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EDUARD ČECH 1893—1960

This year we observe the 100th anniversary of the birthday of Eduard Čech, one of the leading world specialists in topology and differential geometry. To these fields he contributed works of fundamental importance.

He was born on June 29, 1893 in Stračov in northeastern Bohemia. During his high school studies in Hradec Králové he became interested in mathematics and in 1912 he entered the Philosophical Faculty of Charles University in Prague. At that time there were very few opportunities for mathematicians other than to become a high school teacher. For a position of a high school teacher two fields of study were required. Since Čech was not much interested in physics, the standard second subject, he chose descriptive geometry. During his studies at the university he spent a lot of time in the library of the Union of Czech Mathematicians and Physicists and read many mathematical books of his own choice.

In 1914 the First World War broke out and in 1915, after three years of study, Čech had to leave the university for the army of the Austro-Hungarian Empire. He stayed in the army for three years and used this lost time to learn languages, namely German, Italian and Russian. He completed his studies in the school year 1918–1919 and passed the state examination which entitled him to teach mathematics at high schools. In the years 1919–1923 he taught mathematics at several Prague high schools. In 1920 he received a Ph.D. from Charles University.

In the year 1921 his first paper appeared and so began the first Čech's research period devoted to projective differential geometry. It lasted till 1930. It is an important feature of Čech's research activity that he always worked in new, developing fields. Of course, a lot of other people did the same; however, Čech always very soon obtained major results whose importance is not diminished by passing years.

Upon request Čech obtained some funds from the Ministry of Education, took a leave of absence, and spent the school year 1921–1922 in Torino with Guido Fubini. Čech must have impressed Fubini considerably since he offered Čech to become a coauthor of a book on projective differential geometry. Bear in mind that it was an offer made by a well-known scientist to a young Ph.D. who was only a provisional high school teacher at that time. The cooperation between Čech and Fubini was very fruitful.

After his return from Italy Čech submitted his habilitation thesis and in 1922 he became a docent of Charles University. This was just an academic title so he continued teaching at a Prague high school.

At this time Professor Lerch of Masaryk university in Brno died. Brno is the second largest city of the Czech republic, the capital of Moravia, and after the Czechs gained independence from Austria in 1918 the second Czech university was established there in 1919, named after the first president of the republic Tomáš Garrigue Masaryk.

Eduard Čech was offered and accepted the vacant position and in 1923 became extraordinary professor at the Faculty of Natural Sciences of Masaryk University; he became a full professor in 1928. Lerch had held the chair of analysis and the chair of geometry was held by

Professor Seifert. Hence, although geometry was Čech's field of research, Čech had to take over courses in analysis and algebra. He proceeded to master these two fields.

We may observe here one of Čech's basic characteristics. Whenever he was doing something in mathematics, he always strived to achieve thorough understanding of the subject. The result was that even outside of his fields of research he had extensive knowledge and deep insight in many other areas of mathematics.This feature of his personality had also some other consequences. While he was not conceited and talked easily to people with little formal education, he expected in his fellow professors the same qualities he himself possessed. This did not contribute to smooth relations with some people as he was not diplomatic, but, on the contrary, quite forthright in expressing his opinions.

His study of algebra and analysis brought his attention to other fields of mathematics. In particular, he became interested in topology. From 1930 to 1947 he worked in topology and published 31 papers, 19 in algebraic and 12 in general topology. Let us mention his participation at the International Congress of Mathematicians in Zürich in 1932. In 1935 he was invited to attend the prestigious Moscow conference on combinatorial topology and the school year 1935–1936 he spent at the Institute for Advanced Study in Princeton.

After he returned from the U.S.A. he started his famous topological seminar in 1936. Why famous? Up to Čech's seminar, seminars in Czechoslovakia were held only for undergraduate students as part of their studies and mathematical research was done by individuals. The now standard form of small groups working together was started here by Čech. The seminar on general topology was very successful; its participants published 27 papers in the three years it existed. Its work ended in 1939 when the Nazis closed the Czech universities. Čech then continued to meet with the two principal participants, B. Pospíšil and J. Novák, in Pospíšil's flat until Pospíšil was arrested by the Gestapo in 1941. Pospíšil died soon after his release from Nazi prison in 1944. Thus Čech lost his best student.

During the war, Čech worked on his book *Topologické prostory* (*Topological Spaces*) which was later rewritten and published in 1959. At this time he also became deeply interested in high school mathematics. He held seminar for high school teachers and he wrote several high school textbooks on algebra and geometry which are even now remembered for their superior qualities. After the war he became very much involved in a school reform which introduced a unified high school similar to the English comprehensive school. He chaired the commission charged with instruction in mathematics.

After the war Čech moved to Prague and was appointed professor at Charles University in 1945. He remained at the University till the end with the exception of the years 1950–1953 when he was granted a leave of absence.

He became the leading personality in Czech mathematics. In 1947 the Czech Academy of Sciences and Arts established the Mathematical Institute and Čech was appointed its director. This institute had only a secretary and one research assistant, the members of the Institute were employed by the University or by the Czech Technical University. In 1950 the government created the Central Mathematical Institute and Čech again became its director. This institute replaced the former one but this time it was a regular research institute with many research workers and graduate students. When the Czechoslovak Academy of Sciences was founded in 1952, this institute became the Mathematical Institute of the Academy.

In 1953 Čech realized he could not do much more for the Institute and returned to the University. He has indeed done enough. The Institute was well established, its structure and purpose fully determined, and many of the students who would later on become leading Czech scientists already admitted to graduate study at the Institute.

In 1950 Čech started publishing again and he returned to his most favored topic, differential geometry. He published during this last period 21 papers. That does not mean he neglected the welfare of Czech mathematics. Already in 1953 he initiated the creation of the Mathematical Institute of Charles University; the institute was established in 1956 with Eduard Čech as its first director. Unfortunately, his health started to deteriorate and he died on March 15, 1960. Even when already gravely ill, he performed another two important services for Czech mathematics. He founded the journal Commentationes Mathematicae Universitatis Carolinae, the first issue appeared in 1960, and he came with the idea to organize in Prague an international topological conference. The conference took place in 1961 under the name Symposium on General Topology and its Relations to Modern Analysis and Algebra. Since then, every five years there has been a Prague Topological Symposium.

Another feature of Čech's personality is that philology was his hobby. He greatly influenced Czech mathematical terminology and he learned many languages. He wrote papers in French, Italian, German, English and Russian and he continued the study of languages till the very end; before his death he started to learn the Romanian language.

At the very end let us mention some of the honors that came his way: he was a member of the Polish Academy of Sciences, he received honorary doctorates from the University of Warsaw and the University of Bologna, he was a member of the Royal Czech Society of Sciences, Czech Academy of Sciences and Arts, Czechoslovak Academy of Sciences and honorary member of the Union of Czechoslovak mathematicians and physicists. He twice received the State Prize and was awarded the Order of the Republic.

As you may have noticed there are few things in present Czech mathematics which are not due to the activity of Professor Eduard Čech. There are two reasons for his unique position in the history of Czech mathematics, his deep and extensive understanding of modern mathematics and the fact that his decisions were based on the needs of Czech mathematics and not on his personal preferences.

Čech's scientific work consists of 94 original research papers and 11 monographs and textbooks. Although Čech loved geometry and devoted to this field the largest number of his papers, it was his contribution to topology that turned out to be the most significant.

Čech worked in topology for a relatively short period. With a single exception, all his topological papers date to the period 1930–1938.

The specialists in general topology have undoubtedly considered Čech's paper *On bicompact spaces* fundamental. Thus, let us start with it.

The importance of compact spaces in general topology was pointed out by P. S. Alexandrov and P. S. Urysohn in the mid-twenties in their Memoir, where they defined compactness via covers, thus eliminating the metrical context. In 1930, A. N. Tikhonov proved his theorem on the product of compact topological spaces, introduced completely regular spaces and characterized the class of completely regular T_1 spaces (Tikhonov spaces in the present terminology) as the class of subspaces of compact Hausdorff spaces. His proof made use of the embedding of the given completely regular T_1 space into the product of closed intervals.

Čech followed up in an impressive manner. First he introduced the completely regular T_1 modification of an arbitrary topological space, which represented the first example of projective generation in the sense of the present category theory, and presented a general proof of the Tikhonov theorem making use of the Alexandrov–Urysohn's characterization of compactness "every infinite subset has a complete accumulation point, i.e. a point each neighbourhood of which intersects the subset in the set of the same cardinality". The next result is the theorem on existence and uniqueness of the maximal compactification $\beta(S)$ of a Tikhonov space S, now called the Čech-Stone compactification. Čech characterized $\beta(S)$ as a compact space containing S as a dense part and such that all bounded real continuous functions defined on S can be continuously extended to it. He proved that $\text{cl}_{\beta S} T = \beta T$ for $T \subseteq S \subseteq \beta S$ iff T is C^* embedded in S, and characterized βS for a normal space S without mentioning continuous functions (βS is such a compactification that any two disjoint closed subsets of the space S have disjoint closures in the extension). Čech showed that a closed G_δ-set in $\beta S \setminus S$ for a space S which is not countably compact has a cardinality at least continuum. As an easy consequence he obtained that no point in $\beta\mathbb{N}$ is the limit of a convergent sequence, which solved a tantalizing problem from the Memoir. As concerns the spaces with the first axiom of countability, he showed that they may be recognized in their β-compactification. In fact, he proved that in this case S is equal to the set of those points from βS which have countable character in βS. Čech also dealt with the two most important cases, $\beta\mathbb{N}$ and $\beta\omega_1$, the compactification of the discrete set of natural numbers and the compactification of the space of all countable ordinal numbers with the order topology. The remainder $\beta\omega_1 \setminus \omega_1$ is a singleton, and for the cardinality $|\beta\mathbb{N}|$ Čech showed that $2^{\aleph_0} \leq |\beta\mathbb{N}| \leq 2^{2^{\aleph_0}}$; the equality $|\beta\mathbb{N}| = 2^{2^{\aleph_0}}$ is due to his student B. Pospíšil and can be found in the immediately following paper in the same issue of the Annals of Mathematics.

However, this is not all. A new class of spaces is introduced in the third part of the paper. Topologically complete, now Čech complete spaces are defined as G_δ-subsets in some of their compactifications. Čech showed that this is the same as to be a G_δ-set in its own β-compactification. All fundamental theorems on Čech complete spaces follow: the property to be Čech-complete is hereditary to closed subsets, the Baire theorem holds true in Čech complete spaces, a Čech complete subspace of a given space is the intersection of a closed and a G_δ-set, the converse being valid for a Čech complete subspace of a Čech complete space. The main theorem asserts that this notion is well justified, since a metrizable space is Čech complete iff it is metrically complete for a suitable topologically equivalent metric. Hence the Čech complete spaces are a natural generalization of complete metric, as well as of compact or locally compact spaces.

Even now, more than half a century later, reading the paper is a rewarding experience.

Let us mention that in the same year M. H. Stone independently publishes *Applications of Boolean rings to general topology* in the Transactions of the American Mathematical Society. Here the Čech-Stone compactification is introduced in connection with the theory of Stone's representation of Boolean algebras. As opposed to Čech, Stone characterizes βS as a compactification to which any continuous mapping defined on S with values in a compact space can be continuously extended.

In three papers on the dimension theory, published in the years 1931–1933, Eduard Čech laid the foundations of the theory. Although the first results on the dimension of Euclidean spaces belong to Lebesgue, Brouwer, Menger and Urysohn (1911 - 1925), a correct definition of the inductive dimension appears only in the years 1925–1928. This dimension is now known as the small inductive or Menger-Urysohn dimension, denoted by ind. Čech's large inductive dimension, called also Čech-Brouwer's dimension and denoted Ind, is its generalization. The covering property of the n-dimensional Euclidean space formulated in Lebesgue's lemma was used by Čech to define the presently most common dimension dim, also called covering or Čech-Lebesgue's dimension.

Let us recall both Čech's definitions. If $\mathcal{C}$ is a covering of a set M, we say that the order of the covering $\mathcal{C}$ is $\leq n$ if every element of the set M is contained in at most $n+1$ sets of the set $\mathcal{C}$.

A topological space S has a covering dimension $\leq n$ ($n = -1, 0, 1, 2, \ldots$) if every finite covering of the space S has a finite open refinement of an order $\leq n$. $\dim S = n$ if $\dim S \leq n$

but $\dim S \leq n-1$ is not true.

Let us note that the above mentioned Lebesgue's lemma from 1911 asserts that, in the sense of this definition, $\dim[0,1]^n = n$.

The large inductive dimension is defined by induction. $\operatorname{Ind}\emptyset = -1$. $\operatorname{Ind} S \leq n$ if for every closed $F \subset S$ and for every open $U \supset F$ there exists an open V such that $F \subset V \subset \operatorname{cl} V \subset U$ and its boundary $\operatorname{bd} V = \operatorname{cl} V \setminus V$ satisfies $\operatorname{Ind}\operatorname{bd} V \leq n-1$. $\operatorname{Ind} S = n$ if $\operatorname{Ind} S \leq n$ but $\operatorname{Ind} S \leq n-1$ does not hold. In both cases the dimension of a space is infinite if it is $\leq n$ for no n .

For both dimensions Čech proved the sum theorem: if $S = \bigcup_{i=1}^{\infty} F_i$, the sets F_i are closed and the dimension of F_i is $\leq n$, then the dimension of S is $\leq n$ as well. Again for both the dimensions Čech proved that the dimension of a space is greater or equal to the dimension of its closed subspace. The theorems for dim were proved for normal spaces, while for Ind they were established for perfectly normal ones. Let us recall that a space is called perfectly normal if it is normal and each of its closed subsets is G_δ .

Čech conjectured that dim $S = \operatorname{Ind} S$ if S is perfectly normal. All counterexamples given until now were constructed only under the Continuum Hypothesis.

Simultaneously with his work in general topology Čech publishes equally important papers in algebraic topology. In the early thirties the homology theory for finite complexes was essentially completed and attention was turning to more complex objects (J. W. Alexander, S. Lefschetz, P. S. Alexandrov, L. S. Pontryagin).

In his work *Théorie génerale de l'homologie dans un espace quelconque* Čech developed the first comprehensive sufficiently general homology theory.

Already E. Vietoris and P. S. Alexandrov had used a limit process to define homology groups. However, their techniques were applicable only to compact metric spaces. The starting notion of E. Čech was a finite cover of a general set, to which he assigned an abstract simplicial complex. He studied in detail the interrelations of complexes associated with the covers, one of which refines the other, and showed that all projections for a given refining pair of covers determine the same homomorphism of the corresponding homology groups of complexes.

In the context of topological spaces Čech considered the family of all finite open covers directed by the relation of refinement, Čech's homology group being then the inverse limit of homology groups assigned to the individual covers. He developed this theory to considerable depth, constructing homology groups of pairs consisting of a space and its closed subspace and establishing the dependence of the Betti numbers of the union $R_1 \cup R_2 = R$ on the Betti numbers of $R_1, R_2, R_1 \cap R_2$ for closed subspaces R_1, R_2 . Moreover, he proved that in the case of a hereditarily normal space we can start from closed covers and arrive at the same homologies. The paper in which Čech established a new approach to homology was followed in the years 1933–1936 by further articles devoted to applications and further development of the theory.

Further Čech's papers deal with the theory of manifolds. His main aim was to introduce a general notion of a manifold which would include all connected spaces locally homeomorphic to the n-dimensional Euclidean space E_n . The manifold was to be uniquely defined by general topological properties and assumptions expressed in terms of the general homology theory. It was also desirable for these general manifolds to satisfy, after necessary modifications, the duality theorems. This goal was actually achieved, and in addition new results were obtained also for classical duality (for sets in E_n or in S_n). Let us note that S. Lefschetz arrived at analogous results approximately at the same time. Later R. Wilder and other authors started to develop Čech's results, simplifying them by using new methods.

At that time, the cohomology theory did not exist yet. Čech was the first to study cohomological notions under the name of dual cycles. The explicit definition was later introduced by J. W. Alexander and A. N. Kolmogorov.

In the Proceedings of the International Congress of Mathematicians in Zürich in 1932 Čech published a very brief note *Höherdimensionale Homotopiegruppen*, in which he defined higher homotopy groups. In 1961, after Čech's death, P. S. Alexandrov, who had participated in the Zürich congress, commented Čech's contribution in the following way:

"This definition did not meet with the attention it merited; in fact, the commutativity of these groups for dimensions exceeding one was criticised (this was unfounded, as we now know).

Thus, Professor Čech's definition of the homotopy groups was, in 1932, simply not understood – a situation extremely rare in modern mathematics. We must express our admiration at the intuition and talent of Professor Čech, who defined the homotopy groups several years before W. Hurewicz."

Eduard Čech was one of the founders of projective differential geometry, and almost all his papers from geometry belong to this domain. He worked in geometry during two periods of his life, first in the years 1921 through 1930 and then since the end of World War II till his death.

In geometry he dealt with difficult topics. In his work he used his exceptional geometric intuition as well as the ability to carry out extraordinarily complicated calculations. Čech's approach to the study of geometric objects is characterized by three aspects: a systematical study of correspondences between two objects of the same type, special attention devoted to the contact of manifolds and systematical investigation of dual elements.

Immediately at the beginning of his scientific career E. Čech was deeply influenced by the work of Guido Fubini. During Čech's stay in Torino Fubini took advantage of the abilities of young Čech suggesting to him a series of problems. In his papers from this period Čech described a number of geometric properties of various geometric objects. For example he showed that the osculation planes of three Segre curves intersect each other in one straight line, which is now called Čech's line. In another paper he described in detail surfaces whose Segre curves are planar, which was then considered a very difficult problem. This characterization belongs to fundamental results obtained in this area. Through the years 1921-1924 Čech published 25 papers on differential geometry; majority of them concerned curves and surfaces in three-dimensional space. Moreover, together with Fubini they later wrote a joint book on differential geometry *Geometria proiettiva differenziale*, two volumes of which appeared in Bologna in the years 1926 and 1927. In this book great attention is paid to the problem of projective deformation. This problem, formulated by Fubini, was later generalized and studied by Elie Cartan, who obtained a number of interesting results in this direction by using the method of exterior differential systems.

We should mention that Čech and Fubini also published a book in French *Introduction à la géométrie projective différentielle des surfaces* in Paris 1931. It is of interest that its last two chapters, on which Fubini did not collaborate, contain Cartan's methods. Čech compiled here a readable survey of the subject and gave precise formulations of the equations of differential systems in two variables, and also used here the method of specialization of frame.

In the post-war period Čech studied correspondences between n-dimensional projective spaces and line congruences, i.e. two-parameter systems of straight lines in a projective space. He obtained a number of results concerning developable correspondences between congruences of lines in the three-dimensional projective space. A complete description of the theory of correspondences was later given by his student Alois Švec.

We must not omit another feature of Čech's personality, namely his continuous care for students and for teaching mathematics. His books (written in Czech) *Projective Differential Geometry* (1926), *Point Sets* (1936), *Foundations of Analytical Geometry I, II* (1951, 1952) and *Topological Spaces* (1959) have been of great importance for Czech mathematics, because through them the Czech students made their first acquaintance with some branches of modern mathematics and they served not only as textbooks but also as monographs for specialists. The book *Topological spaces*, written essentially during the war, was made to a large extent up to date by appendices written by Josef Novák and Miroslav Katětov. Although the book was written in Czech, numerous citations in works of authors not speaking Czech witness its publicity even abroad.

Both Czech and Slovak mathematics are indebted to Eduard Čech for more than can ever be realized.

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