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On Tensor Fields Semiconjugated with Torse-forming Vector Fields *

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Abstract

The paper deals with tensor fields which are semiconjugated with torse-forming vector fields. The existence results for semitorse-forming vector fields and for convergent vector fields are proved.

Key words: Torse-forming vector fields, Riemannian space, semisymmetric space, T-semisymmetric space.

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1 Introduction

Torse-forming vector fields were introduced by K. Yano [8] in 1944 and their properties in Riemannian spaces have been studied by various mathematicians. For example some properties in Ricci semisymmetric Riemannian spaces have been proved by J. Kowolik in [1]. In T-semisymmetric Riemannian spaces they are studied by the authors in [4] and [5].

This paper is devoted to the study of tensor fields which are semiconjugated with torse-forming vector fields. We are motivated by the work of J. Kowolik [1].

First we give some definitions and notations. V_n denotes an n-dimensional Riemannian space with a metric g and an affine connection ∇. The metric g

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need not be positive definite. TV_n is a space of all tangent vector fields on V_n. In the whole paper we will assume that $n > 2$ and that all functions, vectors and tensor fields are sufficiently smooth. Further $\boldsymbol{\xi}$ will be a non-zero vector field, i.e. $\boldsymbol{\xi}(x) \neq \boldsymbol{o}$ for each $x \in V_n$.

We denote the Riemannian tensor in V_n by R. This tensor is called *harmonic*, if $R^{\alpha}_{ijk,\alpha} = 0$, where "," denotes the covariant derivative. This condition can be written in the form $R_{ij,k} = R_{ik,j}$ where $R_{ij} \equiv R^{\alpha}_{ij\alpha}$ is the Ricci tensor of V_n.

Definition 1 Vector field $\boldsymbol{\xi}$ is called *torse-forming*, if $\nabla_X \boldsymbol{\xi} = \varrho \cdot X + a(X) \cdot \boldsymbol{\xi}$ for all $X \in TV_n$, where ϱ is some function on V_n, a is a linear form on V_n. In the local transcription this formula has the form $\xi^h_{,i} = \varrho\delta^h_i + a_i\xi^h$, where ξ^h are components of the torse-forming field $\boldsymbol{\xi}$, δ^h_i is the Kronecker delta, a_i are components of the form a, which is a covector on V_n.

Definition 2 A torse-forming vector field $\boldsymbol{\xi}$ is called:

- *recurrent*, if $\varrho = 0$,
- *concircular*, if the form a is gradient (or locally gradient), i.e. there exists (locally) a function $\varphi(x)$ such that $a = \partial_i\varphi(x)dx^i$,
- *convergent*, if $\boldsymbol{\xi}$ is concircular and $\varrho = \text{const} \cdot \exp\big(\varphi(x)\big)$,
- *semitorse-forming*, if $R(X, \boldsymbol{\xi})\boldsymbol{\xi} = 0$ for each $X \in TV_n$.

Properties of torse-forming vector fields in the Einsteinian spaces are proved by the authors in [5]. In [2] and [3] J. Mikeš proved that in non-Einsteinian Ricci-symmetric and Ricci-two-symmetric ($R_{ij,kl} = 0$) spaces there are no concircular vector fields which are not recurrent.

In what follows we will need a definition of an operator $R(X, Y) \circ T$ for tensors of the type $(0, q)$ or $(1, q)$.

Let T be a tensor of the type $(0, q)$, which is defined as a q-linear form $T(X_1, X_2, \dots, X_q)$, where $X_1, X_2, \dots, X_q \in TV_n$.

In the space V_n we introduce an operator $R(X, Y) \circ T$ in the following way:

$$R(X, Y) \circ T(X_1, X_2, \dots, X_q) \stackrel{\text{def}}{=} \sum_{s=1}^{q} T(X_1, \dots, X_{s-1}, R(X, Y)X_s, X_{s+1}, \dots, X_q).$$

In the local transcription the tensor $R(X, Y) \circ T$ has a form

$$\sum_{s=1}^{q} T_{i_1 \dots i_{s-1}\alpha i_{s+1} \dots i_q} R^{\alpha}_{i_s jk}.$$

By the Ricci identity we have

$$T_{i_1 \dots i_q,[jk]} = \sum_{s=1}^{q} T_{i_1 \dots i_{s-1}\alpha i_{s+1} \dots i_q} R^{\alpha}_{i_s jk},$$

where $[jk]$ denotes the alternation of the tensor with respect to j and k.

If T is a tensor of the type $(0,0)$ (i.e. an invariant, which is a function or a scalar on V_n), then we put $R(X,Y)\circ T=0$, or locally $T_{,[jk]}=0$.

Similarly we can define an operator $R(X,Y)\circ T$ for a tensor T of the type $(1,q)$:

$$R(X,Y)\circ T(X_1,X_2,\dots,X_q)$$
$$\stackrel{\text{def}}{=}\sum_{s=1}^{q}T(X_1,\dots,X_{s-1},R(X,Y)X_s,X_{s+1},\dots,X_q)-R(X,Y)\big(T(X_1,\dots,X_q)\big).$$

The tensor $R(X,Y)\circ T$ has a local expression

$$\sum_{s=1}^{q}T^h_{i_1\dots i_{s-1}\alpha i_{s+1}\dots i_q}R^\alpha_{i_sjk}-T^\alpha_{i_1\dots i_q}\cdot R^h_{\alpha jk}.$$

By the Ricci identity we have

$$T^h_{i_1\dots i_q,[jk]}=\sum_{s=1}^{q}T^h_{i_1\dots i_{s-1}\alpha i_{s+1}\dots i_q}R^\alpha_{i_sjk}-T^\alpha_{i_1\dots i_q}\cdot R^h_{\alpha jk}.$$

Now we present Kowolik's theorems of [1] in a modified form which is more convenient for us. These theorems will be generalized in the next parts of our paper. First, recall notions used in the theorems.

Definition 3 A Riemannian space V_n is called *semisymmetric*, if

$$R(X,Y)\circ R=0\quad\forall X,Y\in TV_n. \tag{1}$$

We write (1) locally in the form $R^h_{ijk,[lm]}=0$ or

$$R^h_{\alpha jk}R^\alpha_{ilm}+R^h_{i\alpha k}R^\alpha_{jlm}+R^h_{ij\alpha}R^\alpha_{klm}-R^\alpha_{ijk}R^h_{\alpha lm}=0.$$

Definition 4 A Riemannian space V_n is called *Ricci semisymmetric*, if

$$R(X,Y)\circ Ric=0\quad\forall X,Y\in TV_n. \tag{2}$$

We write (2) locally

$$R_{\alpha j}R^\alpha_{ikl}+R_{i\alpha}R^\alpha_{jkl}=0\quad\text{or}\quad R_{ij,[kl]}=0.$$

Simply conformaly recurrent spaces (*s.c.r. spaces*) were defined by W. Roter [7]. These spaces are characterized by the following conditions:

The Riemannian space V_n is a *s.c.r.* space, if and only if:

1. $C_{hijk}\neq 0$, where C_{hijk} is a Weyl tensor of conformal curvature,
2. $C_{hijk,l}=\varphi_l C_{hijk}$,
3. a vector φ_k is locally gradient,
4. the Ricci tensor is a Codazzi tensor.

Remark 1 It holds that each *s.c.r.* space is semisymmetric.

Theorem 1 ([1]) *Let V_n $(n \geq 4)$ be a Ricci semisymmetric space with a harmonic Riemannian tensor. If there is a torse-forming vector field $\boldsymbol{\xi}$ in V_n, then $\boldsymbol{\xi}$ is either concircular or recurrent.*

Theorem 2 ([1]) *If there is a torse-forming vector field $\boldsymbol{\xi}$ in a s.c.r. space V_n $(n \neq 4)$, then $\boldsymbol{\xi}$ is recurrent.*

Let T be a tensor field of the type $(0, q)$ or $(1, q)$ and $\boldsymbol{\xi}$ be a vector field on V_n. By means of the operator $R(X, \boldsymbol{\xi}) \circ T$ let us define the basic notion of our paper:

Definition 5 The tensor field T is *semiconjugated* with the vector field $\boldsymbol{\xi}$, if

$$R(X, \boldsymbol{\xi}) \circ T = 0 \quad \text{for each } X \in TV_n. \tag{3}$$

In the local transcription (3) has the form

$$T^{\cdot}_{\dots,[lm]}\xi^m = 0, \tag{4}$$

where ξ^m are local components of $\boldsymbol{\xi}$.

2 Vector fields semiconjugated with torse-forming vector fields

In this section we will consider 1-covariant vector fields semiconjugated with a torse-forming vector field $\boldsymbol{\xi}$. Denote by $\xi(X)$ a linear form generated by $\boldsymbol{\xi}$, i.e. $\xi(X) \equiv g(X, \boldsymbol{\xi})$.

Theorem 3 *Let T $(\neq 0)$ be a 1-covariant vector field semiconjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$, which is not convergent. Then $\boldsymbol{\xi}$ is semitorse-forming and T is colinear with a form $\xi(X)$.*

Proof Assume that there is a non-zero vector field T and a non-isotropic non-convergent torse-forming vector field $\boldsymbol{\xi}$, which satisfy (4), i.e.

$$T_\alpha R^\alpha_{ij\beta}\xi^\beta = 0, \tag{5}$$

where T_i are local components of T and R^h_{ijk} are components of the Riemannian tensor R. According to [5] we can assume that $\boldsymbol{\xi}$ is normalized, i.e. $g(\boldsymbol{\xi}, \boldsymbol{\xi}) = e = \pm 1$, and the condition

$$\xi_\alpha R^\alpha_{ijk} = g_{ij}c_k - g_{ik}c_j + \xi_i a_{jk} \tag{6}$$

holds, where $a_{jk} \equiv -e\xi_{[j}\,\varrho_{,k]}$ and

$$c_k \equiv \varrho_{,k} + e\varrho^2\xi_k. \tag{7}$$

Since $\boldsymbol{\xi}$ is not convergent, we have $c_i \neq 0$.

Contracting (6) with $T^k \stackrel{\text{def}}{=} T_\alpha g^{\alpha k}$ and using (5) and properties of the Riemannian tensor we get

$$g_{ij} c_k T^k - T_i c_j + \xi_i a_{jk} T^k = 0. \tag{8}$$

If $c_k T^k \neq 0$, then (8) gives rank $\|g_{ij}\| \leq 2$. Since $n > 2$, we have $c_k T^k = 0$ and (8) leads to

$$-T_i c_j + \xi_i a_{jk} T^k = 0. \tag{9}$$

Since $c_j \neq 0$, the condition (9) implies

$$T_i = a\xi_i,$$

where a if a non-zero function.

Substituting $T_i = a\xi_i$ in (6) we see, that either $\boldsymbol{\xi}$ is semitorse-forming vector field or $T_i = 0$. This completes the proof of Theorem 3. □

3 Symmetric 2-covariant tensors semiconjugated with a torse-forming vector field

We will prove the following theorem:

Theorem 4 *Let $n > 2$ and let T $(\neq \gamma g)$ be a 2-covariant symmetric tensor field semiconjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$, which is not convergent. Then it holds that $\boldsymbol{\xi}$ is semitorse-forming in V_n and*

$$T(X, Y) = \gamma \cdot g(X, Y) + \psi \cdot \xi(X) \cdot \xi(Y) \quad \forall X, Y \in TV_n, \tag{10}$$

where γ, ψ are functions on V_n.

Proof Assume that there is a 2-covariant symmetric tensor field T on V_n, which is semiconjugated with a normalised torse-forming vector field $\boldsymbol{\xi}$, which is not convergent. It means that $\boldsymbol{\xi}$ satisfies (6) and $c_i \neq 0$.

Further we have:

$$R(X, \boldsymbol{\xi}) \circ T = 0 \quad \forall X \in TV_n,$$

i.e. locally

$$T_{\alpha j} R^\alpha_{il\beta} \xi^\beta + T_{i\alpha} R^\alpha_{jl\beta} \xi^\beta = 0. \tag{11}$$

If we substitute (6) in (11) and use properties of the Riemannian tensor we get after computation

$$g_{li} T_{\alpha j} c^\alpha - T_{lj} c_i + g_{lj} T_{i\alpha} c^\alpha - T_{il} c_j + \xi_l \omega_{ij} = 0, \tag{12}$$

where ω is some tensor of the type $(0, 2)$ and $c^i \equiv c_\alpha g^{\alpha i}$.

We will prove that

$$T_{\alpha i} c^\alpha = \gamma c_i. \tag{13}$$

Assume, that (13) does not hold. Then there exists a vector ε^i such that

$$c_\alpha \varepsilon^\alpha = 0 \quad \text{and} \quad T_{\alpha\beta}\varepsilon^\alpha c^\beta = 1. \tag{14}$$

Contract (12) with $\varepsilon^i\varepsilon^j$. Since $T_{ij} = T_{ji}$ and (14) holds, we get

$$\varepsilon_l = h\xi_l, \tag{15}$$

where $h \overset{\text{def}}{=} -\frac{1}{2}\omega_{\alpha\beta}\varepsilon^\alpha\varepsilon^\beta$.

If we contract (12) with ε^j, we obtain by means of (14) and (15)

$$g_{li} - T_{l\alpha}\varepsilon^\alpha c_i + \xi_l(hT_{i\alpha}c^\alpha + \omega_{i\beta}\varepsilon^\beta) = 0.$$

This implies that rank $\|g_{ij}\| \le 2$, which contradicts the assumption that (13) does not hold.

By (13) we extract the member $T_{\alpha i}c^\alpha$ in (12). After computation we obtain

$$F_{lj}c_i + F_{il}c_j + \xi_l\omega_{ij} = 0, \tag{16}$$

where

$$F_{ij} \overset{\text{def}}{=} T_{ij} - \gamma g_{ij}. \tag{17}$$

Since $c_i \neq 0$, then there exists φ^i such, that $c_\alpha\varphi^\alpha = 1$.

Contracting (16) with $\varphi^i\varphi^j$ we get $F_{l\alpha}\varphi^\alpha = f \cdot \xi_l$, where $f \overset{\text{def}}{=} -\frac{1}{2}\omega_{\alpha\beta}\varepsilon^\alpha\varepsilon^\beta$.

Similarly, if we contract (16) with φ^j, we get

$$F_{il} = \xi_l\chi_i\,, \tag{18}$$

where $\chi_i \overset{\text{def}}{=} -fc_i - \omega_{i\alpha}\varphi^\alpha$.

Since F_{ij} is a symmetric tensor, the equality (18) implies

$$F_{ij} = \psi \cdot \xi_i\xi_j. \tag{19}$$

By the assumption $F_{ij} \neq 0$, we have $\psi \neq 0$. Substituting (17) to (19) we see, that (10) is true. It remains to prove that the vector field $\boldsymbol{\xi}$ is semitorse-forming.

Therefore we covariantly derive the equality (19) by indices l and m, then we alternate it with respect to l and m and finally we contract it with ξ^m. Since

$$F_{ij,[lm]}\xi^m = 0 \quad \text{and} \quad \psi \neq 0,$$

we reach the formula

$$\xi_{i,[lm]}\xi^m \cdot \xi_j + \xi_i \cdot \xi_{j,[lm]}\xi^m = 0,$$

wherefrom it follows

$$\xi_{i,[lm]}\xi^m = 0.$$

This means that the vector field $\boldsymbol{\xi}$ is semitorse-forming. □

4 Antisymmetric 2-covariant tensors semiconjugated with a torse-forming vector field

The following theorem deals with antisymmetric tensor fields.

Theorem 5 *In a Riemannian space V_n $(n > 3)$ there is no non-zero 2-covariant antisymmetric tensor field T semiconjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$, which is not convergent.*

Proof Assume that there is a 2-covariant anti-symmetric tensor field T on V_n, which is semiconjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$, which is not convergent. It means, that $\boldsymbol{\xi}$ satisfies (6) and $c_i \neq 0$. Similarly as in the proof of Theorem 4 we get, that (11), (12) and (13) are true. Substituting (13) in (12) and using the antisymmetric property of T (i.e. $T_{ij} = -T_{ji}$), we get after computation

$$(T_{li} - \mu g_{li})c_j - (T_{lj} - \mu g_{lj})c_i - \xi_l \omega_{ij} = 0. \tag{20}$$

Since $c_j \neq 0$, then there exists φ^i, for which $\varphi^\alpha c_\alpha = 1$. Contracting (20) with φ^j we find

$$T_{li} - \mu g_{li} = \xi_l \eta_i + \chi_l c_i\,, \tag{21}$$

where η_i and χ_l are some covectors.

Symmetrising (21) we obtain

$$-2\mu g_{li} = \xi_l \eta_i + \xi_i \eta_l + \chi_l c_i + \chi_i c_l. \tag{22}$$

If $n > 4$, we deduce that $\mu = 0$.

Assume that $n = 4$ and $\mu \neq 0$. Then covectors ξ_i, c_i, η_i, χ_i must be linearly independent. Hence their coordinates in a given point x can be chosen in the following way:

$$\xi_i = \delta_i^1, \quad \eta_i = \delta_i^2, \quad c_i = \delta_i^3, \quad \chi_i = \delta_i^4.$$

Then

$$g_{ij} = -\frac{1}{2\mu}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

The inverse matrix g^{ij} has the form

$$g^{ij} = -2\mu \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

We can check that

$$g^{ij}\xi_i\xi_j = 0$$

holds, i.e. $\boldsymbol{\xi}$ is isotropic, a contradiction.

Thus for $n > 3$ the formula (22) implies, that $\mu = 0$. Therefore we can simplify (21) and (22) as follows:

$$T_{ij} = \xi_i \eta_j + \chi_i c_j$$

and

$$\xi_l \eta_i + \xi_i \eta_l + \chi_l c_i + \chi_i c_l = 0. \tag{23}$$

Vectors ξ_i and χ_i are not colinear. Otherwise it should be $T_{ij} = 0$. Therefore there is φ^i such that

$$\xi_\alpha \varphi^\alpha = 1 \quad \text{and} \quad \chi_\alpha \varphi^\alpha = 0.$$

Contracting (23) with $\varphi^i \varphi^l$ we find $\eta_\alpha \varphi^\alpha = 0$ and contracting (23) with φ^l we get $\eta_i = -c_\alpha \varphi^\alpha \cdot \chi_i$. Then (23) has a form

$$(c_i - c_\alpha \varphi^\alpha \xi_i)\chi_l + (c_l - c_\alpha \varphi^\alpha \xi_l)\chi_i = 0.$$

Since $\chi_l \neq 0$, we obtain

$$c_i = c_\alpha \varphi^\alpha \xi_i. \tag{24}$$

Using (7) and (24) we derive

$$\varrho_{,k} = (c_\alpha \varphi^\alpha - e\varrho^2)\xi_k.$$

Hence we have $\varrho = \varrho(\xi)$, where ξ is a scalar field satisfying $\xi_k = \partial_k \xi$. It means that $\boldsymbol{\xi}$ is concircular and, by [3], is convergent. □

5 Main results

By means of Theorem 4 (for symmetric tensors) and Theorem 5 (for antisymmetric tensors) we will prove the following assertion for arbitrary 2-covariant tensors.

Theorem 6 *Let $n > 3$ and let T $(\neq \gamma g)$ be a 2-covariant tensor field semi-conjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$, which is not convergent. Then it holds that $\boldsymbol{\xi}$ is semitorse-forming in V_n and*

$$T(X,Y) = \gamma \cdot g(X,Y) + \psi \cdot \xi(X) \cdot \xi(Y) \quad \forall X, Y \in TV_n,$$

where γ, ψ are functions on V_n.

Proof Assume that there is a 2-covariant tensor field T on V_n, which is semiconjugated with a normalised torse-forming vector field $\boldsymbol{\xi}$, which is not convergent.

Tensor T can be uniquely expressed in the form $T = U + V$, where U is a symmetric part and V is an antisymmetric part of T. It holds

$$U(X,Y) = \frac{1}{2}\big(T(X,Y) + T(Y,X)\big)$$

and
$$V(X,Y) = \frac{1}{2}\big(T(X,Y) - T(Y,X)\big)$$
for any vector fields $X, Y \in TV_n$. Therefore U and V are also semiconjugated with $\boldsymbol{\xi}$. Theorem 5 implies, that $V = 0$. Hence $T \equiv U$ and so T is symmetric and the assertion of Theorem 6 follows from Theorem 4. □

Now we will prove theorems for Riemannian spaces having Riemannian and Ricci tensors semiconjugated with a torse-forming vector field. These theorems generalize Kowolik's results in [1].

Theorem 7 *Let $n > 2$ and let V_n be a non-Einsteinian Riemannian space, where the Ricci tensor is semiconjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$. Then $\boldsymbol{\xi}$ is convergent.*

Proof Assume that the Ricci tensor *Ric* is semiconjugated with a torse-forming vector field $\boldsymbol{\xi}$.

Since *Ric* is a symmetric tensor, we get by Theorem 4
$$Ric(X,Y) = \gamma g(X,Y) + \psi \cdot \xi(X) \cdot \xi(Y) \quad \forall X, Y \in TV_n, \tag{25}$$
where $\xi(X) \overset{\text{def}}{=} g(X, \boldsymbol{\xi})$ and ψ is a function on V_n.

Semitorse-forming fields fulfil $R^h_{\alpha j \beta}\xi^\alpha \xi^\beta = 0$. Contracting it with respect to h and j we obtain $R_{\alpha\beta}\xi^\alpha\xi^\beta = 0$, which can be written in the form
$$Ric(\boldsymbol{\xi}, \boldsymbol{\xi}) = 0.$$

Let us put $X = \boldsymbol{\xi}$ a $Y = \boldsymbol{\xi}$ in (25). Since we can assume that $\boldsymbol{\xi}$ is normalized, i.e. $g(\boldsymbol{\xi},\boldsymbol{\xi}) \equiv \xi(\boldsymbol{\xi}) = e = \pm 1$, we get $\psi = -e\gamma$ and so the formula (25) has the form
$$Ric(X,Y) = \gamma \cdot \big(g(X,Y) - e\xi(X) \cdot \xi(Y)\big) \quad \forall X, Y \in TV_n. \tag{26}$$

Substituting $Y = \boldsymbol{\xi}$ in (26) we obtain
$$Ric(X, \boldsymbol{\xi}) = 0 \quad \forall X \in TV_n.$$
It means that $\boldsymbol{\xi}$ is an eigenvector of the Ricci tensor corresponding to the zero eigenvalue. Therefore $\boldsymbol{\xi}$ is convergent. □

Theorem 8 *Let $n > 2$ and let V_n be a Riemannian space with a non-constant curvature, where the Riemannian tensor is semiconjugated with a non-isotropic torse-forming vector field $\boldsymbol{\xi}$. Then $\boldsymbol{\xi}$ is convergent.*

Proof Assume that a Riemannian space V_n with a non-constant curvature has the Riemannian tensor which is semiconjugated with a torse-forming vector field $\boldsymbol{\xi}$ which is not convergent. Then V_n has the Ricci tensor which is also semiconjugated with $\boldsymbol{\xi}$. Therefore by Theorem 7 the space V_n has to be an Einsteinian space. We can easily see that $\boldsymbol{\xi}$ is concircular.

Then, according to the result of [4] the Riemannian tensor has the form

$$R_{hijk} = K(g_{hj}g_{ik} - g_{hk}g_{ij}),$$

which means that V_n has a constant curvature, a contradiction. We have proved that $\boldsymbol{\xi}$ has to be convergent. □

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