Jerzy Niewiarowski

Convergence of sequences of real functions with respect to small systems

Mathematica Slovaca, Vol. 38 (1988), No. 4, 333--340

Persistent URL: http://dml.cz/dmlcz/130685

Math. Slovaca 38, 1988, No. 4, 333—340

CONVERGENCE OF SEQUENCES OF REAL FUNCTIONS WITH RESPECT TO SMALL SYSTEMS

JERZY NIEWIAROWSKI

Small systems were introduced by Riečan [3] and studied by many authors (see [2 p. 498], for references).

In this paper we define the convergence with respect to a small system. In [4] Wagner defined the convergence with respect to an σ-ideal. We shall study the instances when both the convergence with respect to a small system and that with respect to a suitable σ-ideal are equivalent.

Definition 1. *Let* (X, S) *be a measurable space and* $\{N_r\}_{r=1}^{\infty}$ *a sequence of subfamilies of* S *such that*

(1) $\emptyset \in N_r$ *for* $r = 1, 2, \ldots$

(2) *For each positive integer* r, *there exists a sequence* $\{k_i\}_{i=1}^{\infty}$ *of positive integers such that* $E_i \in N_{k_i}$ $(i = 1, 2, \ldots)$ *implies* $\bigcup_{i=1}^{\infty} E_i \in N_r$.

(3) *For each positive integer* r, *if* $E \in N_r$, $F \subset E$, $F \in S$, *then* $F \in N_r$.

(4) $N_r \supset N_{r+1}$ *for* $r = 1, 2, \ldots$.

(5) *For each positive integer* r, *if* $E \in N_r$, $F \in \bigcap_{r=1}^{\infty} N_r$, *then* $E \cup F \in N_r$.

The sequence $\{N_r\}_{r=1}^{\infty}$ *satisfying all the above properties will be called a small system on* S *and will be denoted by* $\mathcal{N}$.

It is not difficult to verify (see [2, p. 491]) that $N = \bigcap_{r=1}^{\infty} N_r$ is a σ-ideal on S.

Definition 2. *A small system* $\mathcal{N}$ *will be called upper semicontinuous if and only if, for every nonicreasing sequence of the sets* $\{E_i\}_{i=1}^{\infty}$ *the following is true: if there exists a positive integer* r_0 *such that* $E_i \notin N_{r_0}$ *(for* $i = 1, 2, \ldots$*), then* $\bigcap_{i=1}^{\infty} E_i \notin N$.

Let I be an σ-ideal in an σ-field S. We say that I-almost every point of $E \subset X$ has some property (or that this property holds I-almost everywhere, in abbreviation I-a.e., on E) if and only if the set of points in E at which this property does not hold belongs to the σ-ideal I.

Definition 3. *We say that a sequence* $\{f_n\}_{n=1}^{\infty}$ *of* S*-measurable functions defined on* X *converges with respect to* I *to an* S*-measurable function* f *defined on* X *if and* *only if each subsequence* $\{f_{n_m}\}_{m=1}^{\infty}$ *of* $\{f_n\}_{n=1}^{\infty}$ *contains a subsequence* $\{f_{n_{m_p}}\}_{p=1}^{\infty}$ *converging to f I-a.e. on X. We shall use the notation* $f_n \xrightarrow[n \to \infty]{I} f$.

Definition 4. *Let* $\mathscr{N}$ *be a small system on S and* $\{f_n\}_{n=1}^{\infty}$ *a sequence of S-measurable functions defined on X. We assume that the functions f and* f_n $(n = 1, 2, \ldots)$ *are N-a.e. finite* $\left(\text{where } N = \bigcap_{r=1}^{\infty} N_r\right)$. *We say that* $\{f_n\}_{n=1}^{\infty}$ *converges with respect to the small system* $\mathscr{N}$ *to an S measurable function f defined on X if and only if, for each* $\varepsilon > 0$ *and for each positive integer m, there exists a positive integer* $n_{\varepsilon,m}$ *such that, for each positive integer* $n \geqq n_{\varepsilon,m}$, *the set*

$$\{x\colon |f_n(x) - f(x)| \geqq \varepsilon\} \in N_m.$$

We shall use the notation $f_n \xrightarrow[n \to \infty]{\mathscr{N}} f$.

We say that two S-measurable functions f and g are equivalent if and only if f-g vanishes N-a.e. on X.

It is not difficult to observe that both the limit with respect to the small system $\mathscr{N}$ and that with respect to the σ-ideal N are determined up to equivalent functions. In the above definitions we can also suppose that all functions f_n and f are defined only N-a.e. on X (see definition 1(5)).

From now on, we shall use only the properties (1)—(4) from definition 1. We shall assume that all functions f_n, f under consideration are S-measurable and N-a.e. finite $\left(N = \bigcap_{r=1}^{\infty} N_r\right)$.

Remark 1. If $f_n = f$ for every n, then $f_n \xrightarrow[n \to \infty]{N} f$. If $f_n \xrightarrow[n \to \infty]{\mathscr{N}} f$, then, for each subsequence $\{f_{n_m}\}_{m=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$, we have $f_{n_m} \xrightarrow[m \to \infty]{\mathscr{N}} f$.

Lemma 1. *Let* $\mathscr{N}$ *be a small system; then there exists a sequence* $\{k_i\}_{i=1}^{\infty}$ *of positive integers such that, if* $E_i \in N_{k_i}$, *then* $\bigcup_{i=n}^{\infty} E_i \in N_n$ *for each positive integer n.*

Proof. For each positive integer j, there exists a sequence $\{k_i^{(j)}\}_{i=1}^{\infty}$ of positive integers such that, if $E_i \in N_{k_i^{(j)}}$, then $\bigcup_{i=1}^{\infty} E_i \in N_j$. Let $k_i = \max_{j \leqq i} k_i^{(j)}$, $E_i \in N_{k_i}$, $i = 1, 2, \ldots$. If $i \geqq n$, then $k_i \geqq k_i^{(n)}$, hence $E_i \in N_{k_i^{(n)}}$ and $\bigcup_{i=n}^{\infty} E_i \in N_n$.

Theorem 1. *If a sequence* $\{f_n\}_{n=1}^{\infty}$ *of functions converges on X with respect to a small system* $\mathscr{N}$ *to a function f, then there exists a subsequence* $\{f_{n_i}\}_{i=1}^{\infty}$ *of* $\{f_n\}_{n=1}^{\infty}$ *converging to f N-a.e. on X.*

Proof. Let $A = \{x;\ |f(x)| = +\infty\}$, $A_n = \{x; |f_n(x)| = +\infty\}$, $C = A \cup \bigcup_{n=1}^{\infty} A_n$,

$E_n(\varepsilon_i) = \{x \in X - C;\ |f_n(x) - f(x)| \geqq \varepsilon_i\}$, where $\{\varepsilon_i\}_{i=1}^{\infty}$ is a sequence of positive numbers converging on 0. There exists a sequence $\{n_i\}_{i=1}^{\infty}$ such that $E_{n_i}(\varepsilon_i) \in N_{k_i}$, where $\{k_i\}_{i-1}^{\infty}$ is a sequence from Lemma 1, and $n_{i+1} > n_i$ for $i = 1, 2, \ldots$. Let us put $E_i = E_{n_i}(\varepsilon_i)$, $R_n = \bigcup_{i=n}^{\infty} E_i$, $Q = \bigcap_{n=1}^{\infty} R_n$. By Lemma 1, the sets R_n belong to N_n for $n = 1, 2, \ldots$. We assume that $Q \notin N$. There exists an n_0 such that $Q \notin N_{n_0}$, but $R_{n_0} \in N_{n_0}$, $Q \subset R_{n_0}$ and $Q \in S$, so $Q \in N_{n_0}$ — a contradiction. Hence $Q \in N$. If $x_0 \in X - (Q \cup C)$, then there exists an i_0 such that $|f_{n_i}(x_0) - f(x_0)| < \varepsilon_i$ for each positive integer $i \geqq i_0$. Since $Q \cup C \in N$, we obtain that $f_{n_i} \xrightarrow[i \to \infty]{} f$ N-a.e. on X.

Theorem 2. *If a sequence $\{f_n\}_{n=1}^{\infty}$ of functions defined on X converges to f with respect to a small system $\mathcal{N}$, then this sequence converges to f with respect to the σ-ideal $N = \bigcap_{r=1}^{\infty} N_r$.*

Proof. By Remark 1, each subsequence $\{f_{n_m}\}_{m=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$ converges with respect to the small system $\mathcal{N}$ to f and, by Theorem 1, there exists a subsequence $\{f_{n_{m_p}}\}_{p=1}^{\infty}$ of $\{f_{n_m}\}_{m=1}^{\infty}$ converging to f N-a.e. on X, so $f_n \xrightarrow[n \to \infty]{N} f$.

Theorem 3. *Let $\mathcal{N}$ be an upper semicontinuous small system. If a sequence $\{f_n\}_{n=1}^{\infty}$ of functions defined on X converges to f N-a.e. on X, then $\{f_n\}_{n=1}^{\infty}$ converges on f with respect to the small system $\mathcal{N}$.*

Proof. Let C be defined as in the proof of Theorem 1, $B = \{x;\ f_n(x) \underset{n \to \infty}{\not\longrightarrow} f(x)\}$, $Q = B \cup C$. Of course, $Q \in N$.

We have defined the sets $E_k(\varepsilon) = \{x;\ |f_k(x) - f(x)| \geqq \varepsilon\}$ for $k = 1, 2, \ldots$ and each positive ε. Let $R_n(\varepsilon) = \bigcup_{k=n}^{\infty} E_k(\varepsilon)$, $M(\varepsilon) = \bigcap_{n=1}^{\infty} R_n(\varepsilon)$. If $x_0 \notin Q$, then $\lim_{k \to \infty} f_k(x_0) = f(x_0)$ and there exists a positive integer n such that, for each positive $k \geqq n$, $|f_k(x_0) - f(x_0)| < \varepsilon$. Hence $x_0 \notin R_n(\varepsilon)$ and $x_0 \notin M$; so $M \subset Q$. We obtain that $M \in N$ because $M \in S$ and $Q \in N$.

Now suppose that the sequence $\{f_n\}_{n=1}^{\infty}$ does not converge to f with respect to $\mathcal{N}$. There exist a positive ε_0 and a positive integer m_0 such that, for every positive integer i, there exists a positive integer $k_i \geqq i$ such that $E_{k_i}(\varepsilon_0) \notin N_{m_0}$. Hence $R_{k_i}(\varepsilon_0) \notin N_{m_0}$ for $i = 1, 2, \ldots$. The sequence $\{R_{k_i}(\varepsilon_0)\}_{i=1}^{\infty}$ is a nonincreasing sequence of sets and the small system $\mathcal{N}$ is upper semicontinuous, therefore $M \notin N$, which is a contradiction. We obtain that $f_n \xrightarrow[n \to \infty]{\mathcal{N}} f$.

Lemma 2. *If, for each subsequence $\{f_{n_m}\}_{m=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$, there exists a subsequence $\{f_{n_{m_p}}\}_{m=1}^{\infty}$ of $\{f_{n_m}\}_{m=1}^{\infty}$ such that $f_{n_{m_p}} \xrightarrow[p \to \infty]{\mathcal{N}} f$, then $f_n \xrightarrow[n \to \infty]{\mathcal{N}} f$.*

Proof. Suppose that the sequence $\{f_n\}_{n=1}^{\infty}$ does not converge to f with respect to $\mathcal{N}$. Then there exist $\varepsilon_0 > 0$ and a positive integer m_0 such that, for each positive integer i, there exists a positive integer $n_i > i$ such that

$$\{x: |f_{n_i}(x) - f(x)| \geqq \varepsilon_0\} \notin N_{m_0}.$$

By our assumption, there exists a subsequence $\{f_{n_{i_p}}\}_{p=1}^{\infty}$ of $\{f_{n_i}\}_{i=1}^{\infty}$ convergent to f with respect to $\mathcal{N}$. Then there exists some p_{ε_0, m_0} such that $\{x: |f_{n_{i_p}}(x) - f(x)| \geqq \varepsilon_0\} \in N_{m_0}$ for $p \geqq p_{\varepsilon_0, m_0}$, a contradiction.

Theorem 4. *Let $\mathcal{N}$ be an upper semicontinuous small system. If a sequence $\{f_n\}_{n=1}^{\infty}$ converges to f with respect to an σ-ideal $N = \bigcap_{r=1}^{\infty} N_r$, then $\{f_n\}_{n=1}^{\infty}$ converges to f with respect to the small system $\mathcal{N}$.*

Proof. Suppose that $f_n \xrightarrow[n\to\infty]{\mathcal{N}}\!\!\!\!\!\!/\;\; f$; then there exists a subsequence $\{f_{n_m}\}_{m=1}^{\infty}$ with no subsequence converging to f with respect to $\mathcal{N}$. By Theorem 3, no subsequence $\{f_{n_{m_p}}\}_{p=1}^{\infty}$ converges to f N-a.e. — a contradiction.

Corollary 1. *If $\mathcal{N}$ is an upper semicontinuous small system, then both the convergence with respect to a small system $\mathcal{N}$ and that with respect to a σ-ideal $N = \bigcap_{r=1}^{\infty} N_r$ are equivalent.*

Remark 2. If $\mathcal{N}$ is not upper semicontinuous, then the above-mentioned convergences are not equivalent. The convergence with respect to an σ-ideal $N = \bigcap_{r=1}^{\infty} N_r$ does not imply that with respect to $\mathcal{N}$.

Proof. By the assumption there exists a nonincreasing sequence $\{E_i\}_{i=1}^{\infty}$ of sets belonging to S and, for this sequence, there exists a positive integer r_0 such that, for each positive integer i, $E_i \notin N_{r_0}$ and $\bigcap_{i=1}^{\infty} E_i \in N$. Let us put $f_i(x) = \chi_{E_i}(x)$ and $f(x) = 0$ for $x \in X$. If $x \notin \bigcap_{i=1}^{\infty} E_i$, then $f_i(x) \xrightarrow[i\to\infty]{} f(x)$. Hence $f_i(x) \xrightarrow[i\to\infty]{} f(x)$ N-a.e. on X, so $f_i \xrightarrow[i\to\infty]{N} f$. Since, for each i, $\{x: |f_i(x) - f(x)| \geqq \varepsilon\} = E_i \notin N_{r_0}$ where $\varepsilon > 0$, therefore $f_i \xrightarrow[i\to\infty]{\mathcal{N}}\!\!\!\!\!\!/\;\; f$.

Remark 3. Let $\mathcal{N}$, $\mathcal{N}'$ be small systems such that $N = \bigcap_{r=1}^{\infty} N_r = \bigcap_{r=1}^{\infty} N'_r$. The convergence with respect to $\mathcal{N}$ need not be equivalent to the convergence with respect to $\mathcal{N}'$.

Proof. Let (X, S, m) be a measurable space with the finite measure m. We put $N_r = \left\{E \in S: m(E) < \frac{1}{r}\right\}$ for $r = 1, 2, \ldots$. The convergence with respect to $\mathcal{N}$

is the convergence in measure m. The set $N = \bigcap_{r=1}^{\infty} N_r$ is the σ-ideal of sets of null measure. Let $N'_r = N$ for $r = 1, 2, \ldots$. The convergence with respect to $\mathcal{N}'$ is the uniform convergence almost everywhere on X (i.e. there exists a set $A \in N$ such that $f_n \rightrightarrows f$ on $X - A$). It is easy to show when the convergences with respect to the different small systems are equivalent. For this reason, we recall the following definitions and theorems (see [2, p. 492—493]).

Definition [2]. *Let $\mathcal{N}$, $\mathcal{M}$ be two small systems on S and $N = \bigcap_{r=1}^{\infty} N_r$, $M = \bigcap_{r=1}^{\infty} M_r$. We say that $\mathcal{N}$ is the absolutely continuous system with respect to $\mathcal{M}$ if and only if, for each positive integer r, there exists a positive integer n_r such that if $E \in M_{n_r}$, then $E \in N_r$. We use the notation $\mathcal{N} \ll_{\varepsilon} \mathcal{M}$. We say that $\mathcal{N}$ is predominated by a system $\mathcal{M}$ if and only if $M \subset N$. We use the notation $\mathcal{N} \ll \mathcal{M}$.*

Theorem [2]. *If $\mathcal{N} \ll_{\varepsilon} \mathcal{M}$, then $\mathcal{N} \ll \mathcal{M}$.*

Theorem [2]. *Let $\mathcal{N}$, $\mathcal{M}$ be two small systems on S. If $\mathcal{N}$ is the upper semicontinuous system, then the following conditions are equivalent*

(1) $\mathcal{N} \ll_{\varepsilon} \mathcal{M}$

(2) $\mathcal{N} \ll \mathcal{M}$.

Example 1. Let N_r be the family of all measurable subsets of the set $\bigcup_{n=1}^{\infty} \left\langle n, n + \frac{1}{r+3} \right)$ for $r = 1, 2, \ldots$. Let M_r be the family of all measurable subsets of the set $\bigcup_{n=1}^{\infty} \left(n - \frac{1}{r+3}, n \right\rangle$ for $r = 1, 2, \ldots$.

The sequences $\{N_r\}$ and $\{M_r\}$ are small systems such that $\bigcap_{r=1}^{\infty} N_r = \bigcap_{r=1}^{\infty} M_r$, but $\mathcal{N}$ is not absolutely continuous with respect to $\mathcal{M}$ and $\mathcal{M}$ is not absolutely continuous with respect to $\mathcal{N}$.

Theorem 5. *Let $\mathcal{N}$ and $\mathcal{M}$ be two small systems. The convergences with respect to these systems are equivalent if and only if $\mathcal{N} \ll_{\varepsilon} \mathcal{M}$* and *$\mathcal{M} \ll_{\varepsilon} \mathcal{N}$.*

Proof. Let there exist for each positive integer m a positive integer n_m such that if $E \in M_{n_m}$, then $E \in N_m$. Let $f_n \xrightarrow[n \to \infty]{\mathcal{M}} f$, then, for each $\varepsilon > 0$ and for each positive integer m, there exists a positive integer $n_{\varepsilon, m}$ such that

$$\{x\colon |f_n(x) - f(x)| \geqq \varepsilon\} \in M_{n_m}, \quad \text{for} \quad n \geqq n_{\varepsilon, m},$$

so $\{x\colon |f_n(x) - f(x)| \geqq \varepsilon\} \in N_m$ and $f_n \xrightarrow[n \to \infty]{\mathcal{N}} f$.

Now we suppose that $\mathcal{N}$ is not absolutely continuous with respect to $\mathcal{M}$. Then there exists a positive integer n_0 such that for each positive integer n there exists a set E_n such that $E_n \in M_n$ and $E_n \notin N_{n_0}$. Let $f_n(x) = X_{E_n}(x)$, $f(x) \equiv 0$ and

$0 < \varepsilon < 1$. We observe that for each n $\{x\colon |f_n(x) - f(x)| \geqq \varepsilon\} = E_n \notin N_{n_0}$ and $f_n \xrightarrow[n \to \infty]{\mathcal{N}}\!\!\!\!\!\!\!/\;\; f$ but for each positive integer m there exists $n_m = m$ such that for $n \geqq n_m$ $\{x\colon |f_n(x) - f(x)| \geqq \varepsilon\} \in M_m$ and $f_n \xrightarrow[n \to \infty]{\mathcal{M}} f$.

Corollary 2. *If $\mathcal{N}$ is not predominated by $\mathcal{M}$, then the convergence with respect to $\mathcal{M}$ does not imply the convergence with respect to $\mathcal{N}$.*

Corollary 3. *If $\mathcal{N}$, $\mathcal{M}$ are upper semicontinuous small systems such that $\bigcap\limits^{\infty} N_r = \bigcap\limits_{r=1}^{\infty} M_r$, then the convergences with respect to these systems are equivalent.*

Corollary 4. *If N is an σ-ideal, $N_r = N$ for $r = 1, 2, \ldots$ and $\mathcal{N}$ is upper semicontinuous, and $\mathcal{M}$ is a small system such that $\bigcap\limits_{r=1}^{\infty} M_r = N$, then $\mathcal{M}$ is an upper semicontinuous system and the convergences with respect to those systems are equivalent.*

Let $[f]$ denote the class of all S-measurable real functions equivalent to f and $\mathfrak{M}$ denotes the family of these classes of equivalence.

Definition 5. *We say that a sequence $\{[f_n]\}_{n=1}^{\infty}$ of elements $\mathfrak{M}$ converges to $[f] \in \mathfrak{M}$ with respect to the small system $\mathcal{N}$ (abbr. $[f_n] \xrightarrow[n \to \infty]{\mathcal{N}} [f]$) if and only if $f_n \xrightarrow[n \to \infty]{\mathcal{N}} f$.*

Theorem 6. *The space $\mathfrak{M}$ is equipped with the Fréchet topology generated by the convergence with respect to a small system $\mathcal{N}$.*

Proof. By Remark 1, Lemma 2 and Definition 5 the space $\mathfrak{M}$ equipped with the convergence with respect to $\mathcal{N}$ is an L^*-space ([1, p. 90]). We shall show that the following condition is fulfilled. By Definition 5 we can formulate this condition for representatives of suitable classes of equivalence.

If $f_j \xrightarrow[j \to \infty]{\mathcal{N}} f$ and $f_{j,n} \xrightarrow[n \to \infty]{\mathcal{N}} f_j$ for $j = 1, 2, \ldots$, then there exist two sequences of positive integers $\{j_p\}_{p=1}^{\infty}$, $\{n_p\}_{p=1}^{\infty}$ such that $f_{j_p, n_p} \xrightarrow[p \to \infty]{\mathcal{N}} f$.

If the above-mentioned L^*-space fulfils this condition, then it is equipped with the Fréchet topology generated by the convergence with respect to $\mathcal{N}$.

For each positive integer p, there exists positive integers $k_1^{(p)}$ and $k_2^{(p)}$ such that if $E_1 \in N_{k_1^{(p)}}$, $E_2 \in N_{k_2^{(p)}}$, then $E_1 \cup E_2 \in N_p$.

For each positive integer p, there exists a positive integer j_p such that, for each positive integer $j \geqq j_p$, $\left\{x\colon |f_j(x) - f(x)| \geqq \frac{1}{2p}\right\} \in N_{k_1^{(p)}}$. We may assume that $j_{p+j} > j_p$ for $p = 1, 2, \ldots$.

For each positive integer p, there exists a positive integer n_p such that, for each positive integer $n \geqq n_p$, $\left\{x\colon |f_{j_p,n}(x) - f_{j_p}(x)| \geqq \frac{1}{2p}\right\} \in N_{k_2^{(p)}}$. We may assume that $n_{p+1} > n_p$ for $p = 1, 2, \ldots$. Let $\varepsilon > 0$ and m a positive integer. Let us put $p_{\varepsilon,m} = \max\left(\left[\frac{1}{e}\right] + 1, m\right)$; then, for each $p \geqq p_{\varepsilon,m}$, we have the following inclusions:

$$\{x\colon |f_{j_p,n_p}(x) - f(x)| \geqq \varepsilon\} \subset \left\{x\colon |f_{j_p,n_p}(x) - f(x)| \geqq \frac{1}{p}\right\} \subset$$

$$\subset \left\{x\colon |f_{j_p}(x) - f(x)| \geqq \frac{1}{2p}\right\} \cup \left\{x\colon |f_{j_p,n_p}(x) - f_{j_p}(x)| \geqq \frac{1}{2p}\right\}.$$

The last set belongs to N_p, thus it belongs to N_m.

The sets $\{x\colon |f_{j_p,n_p}(x) - f(x)| \geqq \varepsilon\}$ belong to N_m for $p \geqq p_{\varepsilon,m}$ because they are subsets of sets belonging to N_m.

Since the above-mentioned condition is fulfilled, the proof is completed.

Corollary 5. *Let S be the class of sets having the Baire property and N the class of sets of the first category; then there is no upper semicontinuous small system such that $N = \bigcap_{r=1}^{\infty} N_r$.*

Proof. Suppose that there exists such a small system. Then the convergence with respect to it is equivalent to the convergence with respect to an σ-ideal N and generates the Fréchet topology, a contradiction (see [4, p. 93]).

REFERENCES

[1] ENGELKING, R.: General Topology, Warszawa 1977.

[2] NEUBRUNN, T.—RIEČAN, B.: Miera a integrál, Bratislava 1981.

[3] RIEČAN, B.: Abstract formulation of some theorems of measure theory. Mat.-Fyz. Časopis SAV 16, 1966, 268—273.

[4] WAGNER, E.: Sequences of measurable functions. Fund. Math. CXII, 1981, 89—102.

Received August 9, 1985

Institute of Mathematics
University of Łódź
S. Banacha 22, POLAND.

СХОДИМОСТЬ ПО МАЛЫХ СИСТЕМАХ ПОСЛЕДОВАТЕЛЬНОСТИ ДЕЙСТВИТЕЛЬНЫХ ФУНКЦИЙ

Jerzy Niewiarowski

Резюме

В этой работе введено понятие сходимости по малых системах. Доказано, что сходимость по малых системах и сходимость по соответствующем c-идеале эквивалентны в случае полунепрерывной сверху малой системы.

Дальше рассматривается пространство $\mathfrak{M}$ всех классов эквивалентности S-измеримых действительных функций. Доказывается, что сходимость по малых системах преображает это пространство в топологическое пространство.