domains,* by Korányi, and *Jordan triple systems,* by Roos, which unfold two standard approaches to symmetric domains: the Lie-theoretic one, and the algebraic one, respectively. In both cases, the expositions start from the basics, and try to guide the reader all the way through to the problems of current interest. The third approach, relying on "bare hands" computations, is used in Lu's *The heat kernels of noncompact symmetric spaces.* Finally the two remaining papers give an overview of the Gelfand-Gindikin program and of certain generalizations of symmetric spaces, respectively; they are *Function spaces on complex semigroups* by Faraut, and *Graded Lie algebras and pseudo-Hermitian symmetric spaces* by Kaneyuki. It is evident that a considerable effort has been made to keep the papers reasonably self-contained and accessible to anyone who is familiar with the basic facts of differential geometry, functions of several complex variables, and Lie group theory. The book is a very good choice for a newcomer into the field and will make a valuable resource for active researchers in the area as well.

Miroslav Engliš, Praha

R. Estrada, R. P. Kanwal: SINGULAR INTEGRAL EQUATIONS. Birkhäuser, Boston, 2000, xii+427 pages, ISBN 3-7643-4085-1, DM 138,–.

This work focuses on the distributional solutions of singular integral equations, progressing from the basic concepts of the classical theory to the more difficult two-dimensional problems. In the first chapter the authors have included some reference material. They compile the basic principles of Lebesgue integration, improper integrals, singular integrals, Cauchy principal value. They also present the concept of the boundary value of analytic functions. They introduce several spaces of functions and distributions. Chapter 2 deals with the Abel integral equation and related equations. Chapter 3 is devoted to the study of the Cauchy type integral equation. This chapter is concluded by studying the Hilbert equation. In Chapter 4 the authors solve the Carleman type integral equations. In Chapter 5 they present the distributional solutions of the integral equations of Abel, Cauchy and Carleman type. In the process they introduce the concept of the analytic representation of a distribution with compact support. They also consider distributional dual and multiple integral equations of the Cauchy type as well as distributional solutions of these integral equations in the space of generalized periodic functions. In Chapter 6 the authors study the distributional solution of the Carleman equation on the whole line. Chapter 7 is devoted to the study of singular integral equations with logarithmic kernels. Furthermore, the asymptotic behavior of the eigenvalues of the basic integral equation with the logarithmic kernel is studied. The solution of Wiener-Hopf type integral equations is presented in Chapter 8. In Chapter 9 the authors are concerned with the dual and triple integral equations whose kernels are trigonometric and Bessel functions.

Dagmar Medková, Praha

J. M. Appel, A. S. Kalitvin, P. P. Zabrejko: PARTIAL INTEGRAL OPERATORS AND INTEGRO-DIFFERENTIAL EQUATIONS. Marcel Dekker, New York, 2000, x+560 pages, USD 195,–.

The present book is concerned with the study of integral operators and of integro-differential equations of the Barbashin type, i.e. with equations of the form

$$\frac{\partial x(t,s)}{\partial t} = c(t,s)x(t,s) + \int_a^b k(t,s,\sigma)x(t,\sigma)\,\mathrm{d}\sigma + f(t,s).$$

The first chapter is devoted to general Barbashin type equations (in the space C, in Lebesgue spaces and in the so called ideal spaces).

Linear Barbashin equations are presented in the second chapter.

In the third chapter partial integral operators, i.e. operators of the form

$$\begin{aligned} Px(t,s) = c(t,s)x(t,s) &+ \int_T l(t,s,\tau)x(\tau,s)\,\mathrm{d}\tau \\ &+ \int_S m(t,s,\sigma)x(t,\sigma)\,\mathrm{d}\sigma + \int_T\int_S n(t,s,\tau,\sigma)x(\tau,\sigma)\,\mathrm{d}\sigma\,\mathrm{d}\tau, \end{aligned}$$

are described.

The last chapter deals with generalizations and applications of the concepts presented in the previous parts of the book.

The integral operators presented above are studied in the book in all aspects, particular attention being paid to equations containing operators of this type. The book collects the majority of facts concerning these operators acting in various important function spaces. Up to this moment there was no monograph on this interesting topic and especially the contribution of mathematicians from the former Soviet Union is presented in the book form for the first time.

The book will surely serve as a thorough source for references as well as a guide to possible applications of the theory.

Štefan Schwabik, Praha

K. Markov K., L. Preziosi (eds.): HETEROGENEOUS MEDIA. Micromechanics modeling methods and simulations. Birkhäuser, Boston, 2000, ISBN 0-8176-4083-5, hardcover, xiv + 477 pages, DM 178,–.

The book is an ambitious attempt to collect five first class reviews on different aspects of micromechanics of multicomponent systems both from the theoretical and practical view.

First review on the elementary micromechanics of heterogeneous media (written by K. Z. Markov) gathers up different ways of "homogenization", i.e. a model replacement of a heterogeneous solid by a homogeneous one with similar "effective" properties. A short historical overview extending over last two centuries opens this chapter and the impressive stream of names like Cauchy, Poisson, Navier, Maxwell, Rutheford, Lorentz, Einstein, Smoluchowski, Hashin, Kröner, Eshelby, Matheron and many others maps the achievements and difficulties connected with this approach. Beside elastic properties, also thermal, electromagnetic and viscous phenomena are covered.

Processes of diffusion and flow in a two-phase random medium composed of solid phase and pores (voids) occur in a wide range of situations such as heterogeneous catalysis, cell metabolism, migration of atoms and defects and nuclear magnetic resonance. In the second chapter, S. Torquato deals with these phenomena by means of stochastic geometry and variational calculus. In particular, effective steady state diffusion conditions are derived, bounds on the effective diffusion and flow properties and the links between them are found.

The examination of the wave propagation through heterogeneous media (third chapter written by S. K. Kanaun) is based on modern versions of the Maxwell single particle approach and the so called effective field and effective medium methods are introduced. Composite materials with regular lattices of inclusions and granular or polycrystalline matter are the media considered in detail as illustrations.

In the last but one chapter, A. Farina and L. Preziosi focus on the mathematical models applicable to the composite materials manufacturing. The review includes the ensemble average procedures as well as the effective media approach, presents results of 2D and 3D simulations and closes by a discussion of open problems.

Micromechanics of poroelastic rocks is the subject of the last review (R. W. Zimmerman). There is an interaction between the rock deformation induced by pressure within pores and cracks saturated with fluid components (water, air, oil) and the fluid flow through rocks due to pressure changes evoked by natural (e.g. tectonic) or man-made (e.g. drilling of bore-holes) causes. This coupled mechanical and hydrological behaviour is examined in the framework of the theory of poroelasticity treating rocks and fluids as two interpenetrated continua with no or little reference to their microstructure.

The range of applications covered and the display of relevant analytic and stochastic methods is truly impressive and the book can be recommended not only to specialists but also to those who are interested in the recent interdisciplinary trends in the physico-mechanical description of heterogeneous media.

Ivan Saxl, Praha

V. M. Adamyan, I. Gohberg, M. Gorbachuk, V. Gorbachuk, M. A. Kaashoek, H. Langer, G. Popov (eds.): DIFFERENTIAL OPERATORS AND RELATED TOPICS. Proceedings of the Mark Krein International Conference on Operator Theory and Applications, Odessa, Ukraine, August 18–22, 1997, Vol. I, Operator Theory, Advances and Applications, vol. 103, Birkhaüser, Basel, 2000, DM 418,–. OPERATOR THEORY AND RELATED TOPICS. Vol. II of the same Proceedings, Operator Theory, Advances and Applications, vol. 118, Birkhäuser, Basel, 2000, DM 418,–.

The two books are proceedings of the conference held in Odessa in August 1997 which was dedicated to the 90th anniversary of the prominent mathematician Mark Krein.

The conference was focused on the ideas and results of M. Krein. In particular, it concentrated on operator theory and its applications, functional analysis, integral and differential operators, spectral theory, the moment problem, system theory, extension problems and differential equations.

The proceedings contain about 50 original research papers in the above mentioned fields. The first volume is devoted to differential operators; the second part concentrates on operator theory and related topics.

Vladimír Müller, Praha

Jürgen Appell (ed.): RECENT TRENDS IN NONLINEAR ANALYSIS. Festschrift Dedicated to Alfonso Vignoli on the Occasion of His Sixtieth Birthday. Birkhäuser, Basel, 2000, 264 pages, DM 148,–.

This collection of 21 research papers written by 41 authors starts with an introductory note entitled "Alfonso Vignoli – the Researcher, Teacher, and Friend" (authors: the editor, M. Furi and J. Ize) and by a list of Vignoli's publications (2 textbooks, 2 monographs in preparation, 55 papers).

The topics of the 21 papers correspond, roughly speaking, to the broad scientific interests of the honoured person which cover ill-posed problems and regularization, fixed point theorems and applications, nonlinear operator theory, extremal problems and variational calculus, nonlinear spectral and eigenvalue theory, bifurcation and implicit function theorems, topological degree theory and equivariant and related maps.

Alois Kufner, Praha

L. L. Gray: LINEAR DIFFERENTIAL EQUATIONS AND GROUP THEORY FROM RIEMANN TO POINCARÉ. Second Edition, Birkhäuser, Boston, 2000, xx+338 pages, DM 148,–.

The first edition of this book appeared in 1986.

The volume under review is a deep description of the development of differential equations and group theory in the second half of the nineteenth century. It presents not only a historical perspective, mathematics itself is also presented for students as well as for working mathematicians. The book contains plenty of material related to algebra, geometry, and analysis of the nineteenth century.

The starting point are Gauss' 1812 papers on the hypergeometric series and the subsequent work by Jacobi, Kummer and Riemann. The work of L. Fuchs concerning linear differential equations is explained and concluded by the work of Fuchs' successors. Group theory appears in connection with the problem of finding algebraic solutions of the hypergeometric equation (Schwarz, Fuchs, Klein, Gordan, and Jordan). Modular equations are presented in connection with the work of Hermite, Fuchs, and Dedekind. Work on algebraic curves is presented and a brief account of the Jacobi inversion problem and the theory of theta functions is given. Automorphic functions and Poincaré-Klein discussion of the early 1880's conclude the story.

The book presents very technical topics and without the author's thorough explanations it would be very difficult to read the historical material, with all the complicated technical details.

The book shows the author's deep understanding of mathematics of the nineteenth century and represents a fascinating reading for mathematicians as well as historians of science.

Štefan Schwabik, Praha