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ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF THE DIFFERENTIAL EQUATION OF THE FOURTH ORDER II

JOZEF MIKLO

In paper [7] the asymptotic behaviour of solutions of the linear differential equation of the fourth order of the form

$$y^{(iv)} + p(t)y'' + q(t)y' - (-1)^m r(t)y = 0, \quad m = 1,2 \tag{1}$$

was investigated, where the functions $p(t)$, $q(t)$ and $r(t)$ were supposed continuous and continuously differentiable to the order which stands in the Theorems and $r(t) > 0$ on the interval $[a, \infty)$.

In the paper presented an asymptotic behaviour of solutions of the equation of the form

$$y^{(iv)} + p(t)y'' - (-1)^m q(t)y' + r(t)y = 0, \quad m = 1,2 \tag{2}$$

is studied, where the functions $p(t)$, $q(t)$ and $r(t)$ have the same properties as in the equation (1) but $q(t) > 0$ is supposed instead of $r(t) > 0$.

Eight new asymptotic formulae for the linear differential equation of the fourth order are shown. The results in this paper generalize the results in [8]. Theorem 8.1 in [1], p. 92 (in [7] as Theorem I) and Corollary in [2] (in [7] as Theorem II) will be apllied in this paper.

The equation (2) is equivalent to the system of linear differential equations of the first order

$$\mathbf{z}'(t) = \mathbf{A}(t)\,\mathbf{z}(t), \tag{3}$$

where

$$\mathbf{A}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -r(t) & (-1)^m q(t) & -p(t) & 0 \end{pmatrix}$$

and $\mathbf{z}(t) = (y(t), y'(t), y''(t), y'''(t))^T$.

Let $\mathbf{T}(t) = \operatorname{diag}[q(t), q^{2/3}(t), q^{1/3}(t), 1]$ and let

$$\mathbf{z}(t) = \mathbf{T}^{-1}(t)\,\mathbf{w}(t).$$

If $z(t)$ is substituted in (3), then the system (3) has the form

(4) $$\mathbf{w}'(t) = [\mathbf{A}_0 q^{1/3}(t) + \mathbf{A}_1 r(t) q^{-1}(t) + \mathbf{A}_2 p(t) q^{-1/3}(t) + \mathbf{A}_3 q'(t) q^{-1}(t)]\, \mathbf{w}(t),$$

where $\mathbf{A}_3 = \operatorname{diag}[1, 2/3, 1/3, 0]$, $\mathbf{A}_0 = (a_{ij})$, $\mathbf{A}_1 = (b_{ij})$ and $\mathbf{A}_2 = (c_{ij})$ are matrices of the fourth degree such that $a_{12} = a_{23} = a_{34} = 1$, $a_{42} = (-1)^m$ and all the others $a_{ij} = 0$; $b_{ij} = 0$ for $i \neq 4, j \neq 1$, $b_{41} = -1$; $c_{ij} = 0$ for $i \neq 4, j \neq 3$ and $c_{43} = -1$.

Let $\int_a^\infty q^{1/3}(t)\, dt = \infty$, then the function $s = \omega(t) = \int_a^t q^{1/3}(u)\, du$ is defined on the interval $[a, \infty)$ and has an inverse function $t = \alpha(s)$ defined on the interval $[0, \infty)$. By substituting $t = \alpha(s)$ the system (4) has the form

(5) $$\mathbf{x}'(s) = [\mathbf{A}_0 + \mathbf{A}_1 f(s) + \mathbf{A}_2 g(s) + \mathbf{A}_3 h(s)]\, \mathbf{x}(s),$$

where

$$\mathbf{x}(s) = \mathbf{w}(\alpha(s)), f(s) = r(\alpha(s))\, q^{-4/3}(\alpha(s)),$$

$$g(s) = p(\alpha(s))\, q^{-2/3}(\alpha(s)),\ h(s) = q'(\alpha(s))\, q^{-4/3}(\alpha(s)).$$

In order to apply Theorem I (see [1], p. 92 or [7]) the system (5) will be considered in the form

(6) $$\mathbf{x}'(s) = (\mathbf{A}_0 + \mathbf{V}(s) + \mathbf{R}(s))\, \mathbf{x}(s).$$

There are the following alternatives

(A1) $\mathbf{V}(s) = \mathbf{A}_1 f(s) + \mathbf{A}_2 g(s) + \mathbf{A}_3 h(s)$ and $\mathbf{R}(s) = 0$,

(A2) $\mathbf{V}(s) = \mathbf{A}_1 f(s) + \mathbf{A}_2 g(s)$ and $\mathbf{R}(s) = \mathbf{A}_3 h(s)$,

(A3) $\mathbf{V}(s) = \mathbf{A}_1 f(s) + \mathbf{A}_3 h(s)$ and $\mathbf{R}(s) = \mathbf{A}_2 q(s)$,

(A4) $\mathbf{V}(s) = \mathbf{A}_1 f(s)$ and $\mathbf{R}(s) = \mathbf{A}_2 g(s) + \mathbf{A}_3 h(s)$,

(A5) $\mathbf{V}(s) = \mathbf{A}_2 g(s) + \mathbf{A}_3 h(s)$ and $\mathbf{R}(s) = \mathbf{A}_1 f(s)$,

(A6) $\mathbf{V}(s) = \mathbf{A}_2 g(s)$ and $\mathbf{R}(s) = \mathbf{A}_1 f(s) + \mathbf{A}_3 h(s)$,

(A7) $\mathbf{V}(s) = \mathbf{A}_3 h(s)$ and $\mathbf{R}(s) = \mathbf{A}_1 f(s) + \mathbf{A}_2 g(s)$,

(A8) $\mathbf{V}(s) = 0$ and $\mathbf{R}(s) = \mathbf{A}_1 f(s) + \mathbf{A}_2 g(s) + \mathbf{A}_3 h(s)$.

The following designations will be used in Theorems of this paper

$$E(t, t_0) = \exp\left[-(-1)^m \int_{t_0}^t r(u)\, q^{-1}(u)\, du\right],$$

$$E_{1k}(t, t_0) = \exp\left[-\int_{t_0}^t \left[\mu_k q^{1/3}(u) - \frac{(-1)^m}{3}(\mu_k^2 p(u)\, q^{-1/3}(u) + r(u)\, q^{-1}(u))\right] du\right],$$

$$E_{2k}(t,t_0)=\exp\left[-\int_{t_0}^{t}\left(\mu_k q^{1/3}(u)-\frac{(-1)^m}{3}r(u)q^{-1}(u)\right)\mathrm{d}u\right],$$

$$E_{3k}(t,t_0)=\exp\left[-\int_{t_0}^{t}\left(\mu_k q^{1/3}(u)-\frac{(-1)^m}{3}\mu_k^2 p(u)q^{-1/3}(u)\right)\mathrm{d}u\right],$$

$$E_{4k}(t,t_0)=\exp\left[-\int_{t_0}^{t}\mu_k q^{1/3}(u)\,\mathrm{d}u\right],$$

where μ_k, $k=1,2,3,4$ are the roots of the characteristic equation $\mu^4-(-1)^m\mu=0$, $m=1,2$ of the matrix $\mathbf{A}_0$ and $p_k=(1,\mu_k,\mu_k^2,\mu_k^3)^T$ are the characteristic vectors of the matrix $\mathbf{A}_0$.

The symbol $\mathscr{L}[a,\infty)$ will refer to the set of all complexvalued functions which are Lebesgue integrable on the interval $[a,\infty)$.

Applying Theorem I to the system (6) eight asymptotic formulae for the solutions of the equation (2) will be obtained.

Theorem 1. Let $q''(t)q^{-4/3}(t)$, $p'(t)q^{-2/3}(t)$, $p^2(t)q^{-1}(t)$, $r'(t)q^{-4/3}(t)$ and $r(t)q^{-7/3}(t)$ *be in* $\mathscr{L}[a,\infty)$. *Then there is a fundamental system of solutions* $y_k(t)$, $k=1,2,3,4$ *of the equation* (2) *and a number* $t_0\geqq a$ *such that*

$$\text{(F1)}\quad \begin{aligned}&\lim_{t\to\infty}y_1(t)E(t,t_0)=1,\quad \lim_{t\to\infty}y_1^{(j)}(t)q^{-j/3}(t)E(t,t_0)=0,\quad j=1,2,3\\&\lim_{t\to\infty}y_k^{(j)}(t)q^{(2-j)/3}(t)E_{1k}(t,t_0)=\mu_k^j,\quad k=2,3,4,\quad j=0,1,2,3.\end{aligned}$$

If in addition it is supposed that $q'(t)q^{-1}(t)$ *is in* $\mathscr{L}[a,\infty)$, *then there is a fundamental system of solutions* $y_k(t)$, $k=1,2,3,4$ *of the equation* (2) *and a number* $t_0\geqq a$ *such that*

$$\text{(F2)}\quad \begin{aligned}&\lim_{t\to\infty}y_1(t)q(t)E(t,t_0)=1,\quad \lim_{t\to\infty}y_1^{(j)}(t)q^{(3-j)/3}(t)E(t,t_0)=0,\quad j=1,2,3,\\&\lim_{t\to\infty}y_k^{(j)}(t)q^{(3-j)/3}(t)E_{1k}(t,t_0)=\mu_k^j,\quad k=2,3,4,\quad j=0,1,2,3.\end{aligned}$$

Theorem 2. Let $q''(t)q^{-4/3}(t)$, $r'(t)q^{-4/3}(t)$, $r^2(t)q^{-7/3}(t)$ *and* $p(t)q^{-1/3}(t)$ *be in* $\mathscr{L}[a,\infty)$. *Then there is a fundamental system of solutions* $y_k(t)$, $k=1,2,3,4$ *of the equation* (2) *and a number* $t_0\geqq a$ *such that*

$$\text{(F3)}\quad \begin{aligned}&\lim_{t\to\infty}y_1(t)E(t,t_0)=1,\quad \lim_{t\to\infty}y_1^{(j)}(t)q^{-j/3}(t)E(t,t_0)=0,\quad j=1,2,3,\\&\lim_{t\to\infty}y_k^{(j)}(t)q^{(2-j)/3}(t)E_{2k}(t,t_0)=\mu_k^j,\quad k=2,3,4,\quad j=0,1,2,3,\end{aligned}$$

If in addition it is supposed that $q'(t)q^{-1}(t)$ *is in* $\mathscr{L}[a,\infty)$, *then there is a* *fundamental system of solutions* $y_k(t)$, $k = 1, 2, 3, 4$ *of the equation* (2) *and a number* $t_0 \geqq a$ *such that*

$$\text{(F4)}\quad \begin{aligned} &\lim_{t\to\infty} y_1(t)\, q(t)\, E(t, t_0) = 1, \quad \lim_{t\to\infty} y_1(t)\, q^{(3-j)/3}(t)\, E(t, t_0) = 0, \quad j = 1, 2, 3, \\ &\lim_{t\to\infty} y_k^{(j)} q^{(3-j)/3}(t)\, E_{2k}(t, t_0) = \mu_k^j, \quad k = 2, 3, 4, \quad j = 0, 1, 2, 3. \end{aligned}$$

Theorem 3. Let $q''(t)\, q^{-4/3}(t)$, $p'(t)\, q^{-2/3}(t)$, $p^2(t)\, q^{-1}(t)$ *and* $r(t)\, q^{-1}(t)$ *be in* $\mathscr{L}[a, \infty)$. *Then there is a fundamental system of solutions* $y_k(t)$, $k = 1, 2, 3, 4$ *of the equation* (2) *and a number* $t_0 \geqq a$ *such that*

$$\text{(F5)}\quad \begin{aligned} &\lim_{t\to\infty} y_1(t) = 1, \quad \lim_{t\to\infty} y_1(t)\, q^{-j/3}(t) = 0, \quad j = 1, 2, 3, \\ &\lim_{t\to\infty} y_k^{(j)}(t)\, q^{(2-j)/3}(t)\, E_{3k}(t, t_0) = \mu_k^j, \quad k = 2, 3, 4, \quad j = 0, 1, 2, 3. \end{aligned}$$

If in addition it is supposed that $q'(t)\, q^{-1}(t)$ *is in* $\mathscr{L}[a, \infty)$, *then there is a fundamental system of sulutions* $y_k(t)$, $k = 1, 2, 3, 4$ *of the equation* (2) *and a number* $t_0 \geqq a$ *such that*

$$\text{(F6)}\quad \begin{aligned} &\lim_{t\to\infty} y_1(t)\, q(t) = 1, \quad \lim_{t\to\infty} y_1(t)\, q^{(3-j)/3}(t) = 0, \quad j = 1, 2, 3, \\ &\lim_{t\to\infty} y_k^{(j)}(t)\, q^{(3-j)/3}(t)\, E_{3k}(t, t_0) = \mu_k^j, \quad k = 2, 3, 4, \quad j = 0, 1, 2, 3. \end{aligned}$$

Theorem 4. Let $q''(t)\, q^{-4/3}(t)$, $r(t)\, q^{-1}(t)$ and $p(t)\, q^{-1/3}(t)$ *be in* $\mathscr{L}[a, \infty)$. *Then there is a fundamental system of solutions* $y_k(t)$, $k = 1, 2, 3, 4$ *of the equation* (2) *and a number* $t_0 \geqq a$ *such that*

$$\text{(F7)}\quad \begin{aligned} &\lim_{t\to\infty} y_1(t) = 1, \quad \lim_{t\to\infty} y_1^{(j)}(t)\, q^{-j/3}(t) = 0, \quad j = 1, 2, 3, \\ &\lim_{t\to\infty} y_k^{(j)}(t)\, q^{(2-j)/3}(t)\, E_{4k}(t, t_0) = \mu_k^j, \quad k = 2, 3, 4, \quad j = 0, 1, 2, 3. \end{aligned}$$

Theorem 5. *Let* $q'(t)\, q^{-1}(t)$, $r(t)\, q^{-1}(t)$ and $p(t)\, q^{-1/3}(t)$ *be in* $\mathscr{L}[a, \infty)$ *and*

$$\int_a^\infty q^{1/3}(t)\, \mathrm{d}t = \infty.$$

Then there is a fundamental system of solutions $y_k(t)$, $k = 1, 2, 3, 4$ *of the equation* (2) *and a number* $t_0 \geqq a$ *such that*

$$\text{(F8)}\quad \begin{aligned} &\lim_{t\to\infty} y_1(t)\, q(t) = 1, \quad \lim_{t\to\infty} y_1^{(j)}(t)\, q^{(3-j)/3}(t) = 0, \quad j = 1, 2, 3, \\ &\lim_{t\to\infty} y_k^{(j)}(t)\, q^{(3-j)/3}(t)\, E_{4k}(t, t_0) = \mu_k^j, \quad k = 2, 3, 4, \quad j = 0, 1, 2, 3. \end{aligned}$$

Proof of Theorem 1. In this case all assumptions of Theorem I are satisfied. (The proof of this fact is analogous to the proof of Theorem 1 in [7]). Then there are four linearly independent solutions $\boldsymbol{x}_k(s)$ of the system (6) in the case of the alternative (A1) and a number $s_0 \geqq 0$ such that

$$\lim_{s \to \infty} \boldsymbol{x}_k(s) \exp\left[-\int_{s_0}^{s} \lambda_k(u)\,\mathrm{d}u\right] = \boldsymbol{p}_k,$$

where $\lambda_k(s)$, $k = 1, 2, 3, 4$ are the roots of the characteristic equation

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{7}$$

of the matrix $\mathbf{A}_0 + \mathbf{V}(s)$, where

$$a_1 = -2h(s), \quad a_2 = \frac{11}{9}h^2(s) + g(s),$$

$$a_3 = -\frac{2}{9}h^3(s) - \frac{5}{3}h(s)g(s) - (-1)^m,$$

$$a_4 = \frac{2}{3}h^2(s)g(s) - (-1)^m h(s) + f(s), \quad m = 1, 2.$$

Similarly as in [7] it can be proved that the roots $\lambda_k(s)$ of the equation (7) can be expressed in the form

$$\lambda_1(s) = h(s) + (-1)^m f(s) + \gamma_1(s),$$

$$\lambda_k(s) = \mu_k + \frac{1}{3}[h(s) - (-1)^m(\mu_k^2 g(s) + f(s))] + \gamma_k(s),$$

$k = 2, 3, 4$, where $f(s) \to 0$, $g(s) \to 0$, $h(s) \to 0$ and $\gamma_k(s) \to 0$ as $s \to \infty$ and $\gamma_k(s)$ is in $\mathscr{L}[0, \infty)$. Then

$$\begin{aligned}
&\lim_{s \to \infty} \boldsymbol{x}_1(s)\exp\left[-\int_{s_0}^{s}(h(u) + (-1)^m f(u) + \gamma_1(u))\,\mathrm{d}u\right] = \boldsymbol{p}_1\\
&\lim_{s \to \infty} \boldsymbol{x}_k(s)\exp\left[-\int_{s_0}^{s}\left[\mu_k + \frac{1}{3}(h(u) - (-1)^m(\mu_k^2 g(u) + f(u))) + \right.\right.\\
&\qquad\qquad \left.\left. + \gamma_k(u)\right]\mathrm{d}u\right] = \boldsymbol{p}_k, \quad k = 2, 3, 4,
\end{aligned} \tag{8}$$

Denoting $\exp\left[\int_{s_0}^{\infty} \gamma_k(s)\,\mathrm{d}s\right] = B_k$, $k = 1, 2, 3, 4$ and putting $\alpha(u) = v$, i.e., $u = \omega(v)$ and $\mathrm{d}u = q^{1/3}(v)\,\mathrm{d}v$, $u \in [s_0, s]$, $v \in [t_0, t]$, (8) may be written as

$$\lim_{t\to\infty} \boldsymbol{w}_1(t)\, q^{-1}(t)\, E(t, t_0) = \boldsymbol{p}_1 B_1 q^{-1}(t_0),$$

$$\lim_{t\to\infty} \boldsymbol{w}_k(t)\, q^{-1/3}(t)\, E_{1k}(t, t_0) = \boldsymbol{p}_k B_k q^{-1/3}(t_0), \quad k = 2, 3, 4,$$

where the functions $\boldsymbol{w}_k(t)$, $k = 1, 2, 3, 4$ are solutions of the system (4).

Since $\boldsymbol{w}(t) = \mathbf{T}(t)\, \boldsymbol{z}(t)$ and the system (3) is linear there are linearly independent solutions $\boldsymbol{z}_k(t)$, $k = 1, 2, 3, 4$ of (3) such that

$$\begin{aligned} &\lim_{t\to\infty} \mathbf{T}(t)\, \boldsymbol{z}_1(t)\, q^{-1}(t)\, E(t, t_0) = \boldsymbol{p}_1 \\ &\lim_{t\to\infty} \mathbf{T}(t)\, \boldsymbol{z}_k(t)\, q^{-1/3}(t)\, E_{1k}(t, t_0) = \boldsymbol{p}_k, \quad k = 2, 3, 4. \end{aligned} \tag{9}$$

Substituting $\mathbf{T}(t) = \operatorname{diag}(q(t), q^{2/3}(t), q^{1/3}(t), 1)$ and

$$\boldsymbol{z}_k(t) = (y_k(t), y_k'(t), y_k''(t), y_k'''(t))^T, \quad k = 1, 2, 3, 4$$

in (9), becomes

$$\begin{aligned} &\lim_{t\to\infty} \operatorname{diag}(y_1(t), y_1'(t)\, q^{-1/3}(t), y_1''(t)\, q^{-2/3}(t), y_1'''(t)\, q^{-1}(t))\, E(t, t_0) = \\ &= (1, 0, 0, 0)^T \\ &\lim_{t\to\infty} \operatorname{diag}(y_k(t)\, q^{2/3}(t), y_k'(t)\, q^{1/3}(t), y_k''(t), y_k'''(t)\, q^{-1/3}(t))\, E_{1k}(t, t_0) = \\ &= (1, \mu_k, \mu_k^2, \mu_k^3)^T, \quad k = 2, 3, 4. \end{aligned} \tag{10}$$

Then the formula (F1) follows directly from (10). Therefore the first part of Theorem 1 is proved.

The formulae (F2)—(F8) may be proved analogously.

Corollary. The formulae (F1)—(F8) imply the corresponding formulae (F′1)—(F′8) for the general solution of the equation (2):

$$\text{(F}'1) \qquad y = \left[c_1 E^{-1}(t, t_0) + q^{-2/3}(t) \sum_{k=2}^{4} c_k \mu_k E_{1k}^{-1}(t, t_0)\right](1 + o(1))$$

$$\text{(F}'2) \qquad y = q^{-1}(t)\left[c_1 E^{-1}(t, t_0) + \sum_{k=2}^{4} c_k \mu_k E_{1k}^{-1}(t, t_0)\right](1 + o(1)),$$

$$\text{(F}'3) \qquad y = \left[c_1 E^{-1}(t, t_0) + q^{-2/3}(t) \sum_{k=2}^{4} c_k \mu_k E_{2k}^{-1}(t, t_0)\right](1 + o(1)),$$

$$\text{(F}'4) \qquad y = q^{-1}(t)\left[c_1 E^{-1}(t, t_0) + \sum_{k=2}^{4} c_k \mu_k E_{2k}^{-1}(t, t_0)\right](1 + o(1)),$$

$$\text{(F}'5) \qquad y = \left[c_1 + q^{-2/3}(t) \sum_{k=2}^{4} c_k \mu_k E_{3k}^{-1}(t, t_0)\right](1 + o(1)),$$

$$\text{(F}'6) \qquad y = q^{-1}(t)\left[c_1 + \sum_{k=2}^{4} c_k\mu_k E_{3k}^{-1}(t, t_0)\right](1 + o(1)),$$

$$\text{(F}'7) \qquad y = \left[c_1 + q^{-2/3}(t) \sum_{k=2}^{4} c_k\mu_k E_{4k}^{-1}(t, t_0)\right](1 + o(1)),$$

$$\text{(F}'8) \qquad y = q^{-1}(t)\left[c_1 + \sum_{k=2}^{4} c_k\mu_k E_{4k}^{-1}(t, t_0)\right](1 + o(1))$$

where c_1, c_2, c_3, c_4 are arbitrary numbers and the symbol $o(1)$ denotes a function which converges to zero as $t \to \infty$.

Remark. The equation $y^{(iv)} + a^3y' = 0$, $a > 0$ satisfies the hypothesis of Theorems 1—5, thus from each formula (F′1)—(F′8) it follows that the general solution of this equation is of the form

$$y = [c_1 + c_2e^{\pm at} + e^{+at/2}(c_3 \cos(a\sqrt{3}t/2) + c_4 \sin(a\sqrt{3}t/2))](1 + o(1))$$

This equation has constant coefficients and therefore its general solution is

$$y = c_1 + c_2e^{\pm at} + e^{\mp at/2}(c_3 \cos(a\sqrt{3}t/2) + c_4 \sin(a\sqrt{3}t/2))$$

and so $o(1) \equiv 0$.

Example. If $p(t)q^{-1/3}(t)$ and $r(t)q^{-1}(t)$ are in $\mathscr{L}[a, \infty)$, $a > 0$, where $q(t) = \left(\frac{2t}{t+1}\right)^3$, then the equation

$$y^{(iv)} + p(t)y'' - q(t)y' + r(t)y = 0$$

satisfies the assumptions of Theorem 5 and therefore its general solution has the form

$$y = \left(\frac{t+1}{2t}\right)^3\left[c_1 + c_2\frac{e^{2t}}{(t+1)^2} + (t+1)e^{-t}(c_3 \cos 3(t - \ln(t+1)) + \right.$$
$$\left. + c_4 \sin 3(t - \ln(t+1)))\right](1 + o(1)),$$

where c_1, c_2, c_3 and c_4 are arbitrary numbers.

From this example it may be seen that the coefficients do not satisfy the assumptions of theorems in [3], [4] and therefore this paper gives new results on the asymptotic behaviour of solutions of the linear differential equation of the fourth order.

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АСИМПТОТИЧЕСКИЕ ПОВЕДЕНИЯ РЕШЕНИЙ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ЧЕТВЕРТОГО ПОРЯДКА II

Jozef Miklo

Резюме

В работе рассметриваются асимптотические поведения решений уравнения (2) при $t \to \infty$, если несобственные интегралы от некоторых дробей функций p, q и r являются конечными.