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GENERALIZED TOPOLOGICAL SPACES

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The properties of structures defined by a given set X and a relation, respectively relations defined on a class of subsets of X and satisfying some conditions are often studied. Such structures are given for example in [1], [2], [3], [5], [7] and [8]. The best known structures of such type are topological spaces defined by a closure operation [6].

In the present paper we introduce a new class of spaces, called r-spaces, as a generalization of topological spaces.

We shall use the notations from [4] and 2^X will denote the class of all subsets of X. The notation $A \subseteq B$ means that A is a subset of B and if A is a proper subset of B we write $A \subset B$. Specific terms will be explained when used for the first time.

Let X be a nonempty set and ϱ be a relation on 2^X. Let us consider the following properties of ϱ:

R_1) for each subset A of X there is a subset B of X such that $A\varrho B$
R_2) $\emptyset\varrho\emptyset$
R_3) if $A\varrho B$, then $A \subseteq B$
R_4) if $A\varrho B$, then $B\varrho B$
R_5) if $A \subseteq B$ and $B\varrho B$, then there is a subset C of X such that $A\varrho C$ and $C \subset B$
R_6) if $A\varrho B$, then there is no subset C of X such that $A \subseteq C \subseteq B$ and $C\varrho C$.

Remark 1. It is easy to prove that the properties R_1—R_6 are independent.

Remark 2. Let $(X, \mathcal{T})$ be a topological space, where $\mathcal{T}$ is the class of closed sets. Let us define a relation ϱ on 2^X as follows:

$$A\varrho B \text{ iff } B \text{ is the closure of } A.$$

It is clear that ϱ satisfies R_1—R_6.

Definiction 1. *The relation ϱ with the properties R_1—R_6 will be called a relation of closure on 2^X.*

The pair (X, ϱ) is called an r-space if X is a nonempty set and ϱ is a relation of closure on 2^X.

Let (X, ϱ) be an r-space. If for the subsets A, B of X we have $A\varrho B$, then we say that B is a closure of A. A set $A \subseteq X$ satisfying $A\varrho A$ will be called a closed set. The complement of a closed set will be called an open set. A set $A \subseteq X$ is said to be an interior of $B \subseteq X$ if $X-A$ is a closure of $X-B$. The relation σ defined by

$$A \sigma B \quad \text{iff} \quad (X-B)\varrho(X-A)$$

is called the relation of the interior relative to ϱ.

From Remark 2 we can see that each topological space is an r-space. Unlike topological spaces in an r-space a set may have more than one closure — see the next example.

Example 1. Let $(X, \mathcal{T}_1)$ and $(X, \mathcal{T}_2)$ be two topological spaces, where $\mathcal{T}_1$ and $\mathcal{T}_2$ are classes of closed sets in $(X, \mathcal{T}_1)$ and $(X, \mathcal{T}_2)$, respectively. Let $A \subseteq X$. Let B be the closure of A in $(X, \mathcal{T}_1)$ and let C be the closure of A in $(X, \mathcal{T}_2)$. Define a relation ϱ on 2^X as follows

a) if $B \subseteq C$, then $A\varrho B$
b) if $C \subseteq B$, then $A\varrho C$
c) if $B \not\subseteq C$ and $C \not\subseteq B$, then $A\varrho B$ and $A\varrho C$.

The relation ϱ satisfies the properties R_1—R_6. It is clear that in the case c) the set A has two distinct closures B and C.

In what follows we shall give another characterizations of r-spaces.

Since the notion of the relation σ of the interior relative to ϱ is dual to the relation ϱ of the closure, from the properties R_1—R_6 it follows:

Theorem 1. *Let (X, ϱ) be an r-space. Let σ be the relation of the interior relative to ϱ. Then ϱ satisfies the following conditions*

K_1) *for each subset A of X there is a subset B of X such that $B\sigma A$*
K_2) $X\sigma X$
K_3) *if $B\sigma A$, then $B \subseteq A$*
K_4) *if $B\sigma A$, then $B\sigma B$*
K_5) *if $B \subseteq A$ and $B\sigma B$, then there is a subset C of X such that $C\sigma A$ and $B \subseteq C$*
K_6) *if $B\sigma A$, then there is no subset C of X such that $B \subset C \subseteq A$ and $C\sigma C$.*

Theorem 2. *Let σ be a relation on 2^X satisfying conditions K_1—K_6. Let us define a relation ϱ on 2^X as follows*

$$A\varrho B \quad \text{iff} \quad (X-B)\sigma(X-A).$$

Then ϱ is a relation of closure on 2^X, (X, ϱ) is an r-space and σ is the relation of the interior relative to ϱ.

Let $\mathcal{F}$ be a nonempty class of subsets of a set X, let $A \subseteq X$ and let $x \in X$. Throughout this paper the symbols ${}_A\mathcal{F}$, ${}^A\mathcal{F}$ and $\mathcal{F}(x)$ denote

(1) $${}_A\mathcal{F} = \{B \in \mathcal{F} : A \subseteq B\},$$
(2) $${}^A\mathcal{F} = \{B \in \mathcal{F} : B \subseteq A\},$$
(3) $$\mathcal{F}(x) = \{B \in \mathcal{F} : x \in B\}.$$

Theorem 3. *Let (X, ϱ) be an r-space. Then the class of closed subsets of X, i.e., the class $\mathcal{T}=\{A \subseteq X: A\varrho A\}$ satisfies the following conditions:*

Ω_1: $\emptyset, X \in \mathcal{T}$

Ω_2: *for every $A \subseteq X$ and $B \in {}_A\mathcal{T}$ there is a minimal element C of ${}_A\mathcal{T}$ such that $A \subseteq C \subseteq B$.*

Moreover, $A\varrho B$ iff B is a minimal element of the class ${}_A\mathcal{T}$.

Proof. From the property R_2 it follows that $\emptyset \in \mathcal{T}$ and from the properties R_1 and R_3 we have $X \in \mathcal{T}$. Let $A \subseteq X$. According to R_1, R_3 and R_4 the class ${}_A\mathcal{T}$ is nonempty. If $B \in {}_A\mathcal{T}$ then by R_5 there is a subset C of X such that $A\varrho C$ and $C \subseteq B$. By R_3 $A\varrho C$ implies $A \subseteq C \subseteq B$. $C \in \mathcal{T}$ according to R_4, hence $C \in {}_A\mathcal{T}$ and by R_6 the set C is a minimal element in ${}_A\mathcal{T}$. To show that $A\varrho B$ iff B is a minimal element of the class ${}_A\mathcal{T}$ suppose first $A\varrho B$. Then by R_4 $B \in \mathcal{T}$ and from R_6 it follows that B is a minimal element of ${}_A\mathcal{T}$. To prove the converse suppose that B is a minimal element of ${}_A\mathcal{T}$. Since $B \in {}_A\mathcal{T}$, then $B \in \mathcal{T}$, i.e. $B\varrho B$ and $A \subseteq B$. From R_5 it follows that there is a subset C of X such that $A\varrho C$ and $C \subseteq B$. From $A\varrho C$ we have $C \in {}_A\mathcal{T}$ by R_3 and R_4. The minimality of B implies $C=B$. Hence $A\varrho B$.

Theorem 4. *Let X be a nonempty se and $\mathcal{T}$ be a class of subsets of X satisfying Ω_1 and Ω_2. Let us define a relation ϱ on 2^X as follows*

(4) *$A\varrho B$ iff B is a minimal element of the class ${}_A\mathcal{T}$. Then ϱ satisfies R_1—R_6 and $\mathcal{T}$ is precisely the class of all closed subsets of the r-space (X, ϱ), i.e., $\mathcal{T}=\{A \subseteq X: A\varrho A\}$.*

Proof. First we prove the property R_1. For every $A \in 2^X$ the class ${}_A\mathcal{T}$ is nonempty, as it contains X according to Ω_1. Then by Ω_2 there is an element $B \in {}_A\mathcal{T}$ such that $A\varrho B$.

Since $\emptyset \in \mathcal{T}$, it is clear that $\emptyset$ is a minimal element of ${}_\emptyset\mathcal{T}$ and so we have R_2.

The property R_3 follows from the fact that for every element $B \in 2^X$ for which $A\varrho B$ we have $B \in {}_A\mathcal{T}$ by the definition of ϱ. But ${}_A\mathcal{T}$ contains only subsets of X containing A.

$A\varrho B$ implies that $B \in {}_A\mathcal{T}$, hence $B \in \mathcal{T}$. From this it follows that B is a minimal element of ${}_B\mathcal{T}$, i.e., $B\varrho B$ holds. This proves R_4.

To prove R_5 suppose that $A \subseteq B$ and $B\varrho B$. Then $B \in {}_B\mathcal{T}$ and so $B \in \mathcal{T}$. Since $A \subseteq B$, we have $B \in {}_A\mathcal{T}$. According to Ω_2 there is a minimal element C of ${}_A\mathcal{T}$ such that $A \subseteq C \subseteq B$. Therefore, by the definition of ϱ we have $A\varrho C$ and $C \subseteq B$.

The last property R_6 follows immediately from the definition of ϱ: if $A\varrho B$, then B is a minimal element of ${}_A\mathcal{T}$. Hence there is no $C \in \mathcal{T}$ such that $A \subseteq C$ and $C \subset B$.

We complete the proof by showing that $\mathcal{T}=\{A \subseteq X: A\varrho A\}$. Let $A \in \mathcal{T}$ holds. Then A is a minimal element of ${}_A\mathcal{T}$. Hence by (4) $A\varrho A$ is true. This means that $\mathcal{T} \subseteq \{A \subseteq X: A\varrho A\}$. Now let $A\varrho A$ hold. Then by (4) A is a minimal element of ${}_A\mathcal{T}$. Hence we have $A \in \mathcal{T}$. This means that $\{A \subseteq X: A\varrho A\} \subseteq \mathcal{T}$. From these inclusions we get $\mathcal{T}=\{A \subseteq X: A\varrho A\}$.

By duality we get the following theorems.

Theorem 5. *Let (X, ϱ) be an r-space and let T be the class of all closed subsets of X. Then the class of all open subsets of X, i.e., the class $\mathscr{D} = \{A \subseteq X: (X-A) \in \mathscr{T}\}$ satisfies the following conditions:*
Ω_1': $\emptyset, X \in \mathscr{D}$
Ω_2': *for every $A \subseteq X$ and $B \in {}^A\mathscr{D}$ there is a maximal element C of ${}^A\mathscr{D}$ such that $B \subseteq C \subseteq A$.*

Theorem 6. *Let X be a nonempty set and $\mathscr{D}$ be a class of subsets of X satisfying Ω_1' and Ω_2'. Then the class*

$$\mathscr{T} = \{A \subseteq X: (X-A) \in \mathscr{D}\}$$

satisfies Ω_1 and Ω_2 and $\mathscr{D}$ is precisely the class of all open subsets of the r-space (X, ϱ), where ϱ is the relation on 2^X defined by (4). The relation σ defined on 2^X by
(5) *$A\sigma B$ iff A is a maximal element of the class ${}^B\mathscr{D}$ is a relation of the interior relative to ϱ.*

Remark 3. From the above theorem it is clear that if $\mathscr{D}$ is a class of subsets of X satisfying Ω_1' and Ω_2', then there is a unique r-space having $\mathscr{D}$ for the system of all open sets.

Example 2. Let G be a universal algebra. Let $\mathscr{D}$ be the class that consists of all subalgebras of G and of the empty set. We shall prove that $\mathscr{D}$ satisfies Ω_1' and Ω_2'.

The proof of Ω_1' follows easily from the definition of $\mathscr{D}$. To prove Ω_2' let A be a subset of G. The system ${}^A\mathscr{D}$ is partially ordered by the relation of inclusion $\subseteq$. Since $\emptyset \in {}^A\mathscr{D}$, it is clear that ${}^A\mathscr{D}$ is nonempty. Moreover it is clear that the union of an arbitrary chain of subalgebras of G belonging to ${}^A\mathscr{D}$ is a subalgebra of G belonging to ${}^A\mathscr{D}$. Hence by the Kuratowski—Zorn Theorem it follows that each element $B \in {}^A\mathscr{D}$ is contained in a maximal element $C \in {}^A\mathscr{D}$. This means that Ω_2' is satisfied.

Example 3. Let X be a partially ordered set and let $\mathscr{D}$ be the class of all convex subsets of X (see, e.g., Fuchs [9]). Then $\mathscr{D}$ satisfies Ω_1' and Ω_2'.

Example 4. Let X be a connected topological space. Let $\mathscr{D}$ be the class of all connected subsets of X. Then $\mathscr{D}$ satisfies Ω_1' and Ω_2'.

In Examples 3 and 4 the proof of the properties Ω_1' and Ω_2' is analogous to the proof of the corresponding parts in Example 2. We recall that the proof of Ω_2' depends largely on the use of the statement: the union of an arbitrary chain of elements of ${}^A\mathscr{D}$ is an element of ${}^A\mathscr{D}$. But in Example 3 this statement is clear and in Example 4 it follows from Theorem 21 of [4].

Remark 4. It is easy to see that if we consider the class $\mathscr{D}$ described in Examples 2 and 3, then the intersection of an arbitrary class of elements of $\mathscr{D}$ is an element of $\mathscr{D}$. But the class $\mathscr{D}$ described in Example 4 does not satisfy this condition.

Definition 2. *Let (X, ϱ) be an r-space. A preneighbourhood of $x \in X$ is a subset of X of the form $\{x\} \cup A$, where A is an open set. By a neighbourhood of a point $x \in X$ we mean any open subset of X containing x.*

Remark 5. If (X, ϱ) is an r-space and $\mathscr{D}$ is the class of all open subsets of X, then by (3) $\mathscr{D}(x)$ denotes for each $x \in X$ the class of all neighbourhoods of x.

As an immediate consequence of Theorem 5 and Definition 2 we have the following

Theorem 7. *Let (X, ϱ) be an r-space and $\mathscr{D}$ be the class of all open subsets of X. Then $\{\mathscr{D}(x)\}_{x \in X}$ satisfies the following conditions:*

N_1) *There is a point $x \in X$ such that $X \in \mathscr{D}(x)$.*

N_2) *If $V \in \mathscr{D}(x)$, then $x \in V$.*

N_3) *If $V \in \mathscr{D}(x)$ and $y \in V$, then $V \in \mathscr{D}(y)$.*

N_4) *Let A be a subset of X, let $x \in A$ and let $V \in \mathscr{D}(x)$ such that $V \subseteq A$. Then there is a maximal element V_m in ${}^A\mathscr{D}(x)$ such that $V \subseteq V_m \subseteq A$.*

Moreover, $\mathscr{D} = \{\emptyset\} \cup \bigcup_{x \in X} \mathscr{D}(x)$ holds.

Theorem 8. *Let X be a nonempty set. For each $x \in X$ let $\mathscr{D}(x)$ be a class of subsets of X such that $\{\mathscr{D}(x)\}_{x \in X}$ satisfies the conditions* N_1, N_2, N_3 *and* N_4 *of Theorem 7. Then the class $\mathscr{D}_1 = \bigcup_{x \in X} \mathscr{D}(x) \cup \{\emptyset\}$ of the subsets of X satisfies the conditions Ω_1' and Ω_2' of Theorem 5. Moreover let (X, ϱ) be the r-space (uniquely) determined by $\mathscr{D}_1$ (Theorem 6). Then for every $x \in X$ the system of all neighbourhoods of x in the r-space (X, ϱ) is the system $\{\mathscr{D}(x)\}_{x \in X}$, i.e., $\mathscr{D}(x) = \mathscr{D}_1(x)$ for each $x \in X$.*

Proof. The proof of Ω_1' follows easily from the definition of $\mathscr{D}_1$ and from N_1. To prove Ω_2' let A be a subset of X and let $V \in \mathscr{D}_1$, where $V \subseteq A$. Suppose first $V \neq \emptyset$. From this and from the fact that $\mathscr{D}_1 = \bigcup_{x \in X} \mathscr{D}(x) \cup \{\emptyset\}$ it follows that there is an element $x \in X$ such that $V \in \mathscr{D}(x)$. Then by N_4 there is a maximal element V_m of ${}^A\mathscr{D}(x)$ such that $V \subseteq V_m \subseteq A$. It remains to be proved that there is no element V_1 of $\mathscr{D}_1$ such that $V_m \subset V_1 \subseteq A$. Suppose that this is not true, i.e., there is $V_1 \in \mathscr{D}_1$ with $V_m \subset V_1 \subseteq A$. Since $V_1 \in \mathscr{D}_1$ and $V_1 \neq \emptyset$, we see at once that there is an $y \in X$ such that $V_1 \in \mathscr{D}(y)$. But by N_2 we have $x \in V$ and since $V \subseteq V_1$, then $x \in V_1$. And so by N_3 $V_1 \in {}^A\mathscr{D}(x)$, which is impossible, since $V_m \subset V_1$ and V_m is a maximal element in ${}^A\mathscr{D}(x)$. *Now let* $V = \emptyset$. Consider two cases:

1) There does not exists $V_1 \in {}^A\mathscr{D}_1$ such that $V_1 \neq \emptyset$. In this case it is clear that the empty set is the maximal element of $\mathscr{D}_1$ contained in A.

2) There is $V_1 \in {}^A\mathscr{D}_1$ such that $V_1 \neq \emptyset$. This case has already been discussed in the first half of the proof.

We have shown that $\mathscr{D}_1$ satisfies Ω_1' and Ω_2'. According to Remark 3 there is a unique r-space (X, ϱ) having $\mathscr{D}_1$ for the system of all open sets. Now it remains to be shown that $\{\mathscr{D}(x)\}_{x\in X}$ is the class of all neighbourhoods of the r-space (X, ϱ), i.e., that $\mathscr{D}(x)=\mathscr{D}_1(x)$ for each $x\in X$. Let first $x\in X$ and $V\in\mathscr{D}(x)$. Then by N_2 and definition of $\mathscr{D}_1$ we have $x\in V$ and $V\in\mathscr{D}_1$. Thus $V\in\mathscr{D}_1(x)$. If $V\in\mathscr{D}_1(x)$, where $x\in X$, then $V\in\mathscr{D}_1$ and $x\in V$. Since $V\in\mathscr{D}_1$ and $\mathscr{D}_1=\bigcup_{x\in X}\mathscr{D}(x)\cup\{\emptyset\}$, then there is a $y\in X$ such that $V\in\mathscr{D}(y)$. But then $V\in\mathscr{D}(x)$ by N_3.

In the theorems of this chapter we have proved that an r-space can be described in several ways:

1) by a relation $\varrho\subseteq 2^X\times 2^X$ sati fying R_1—R_6;
2) by a relation $\sigma\subseteq 2^X\times 2^X$ satisfying K_1—K_6;
3) by a class $\mathscr{T}\subseteq 2^X$ satisfying Ω_1 and Ω_2;
4) by a class $\mathscr{D}\subseteq 2^X$ satisfying Ω_1' and Ω_2';
5) by a class $\{\mathscr{D}(x)\}_{x\in X}$ satisfying N_1—N_4.

We have also seen that in every r-space there are always uniquely defined:

1) the class of all closed sets;
2) the class of all open sets;
3) the relation of closure;
4) the relation of the interior;
5) the class of all neighbourhoods.

If it does not cause ambiguity we often refer to the r-space as X instead of the more proper form (X, ϱ). We shall be explicit in cases where precision is necessary (for example if we are considering two different relations of closure for the same set X).

Some properti s of closures and closed sets

Theorem 9. *Let there be given an r space X and let $\mathscr{T}$ be the class of all closed subsets of X. Then*

a) *if B and C are closures of a set $A\subseteq X$ and $B\neq C$, then $B\nsubseteq C$ and also $C\nsubseteq B$ (i.e. $(B-C)\neq\emptyset$ and $(C-B)\neq\emptyset$);*
b) *if B is a closure of a set $A\subseteq X$ and $A\subseteq C\subset B$, then B is also a closure of C;*
c) *each closed subset of X has only one closure.*

Proof.

a) since B and C are closures of A, then $B, C\in{}_A\mathscr{T}$. If $B\subset C$ $(C\subset B)$, then $C(B)$ cannot be a minimal element of ${}_A\mathscr{T}$, which contradicts(4);
b) if B is a closure of A, then B is a minimal element of ${}_A\mathscr{T}$. Since $A\subseteq C\subseteq B$ then B belongs to ${}_C\mathscr{T}$ and it is clear that B is a minimal element also of ${}_C\mathscr{T}$;
c) if A is closed, then $A\in{}_A\mathscr{T}$ and A is even the smallest element of ${}_A\mathscr{T}$. Therefore by (4) A has only one clo ure and A is the unique closure of A.

Theorem 10. *Let X be an r-space and $\{A_s\}_{s\in S}$ is a class of closed subsets of X. Then all sets of $\{A_s\}_{s\in S}$ are closures of the same set iff A_s is a closure of the set $\bigcap_{s\in S} A_s$, for each $s\in S$.*

Proof. If all sets of $\{A_s\}_{s\in S}$ are closures of $\bigcap_{s\in S} A_s$, they are evidently closures of the same set. On the contrary, if all the sets of $\{A_s\}_{s\in S}$ are closures of the same set $B\subseteq X$, then by R_3 $B\subseteq\left(\bigcap_{s\in S} A_s\right)$. According to b) of Theorem 9 each set of $\{A_s\}_{s\in S}$ must be a closure of $\bigcap_{s\in S} A_s$.

Corollary 1. *If X is an r-space, $A, B\subseteq X$, A and B are closures of the same set and $A\neq B$, then $A\cap B$ cannot be closed.*

Proof. By Theorem 10 A and B are closures of $A\cap B$. But by c) of Theorem 9 each closed set has only one closure. Therefore $A\cap B$ cannot be closed.

We shall now characterize closed sets, closures and interiors of sets in terms of neighbourhoods and preneighbourhoods.

Let X be an r-space, $\{\mathscr{D}(x)\}_{x\in X}$ be the class of all neighbourhoods and $\mathscr{D}$ be the class of all open subsets of X.

Theorem 11. *Let X be an r-space, $A\subseteq B\subseteq X$* and let A be *open. Then A is an interior of B iff for each $x\in X-A$ and each neighbourhood V of x containing the preneighbourhood $V_1=\{x\}\cup A$ of x we have $V\cap(X-B)\neq\emptyset$.*

Proof. By (5) A is an interior of B iff A is a maximal element of the class ${}^B\mathscr{D}$. But this is if and only if for each open subset V of X containing $\{x\}\cup A$, where $x\in B-A$, we have $V\cap(X-B)\neq\emptyset$.

Since closed sets are complements of open sets, the dual statement of Theorem 11 also holds.

Corollary 2. *Let X be an r-space, $A\subseteq B\subseteq X$ and B is closed. Then B is a closure of A iff $V\cap A\neq\emptyset$ for each $x\in B$ and each neighbourhood V of x containing the preneighbourhood $V_1=\{x\}\cup(X-B)$* of x.

Theorem 12. *Let X be an r-space and $A\subseteq X$. Then A is an open set iff for each $x\in A$ and each preneighbourhood V_1* of x such *that $V_1\subseteq A$ there is a neighbourhood V_2 of x satisfying $V_1\subseteq V_2\subseteq A$.*

Proof. If A is open, then for each $x\in A$ the set A is a neighbourhood of x. This proves one half of the theorem. To prove the other half suppose that A is not open. Then there is an interior C of A such that $C\subset A$ and C is open. Let $x\in A-C$. Then the set $V_1=\{x\}\cup C$ is a preneighbourhood of x and we have $C\subset V_1\subseteq A$. Since C is an interior of A, C is a maximal element of ${}^A\mathscr{D}$. Therefore for the preneighbourhood V_1 of x there is no neighbourhood V_2 of x such that $V_1 \subseteq V_2 \subseteq A$.

As a dual consequence of Theorem 12 we have the following coıollary.

Corollary 3. *Let X be an r-space and $B \subseteq X$. Then B is a closed set ıff for each $x \notin B$ and each preneighbourhood V_1 of x such that $V_1 \cap B = \emptyset$ there is a neighbourhood V_2 of x such that $V_1 \subseteq V_2$ and $V_2 \cap B = \emptyset$*

REFERENCES

[1] CSÁSZÁR, Á.: Grundlagen der allgemeinen Topologie. Budapest 1963.

[2] ČECH, E.: Topologické prostory. Čas. pěst. mat., 66, 1936—37 D 225—264.

[3] HAUSDORFF, F.: Gestufte Räume. Fundamenta mathematicae, 25 1935 486—502.

[4] KELLEY, J. L.: General Topology. D. Van Nostrand company, New York 1955

[5] KOUTSKÝ, K.: Určenost topologických prostorů pomocı uplných systemů okolı. Spisy prır. fak MU, 1956, 374, 1—11.

[6] КУТАТОВСИЙ, К.: Топология I, II. Москва 1969.

[7] NETZER, N.: Konvergenz in verallgemeinerten topologischen Raumen. Math Nachr, 77, 245—262.

[8] NOVÁK, J : On convergence spaces and their sequentıal envelopes. Czech Math. Journ., 15(90), 1965, 74—100.

[9] FUCHS, L.: Partially Ordered Algebraic Systems. Pergamon press, New York 1963

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ОБОБЩЕННЫЕ ТОПОЛОГИЧЕСКИЕ ПРОСТРАНСТВА

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Резюме

В настоящей работе изучаются структуры, называемые r-пространствами, определенными как пара (X, ϱ), где X — непустое множество и ϱ — отношение в 2^X, исполняющее условия R_1—R_6. Эти структуры являются обобщением топологических пространств.