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ADDENDUM AND ERRATUM TO THE PAPER "ON THE SIZE OF A MAXIMAL INDUCED TREE IN A RANDOM GRAPH"

MICHAŁ KAROŃSKI—ZBIGNIEW PALKA

Due to final remark in our paper [1] regarding lower bound on the size of a maximal induced tree we shall briefly prove the following stronger version of the Theorem 3.

Theorem 3′. *Let $p_1 \leq p < 1$, $n \geq n_1$ and $l = l(n, p)$ be the threshold function given by the formula* (1). *Then for any integer k such that,* $2 \leq k < l(n, p)$

$$\text{Prob}\,(\alpha_{n,p} \leq k) \leq \frac{1 - b^{1-k}}{b(b-1)},$$

where

$$b = b(n, p) = (n\lambda)^f, \quad f = f(\delta) = d^\delta - 1$$

and

$$\delta = \delta(n, k, p) = l(n, p) - k.$$

Proof. Let Z_k denote the number of maximal trees of the size k, $k \geq 2$. Then by Bool's inequality and formula (6) we get

$$\text{Prob}\,(\alpha_{n,p} \leq k) \leq \sum_{j=2}^{k} E(Z_j) \leq \sum_{j=2}^{k} (n\lambda \exp(-npq^{j-1}))^j \leq$$

$$\leq \sum_{j=2}^{k} (n\lambda \exp(-npq^{k-1}))^j.$$

Now we shall notice that from the definition of the threshold function $l(n, p)$ we have $n\lambda \exp(-npq^{k-1}) = b^{-1}$. Moreover, from the proof of Theorem 1, it follows that if $k < l(n, p)$ then $b^{-1} < 1$ and one can get the result immediately. Now from Theorems 3′ and 4 we shall get the following corollary.

Corollary. *For every p, $p_1 \leq p < 1$ and every $\varepsilon_0 > 0$, probability of the event that* *a random graph $G_{n,p}$ contains a maximal induced tree of the size which not belongs to the interval*

$$\langle [(n, p) - \varepsilon_0],\ \{u(n, p) + \varepsilon_0\} \rangle$$

tends to zero as $n \to \infty$.

Finally we would like to correct the statement that Theorem 2 and 4 hold for $n \geq 6$ whereas in fact it is true for $n \geq n_3$ where $n_3 = n_3(p)$ is the least integer such that the inequality $2 \leq u(n, p) \leq n$ holds.

REFERENCES

[1] KAROŇSKI, M.—PALKA, Z.: On the size of maximal induced tree in a random graph. Math. Slovaca 30, 1980, 151—155.

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