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Addendum and erratum to the paper "On the size of a maximal induced tree in a random graph"

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**ADDENDUM AND ERRATUM TO THE PAPER  
“ON THE SIZE OF A MAXIMAL INDUCED TREE  
IN A RANDOM GRAPH”**

MICHAŁ KAROŃSKI—ZBIGNIEW PALKA

Due to final remark in our paper [1] regarding lower bound on the size of a maximal induced tree we shall briefly prove the following stronger version of the Theorem 3.

**Theorem 3'.** *Let  $p_1 \leq p < 1$ ,  $n \geq n_1$  and  $l = l(n, p)$  be the threshold function given by the formula (1). Then for any integer  $k$  such that,  $2 \leq k < l(n, p)$*

$$\text{Prob}(\alpha_{n,p} \leq k) \leq \frac{1 - b^{1-k}}{b(b-1)},$$

where

$$b = b(n, p) = (n\lambda)^f, \quad f = f(\delta) = d^\delta - 1$$

and

$$\delta = \delta(n, k, p) = l(n, p) - k.$$

**Proof.** Let  $Z_k$  denote the number of maximal trees of the size  $k$ ,  $k \geq 2$ . Then by Bool's inequality and formula (6) we get

$$\begin{aligned} \text{Prob}(\alpha_{n,p} \leq k) &\leq \sum_{j=2}^k E(Z_j) \leq \sum_{j=2}^k (n\lambda \exp(-npq^{j-1}))^j \leq \\ &\leq \sum_{j=2}^k (n\lambda \exp(-npq^{k-1}))^j. \end{aligned}$$

Now we shall notice that from the definition of the threshold function  $l(n, p)$  we have  $n\lambda \exp(-npq^{k-1}) = b^{-1}$ . Moreover, from the proof of Theorem 1, it follows that if  $k < l(n, p)$  then  $b^{-1} < 1$  and one can get the result immediately. Now from Theorems 3' and 4 we shall get the following corollary.

**Corollary.** *For every  $p$ ,  $p_1 \leq p < 1$  and every  $\varepsilon_0 > 0$ , probability of the event that*

a random graph  $G_{n,p}$  contains a maximal induced tree of the size which not belongs to the interval

$$\langle [(n, p) - \varepsilon_0], \{u(n, p) + \varepsilon_0\} \rangle$$

tends to zero as  $n \rightarrow \infty$ .

Finally we would like to correct the statement that Theorem 2 and 4 hold for  $n \geq 6$  whereas in fact it is true for  $n \geq n_3$  where  $n_3 = n_3(p)$  is the least integer such that the inequality  $2 \leq u(n, p) \leq n$  holds.

#### REFERENCES

- [1] KAROŃSKI, M.—PALKA, Z.: On the size of maximal induced tree in a random graph. *Math. Slovaca* 30, 1980, 151—155.

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