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ONE-VARIABLE EQUATIONALLY COMPACT DISTRIBUTIVE LATTICES

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The lattices of the title have been characterized by D. A. Kelly [K] and (independently) by R. Beazer [B] as those which are complete and (bi-) infinitely distributive. We have obtained this characterization as a corollary of our characterization of one-variable equationally compact semilattices with pseudocomplementation which satisfy a certain partial distributive law [BF, Corollary 7]. This Note is to point out that the characterization can also be deduced via our preceding paper [BFK], which route yields somewhat more: a partial positive answer to Mycielski's question (see [T] for a discussion of this and related matters) in that the equationally compact lattice is exhibited as a retract of a topologically compact containing semilattice on which it acts as continuous endormorphisms.

We start by recalling the "regular left representation" of a semigroup [C-PI p. 9]: This assigns to every s in the semigroup S the transformation of left translation by s: $\lambda_s(x)=sx$. In terms of it associativity may be expressed as $\lambda_{st}=\lambda_s\lambda_t$, and distributivity (say of $\vee$ over $\wedge$) as the requirement that (taking the semigroup operation to be $\vee$) each λ_s be an endomorphism (of the $\wedge$-semilattice structure). Thus every distributive lattice may be construed as a semilattice (for the single operation $\wedge$) on which there acts a set (here indexed by its elements) of endomorphisms: a structure we have dubbed in [BFK] with the acronym SENDO.

In order to apply our SENDO result we must verify that this way of construing a distributive lattice yields the same one-variable equations as does that using the two lattice operations as the term generators. Since $\lambda_s(t)=s\vee t$, every SENDO term may be expressed as a lattice term. Conversely, every lattice term in a distributive lattice may be put into conjunctive form; each of the conjucts may be reduced, using idempotence, to a disjunction of distinct monomials: thus if the term is one-variable, then at most one of these can fail to be an element of S; whence each of these disjuncts can be expressed as the result of operating on the variable or on an element of S with a composition of λ's — and this achieves the re-expression as a SENDO term.

We can now read off from the theorem in [BFK] (taking into account the following Notes (2) and (4), and the simplification in (ii) which results from having a distributive lattice on which the endomorphisms are just $\vee$ with fixed elements):

Let S be a distributive lattice. Then the following are equivalent:
(i) S is one-variable equationally compact.
(ii) S is complete and satisfies both infinite distributive laws.
(iii) S is a retract (in the algebraic sense) of a compact, Hausdorff, zero-dimensional topological semilattice to which its regular representation extends as an action by continuous endomorphisms.

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ЭКВАЦИОНАЛЬНО КОМПАКТНЫЕ ДИСТРИБУТИВНЫЕ РЕЩЕТКИ С ОДНОЙ ПЕРЕМЕННОЙ

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Резюме

Приводится новое доказательство теоремы, в которой охарактеризованы эквационально компактные дистрибутивные решетки с одной переменной.