Mathematica Slovaca

Mária Kráľová

Constant ratio-function of Lindenmayer systems

Mathematica Slovaca, Vol. 35 (1985), No. 3, 283--294

Persistent URL: `http://dml.cz/dmlcz/130330`

Math. Slovaca 35, 1985, No. 3, 283–294

CONSTANT RATIO-FUNCTION OF LINDENMAYER SYSTEMS

MÁRIA KRÁĽOVÁ

1. Introduction

Mathematical models describing the process of a biological development of simple organisms, which are called Lindenmayer systems or L-systems, have been in the past few years in the centre of attention of both formal language theorists and mathematicians. So far two special functions have been investigated — the growth function and the letter occurrence function — as the functions of L-systems of biological origin.

The first paper dealing with the analysis of growth functions was Szilard's [6]. The growth function is defined as a function over the set of natural numbers, whose value is the length of the t-th word of a developmental sequence for any number t. The reader can find a detailed survey of results concerning the growth functions, e.g. in [7]. Then for a given letter the letter occurrence function associates with the natural number t the number of occurrences of this letter in the t-th word produced by the given L-systems. The reader who want to obtain a deeper insight into this topic is referred to [7].

This is the first paper in which mathematical properties of ratio-functions are investigated. Ratio-functions are new special functions of L-systems, which can be widely used in modelling biological reality and they have the following motivation:

During the past two decades the most popular method of studying the cell cycle in autoradiography has been the method of fraction labelled mitosis *(FLM)*. Experimentally it starts by introducing a radioactive molecule into the cell population. Then specimens of tissue or cell culture are taken at fixed intervals of time and microautoradiographs are prepared. The number of labelled mitotic cells and the total number of cells are counted. Thus the *FLM*-curve is a curve defined on the basis of the cell mitotic cycle, which is important for determining the increase of the cell population.

In [2] a $D0L$-system is used in this method as a model of the cell population development. In that model a new type of the function (we shall call it the ratio-function) corresponds to the *FLM*-curve. It is defined as follows:

For a $D0L$-system $G=(W, \delta, w)$ and its subaxiom w' the ratio-function associates with a natural number t the ratio of the letter occurrence function of a given L-system with the axiom restricted to the subaxiom w' and the letter occurrence function of a given L-system.

In this paper we shall characterize L-systems with constant ratio-functions using the same method as in [3], i.e. by properties of levels in L-systems.

2. Preliminaries

In this section we give a brief survey of the most needed notions and notations used in the paper. Throughout the paper we shall deal only with deterministic Lindenmayer systems without interactions ($D0L$-systems)

The set of natural numbers $\{1, 2, \ldots\}$ will be denoted by N, the set of nonnegative integers $\{0, 1, \ldots\}$ by Z^+ and the set of nonnegative reals by $\mathcal{R}^+$.

Definition 2.1. *A deterministic Lindenmayer system without interactions is a triple $G=(W, \delta, w)$ consisting of a finite nonempty set W, called alphabet, a total mapping δ: $W \to W^*$ (it determines the production rules) and an initial word $w \in W$, called axiom.*

We remark that the set of all words over some finite alphabet W is denoted by W^* and ε denotes the empty word.

We can extend the domain of function δ to W^* in a natural way, i.e.

$$\delta(\varepsilon)=\varepsilon$$
$$\delta(ab)=\delta(a)\delta(b)$$

for $a \in W$, $b \in W^*$.

Next we define

$$\delta^t(b)=\delta(\delta^{t-1}(b))$$

for each $t \in N$, $t \geq 2$ and $b \in W^*$.

Definition 2.2. *Let $G=(W, \delta, w)$ be a $D0L$-system and let $a, b \in W$. Then*

$$a \rhd_G b \ \textit{ iff } \ \delta(a)=xby \quad \textit{for some} \quad x, y \in W^*,$$

$\rhd_G^+$, $\rhd_G^*$ are the transitive closure and the reflexive and transitive closure of $\rhd_G$, respectively,

$$a \equiv_G b \ \textit{ iff } \ a \rhd_G^* b \quad \textit{and} \quad b \rhd_G^* a.$$

Definition 2.3. *Let $G=(W, \delta, w)$ be a $D0L$-system and let $a \in W$. An equivalence class*

$$[a]_G=\{b \in W;\ b \equiv_G a\}$$

is called the level of the $D0L$-system G generated by a.

Remark 2.1. The subscript G will be omitted in notations always when it is clear which G is considered.

We conclude this section by mentioning some structural properties of letters in the $D0L$-system $G=(W, \delta, w)$.

Definition 2.4. *Let $G=(W, \delta, w)$ be a $D0L$-system. A letter $a \in W$ is* mortal */$a \in M$/ if $\delta^j(a)=\varepsilon$ for some $j \in Z^+$;* recursive */$a \in R$/ if $\delta^j(a) \in W^*aW^*$ for some $j \in N$;* monorecursive */$a \in MR$/ if $\delta^j(a) \in M^*aM^*$ for some $j \in N$;* expanding */$a \in E$/ if $\delta^j(a) \in W^*aW^*aW^*$ for some $j \in N$;* accessible *from a word $b \in W$ /$a \in U(b)$/ if $\delta^i(b) \in W^*aW^*$ for some $j \in N$;* z-mortal *for $z \in W$ /$a \in z$-M/ if there exists a $j_0 \in N$ such that $\#_z(\delta^j(a))=0$ for all $j \geq j_0$. /The symbol $\#_a(w)$ denotes the number of occurrences of the letter a in the word w./*

Definition 2.5. *Let $G=(W, \delta, w)$ be a $D0L$-system. A letter $a \in W$ is called* mortal with the index of mortality *t /$a \in M^{(t)}$/,* recursive with the index of recursivity *t /$a \in R^{(t)}$/,* monorecursive with the index of monorecursivity *t /$a \in MR^{(t)}$/,* expanding with the index of expansion *t /$a \in E^{(t)}$/,* z-mortal with the index of z-mortality *t /$a \in z$-$M^{(t)}$/ if t is the smallest number for which the condition of mortality, recursivity, monorecursivity, expansion, z-mortality, respectively, is satisfied. The number t is called* the index of mortality, recursivity, monorecursivity, expansion, z-mortality, *respectively, of the letter a.*

Definition 2.6. *The level $[a]$ is called mortal, recursive, monorecursive, expanding, z-mortal if the letter a is mortal, recursive, monorecursive, expanding, z-mortal, respectively.*

Definition 2.7. *The level $[a]$ is called monorecursive with the index of monorecursivity t if the letter a is monorecursive with the index of monorecursivity t.*

Remark 2.2. If $[a]$ is a monorecursive level of the $D0L$-system, then each $b \in [a]$ is monorecursive.

If $[a]$ is a monorecursive level with the index of monorecursivity t of the $D0L$-system, then each $b \in [a]$ is a monorecursive letter with the index of monorecursivity t.

3. Ratio-function, its characterization and properties

Definition 3.1. *Let $G=(W, \delta, w)$ be a $D0L$-system. A word w' is called the subaxiom of the axiom w if the relation*

$$w \in W^*w'W^*$$

is satisfied.

It is easy to see that the system $G=(W, \delta, w')$ is again a $D0L$-system.

Definition 3.2. *Let $G=(W, \delta, w)$ be a D0L-system and let w' be a subaxiom of the axiom w. A quadruple $G'=(W, \delta, w, w')$ is called a D0L-system with a subaxiom.*

Definition 3.3. *Let $G'=(W, \delta, w, w')$ be a D0L-system with a subaxiom and let $a \in W$. Then the function r_a: $Z^+ \to \mathcal{R}^+$, defined by*

$$r_a(j)=\frac{\#_a(\delta^j(w'))}{\#_a(\delta^j(w))}$$

if $\#_a(\delta^j(w)) \neq 0$ and not defined if $\#_a(\delta^j(w))=0$, is called the ratio-function of G' determined by a.

Theorem 3.1. *For each number $\frac{p}{q}$, where $p \in Z^+$, $q \in N$, $p \leq q$, there exists a D0L-system with a subaxiom $G'=(W, \delta, w, w')$ such that*

$$r_a(j)=\frac{p}{q} \quad \text{for all} \quad j \in Z^+.$$

Proof. The $D0L$-system with a subaxiom $G'=(W, \delta, w, w')$ is constructed as follows

$$W=\{a\}, \quad \delta(a)=a, \quad w=a^q$$

and for $p=0$ we put $w'=\varepsilon$, for $p \neq 0$ we put $w'=a^p$.

The following lemmas will be used in the proofs of theorems.

Lemma 3.1. *Let $G=(W, \delta, w)$ be a D0L-system and let a, $b \in W$ such that $a \neq b$. A letter b is a-mortal with the index of a-mortality 1 iff $a \notin U(b)$.*

Proof. It follows easily from the definitions.

Lemma 3.2. *Let $G=(W, \delta, w)$ be a D0L-system and let $[a]$ be its monorecursive level with the index of monorecursivity 1. Then $[a]$ consists of exactly one element.*

Proof. Suppose that the assumptions of the lemma are satisfied. Evidently $a \in [a]$. Since $a \in MR^{(1)}$, we obtain

$$\delta(a)=x_1 a y_1, \tag{1}$$

where x_1, $y_1 \in M$.

Suppose further that $b \in [a]$, $b \neq a$ and $b \in MR^{(1)}$. Hence there holds

$$\delta(a)=x_2 b y_2, \tag{2}$$

where x_2, $y_2 \in W$. Relations (1), (2) imply $b \in M$, because G is a $D0L$-system. This contradiction proves our lemma.

Lemma 3.3. *Let $G=(W, \delta, w)$ be a D0L-system. Let $[a]$ be its monorecursive* *level with the index of monorecursivity* 1 *and let* $b \in W$, $b \notin [a]$, *be such that* b *is* a*-mortal. Then* b *is* a*-mortal with the index of* a*-mortality* 1.

Proof. Let G be a given $D0L$-system and let the other assumptions of the lemma be satisfied too. Then we obtain $b \neq a$ and

$$\#_a(\delta^j(a)) > 0 \quad \text{for all} \quad j \in N.$$

Since we assume that b is an a-mortal letter, we have: There exists a $j_0 \in N$ such that $\#_a(\delta^j(b)) = 0$ for $j \geq j_0$. Assume further that $j_0 \neq 1$. It means:

There exists a $s \in N$, $1 < s < j_0$ such that $\#_a(\delta^s(b)) > 0$. Then

$$\#_a(\delta^j(b)) > 0 \quad \text{for all} \quad j \geq s$$

by the condition that $a \in MR^{(1)}$. However, it is a contradiction with the a-mortality of the letter b.

Theorem 3.2. *Let* $G' = (W, \delta, w, w')$ *be a* $D0L$*-system with a subaxiom, let* $[a]$ *be its monorecursive level with the index of monorecursivity* 1 *and let for each* $b \in W$, $b \neq a$ *one of the following conditions hold:*

(i) b *is* a*-mortal,*

(ii) b *is a nonrecursive letter and* $a \in U(b)$.

Then there exists a number j_0 *such that*

$$r_a(j) = \frac{p}{q} \quad \text{for all} \quad j \geq j_0, \ j \in N,$$

where $p \in Z^+$, $q \in N$.

Proof. Suppose that $[a]$ is a monorecursive level with the index of monorecursivity 1 of the given $D0L$-system with a subaxiom G' and let $b \neq a$. Then $[b] \neq [a]$ according to lemma 3.2. For the case (i) suppose now that b is a-mortal. By lemma 3.3 $b \in a\text{-}M^{(1)}$ hold. It is equivalent to $a \notin U(b)$ by lemma 3.1. It means that

$$\#_a(\delta^j(b)) = 0 \quad \text{for all} \quad j \in N.$$

For the case (ii) let b_s, $s = 1, 2, \ldots, l$ be all letters of the alphabet W of the given $D0L$-system with a subaxiom G' which are nonrecursive and $a \in U(b_s)$.

It is clear that for every $k > l$ and every b_s, $s = 1, 2, \ldots, l$ there holds

$$\#_a(\delta^{k+1}(b_s)) = \#_a(\delta^k(b_s)).$$

Put $j_0 = l$.

Since $a \in U(b_s)$ for all $s = 1, 2, \ldots, l$ and $a \in MR^{(1)}$ we obtain

$$\#_a(\delta^{j_0}(b_s)) \neq 0.$$

Hence according to (i) there holds

$$\#_a(\delta^{j_0}(w)) = \#_a(w) + \sum_{s=1}^{l} \#_{b_s}(w) \, \#_a(\delta^{j_0}(b_s)).$$

Then the ratio-function is given as

$$r_a(j_0)=\frac{\#_a(w')+\sum_{s\in A}\#_{b_s}(w')\,\#_a(\delta^{j_0}(b_s))}{\#_a(w)+\sum_{s=1}^{l}\#_{b_s}(w)\,\#_a(\delta^{j_0}(b_s))},$$

where

$$A=\{s\,;\;b_s\in w'\}.$$

Since $a\in MR^{(1)}$, we have

$$\#_a(\delta^{j}(b_s))=\#_a(\delta^{j_0}(b_s)),\quad j\geq j_0,\ j\in N.$$

Then

$$r_a(j)=r_a(j_0)\quad\text{for all}\quad j\geq j_0.$$

The next theorem is an extension of the previous assertion for the $D0L$-system with a subaxiom with the monorecursive level with the index of monorecursivity t, $t>1$.

Theorem 3.3. *Let $G'=(W,\delta,w,w')$ be a $D0L$-system with a subaxiom, let $[a]$ be its monorecursive level with the index of monorecursivity t, $t>1$, $t\in N$ and let for any $b\in W$, $b\neq a$, one of the following conditions holds:*
(i) $a\notin U(b)$,
(ii) $a\in U(b)$ and b is either a monorecursive or a nonrecursive letter.
Then there exists a number j_0 such that for $j\geq j_0$, $j\in N$ the ratio-function $r_a(j)$ is a periodic function with the period t.

Proof. Let $b\in W$, $b\neq a$. We shall prove that there exist $j_0\in N$ and for any $l=0,1,\ldots,t-1$ some numbers $p_l\in Z^+$, $q_l\in N$ such that

$$r_a(j_0+nt+l)=\frac{p_l}{q_l}\quad\text{for any}\quad n\in N.$$

For b such that $a\notin U(b)$ we have $b\in a\text{-}M^{(1)}$ according to lemma 3.1. Hence

$$\#_a(\delta^n(b))=0\quad\text{for}\quad n\in N.$$

The property $a\in MR^{(t)}$ implies:
There exists a finite sequence $b_1,b_2,\ldots,b_{t-1}\in W$ such that

$$\begin{aligned}&\delta(a)\in M^*b_1M^*,\\&\delta^2(a)\in M^*b_2M^*,\\&\vdots\\&\delta^{t-1}(a)\in M^*b_{t-1}M^*,\\&\delta^t(a)\in M^*aM^*.\end{aligned}$$

We remark that $b_1,b_2,\ldots,b_{t-1}$ are all monorecursive letters of the alphabet W from which a is accessible.

There holds

$$\#_a(\delta^{nt-s}(b_s)) \neq 0$$

for all b_s, $s = 1, 2, \ldots, t-1$ and $n \in N$.

Suppose now that c_k, $k = 1, 2, \ldots, m$ are all letters of the alphabet W satisfying the conditions:

$$a \in U(c_k) \quad \text{and } c_k \text{ are nonrecursive.}$$

This implies:
for each $c_k \in W$, $k = 1, 2, \ldots, m$, there exists the smallest number j_k such that

$$\delta^{j_k}(c_k) \in W^* a W^*.$$

Denote now

$$A = \{k\,;\; k = 1, 2, \ldots, m,\ j_k \,(\operatorname{mod} t) = 0\}$$

and put

$$j_0 = \max_{k \in A} j_k + t.$$

Then

$$\sum_{k=1}^{m} \#_a(\delta^{j_0}(c_k)) = \sum_{k \in A} \#_a(\delta^{j_0}(c_k)).$$

If we consider that $a \in MR^{(t)}$, we have

$$\#_a(\delta^{j_0+nt}(c_k)) = \#_a(\delta^{j_0}(c_k)),$$

where $k = 1, 2, \ldots, m$, $n \in N$.

Let the axiom w be given by

$$w = x_1 x_2 \ldots x_h, \quad h \in N.$$

Denote

$$B = \{x_i\,;\; i = 1, 2, \ldots, h,\ x_i = c_k,\ k \in A\}$$

and

$$B' = \{x_i \in B\,;\; \#_{x_i}(w') \neq 0\}.$$

Hence

$$r_a(j_0 + nt) = \frac{\#_a(w') + \sum_{x_i \in B'} \#_a(\delta^{j_0}(x_i))}{\#_a(w) + \sum_{x_i \in B} \#_a(\delta^{j_0}(x_i))}, \quad n \in N$$

and the theorem is proved for $l = 0$.

We outline now the proof of this theorem for $l = 1, 2, \ldots, t-1$.

Denote

$$A_l = \{k\,;\; k = 1, 2, \ldots, m,\ j_k \,(\operatorname{mod} t) = l\},$$
$$B_l = \{x_i\,;\; x_i = c_k,\ k \in A_l\},$$
$$B_l' = \{x_i \in B_l\,;\; \#_{x_i}(w') \neq 0\}.$$

Then

$$r_a(j_0+nt+l)=\frac{\#_{b_{t-l}}(w')+\sum_{x_i\in B_l'}\#_a(\delta^{j_0+l}(x_i))}{\#_{b_{t-l}}(w)+\sum_{x_i\in B_l}\#_a(\delta^{j_0+l}(x_i))}$$

for $n\in N$.

It is clear, that it suffices now to put

$$p_l=\#_{b_{t-l}}(w')+\sum_{x_i\in B_l'}\#_a(\delta^{j_0+l}(x_i))$$

and

$$q_l=\#_{b_{t-l}}(w)+\sum_{x_i\in B_l}\#_a(\delta^{j_0+l}(x_i)).$$

To illustrate this theorem we give a simple example.

Example. Let $G'=(W,\delta,w,w')$ be a $D0L$-system with a subaxiom given by

$$W=\{a,b_1,b_2,b_3,c_1,c_2,c_3,c_4,c_5\},$$
$$w=b_1ab_2b_3c_1c_5,$$
$$w'=ab_2b_3c_1,$$

$$\begin{array}{ll}\delta\colon\ \delta(a)=b_1, & \delta(c_1)=c_2b_1,\\ \quad\ \delta(b_1)=b_2, & \delta(c_2)=c_3,\\ \quad\ \delta(b_2)=b_3, & \delta(c_3)=c_4,\\ \quad\ \delta(b_3)=a, & \delta(c_4)=c_5,\\ & \delta(c_5)=b_1.\end{array}$$

Let us table the beginning of the derivations for this $D0L$-system following one another with all the letters from W.

j	$\delta^j(a)$	$\delta^j(b_1)$	$\delta^j(b_2)$	$\delta^j(b_3)$	$\delta^j(c_1)$	$\delta^j(c_2)$	$\delta^j(c_3)$	$\delta^j(c_4)$	$\delta^j(c_5)$
0	a	b_1	b_2	b_3	c_1	c_2	c_3	c_4	c_5
1	b_1	b_2	b_3	a	c_2b_1	c_3	c_4	c_5	b_1
2	b_2	b_3	a	b_1	c_3b_2	c_4	c_5	b_1	b_2
3	b_3	a	b_1	b_2	c_4b_3	c_5	b_1	b_2	b_3
4	a	b_1	b_2	b_3	c_5a	b_1	b_2	b_3	a
5	b_1	b_2	b_3	a	b_1b_1	b_2	b_3	a	b_1
6	b_2	b_3	a	b_1	b_2b_2	b_3	a	b_1	b_2
7	b_3	a	b_1	b_2	b_3b_3	a	b_1	b_2	b_3
8	a	b_1	b_2	b_3	aa	b_1	b_2	b_3	a
9	b_1	b_2	b_3	a	b_1b_1	b_2	b_3	a	b_1
10	b_2	b_3	a	b_1	b_2b_2	b_3	a	b_1	b_2
11	b_3	a	b_1	b_2	b_3b_3	a	b_1	b_2	b_3
12	a	b_1	b_2	b_3	aa	b_1	b_2	b_3	a
$\vdots$									

From the table above it is easy to see that $a \in MR^{(4)}$, $a \in U(b_i)$ and $b_i \in MR$, $i = 1, 2, 3$, $a \in U(c_k)$, $c_k \notin R$, $k = 1, 2, \dots, 5$. Now we can easily determine j_k, $k = 1, 2, \dots, 5$:

$$j_1 = 4, \quad j_2 = 7, \quad j_3 = 6, \quad j_4 = 5, \quad j_5 = 4.$$

The set A from the proof of theorem 3.3 is given by

$$A = \{1, 5\}.$$

We have $j_0 = 8$.

The ratio-function $r_a(j)$ for the given $D0L$-system with a subaxiom $G' = (W, \delta, w, w')$ is

$$r_a(4n+8) = \frac{3}{4},$$
$$r_a(4n+9) = 1,$$
$$r_a(4n+10) = 1,$$
$$r_a(4n+11) = 0, \quad n = 0, 1, 2, \dots .$$

The value of the ratio-function $r_a(j)$ can be seen also from the following table.

j	0	1	2	3	4	5	6	7	8	9	10	11	12
$\#_a(\delta^j(w))$	1	1	1	1	3	1	1	1	4	1	1	1	4
$\#_a(\delta^j(w'))$	1	1	1	0	2	1	1	0	3	1	1	0	3

Up to now we have determined such $D0L$-system with a subaxiom which have at least one monorecursive level. Now we present a theorem stating a secure constant ratio-function for a $D0L$-system with a subaxiom has an expanding level.

Theorem 3.4. *Let $G' = (W, \delta, w, w')$ be a $D0L$-system with a subaxiom, let a be its expanding letter with the index of expansion* 1 *and let there for any $b \in W$, $b \neq a$ hold*

either that $a \notin U(b)$
or $a \in U(b)$ and b is nonrecursive.

Then there exists a number $j_0 \in Z^+$ such that

$$r_a(j) = \frac{p}{q} \quad \text{for all} \quad j \geq j_0,$$

where $p \in Z^+$, $q \in N$.

Proof. Suppose that $a \in W$ is an expanding letter with the index of expansion 1 of the given $D0L$-system with a subaxiom G'. Let there for $b \in W$ hold $b \neq a$ and $a \notin U(b)$.

Put

$$\#_a(w) = q$$
$$\#_a(w') = p$$

and

$$\#_a(\delta(a)) = m.$$

Then

$$\#_a(\delta^j(a)) = m^j \quad \text{for} \quad j \in Z^+,$$

because $a \in E^{(1)}$, and from the definition of the ratio-function it follows that

$$r_a(j) = \frac{pm^j}{qm^j} = \frac{p}{q} \quad \text{for all} \quad j \in Z^+.$$

So, in this case $j_0 = 0$.

Now we shall prove the assertion of the theorem if there exists b such that $a \in U(b)$ and b is a nonrecursive letter. The nonrecursivity of the letter b implies $b \notin U(a)$.

Let $b_1, b_2, \ldots, b_s$ be all letters of the axiom w such that $b_i \neq a$, $a \in U(b_i)$ and b_i are nonrecursive for $i = 1, 2, \ldots, s$.

Put

$$j_0 = s$$

and denote by

$$n_i = \#_a(\delta^{j_0}(b_i)) \quad \text{for all} \quad i = 1, 2, \ldots, s.$$

Hence by the notation

$$\#_a(\delta(a)) = m$$

there holds

$$r_a(j_0) = \frac{k'm^{j_0} + \sum_{i \in A} \#_{b_i}(w')n_i}{km^{j_0} + \sum_{i=1}^{s} \#_{b_i}(w)n_i},$$

where we denote by

$$A = \{i;\ b_i \in w'\},$$
$$k' = \#_a(w'),$$
$$k = \#_a(w).$$

Then generally

$$r_a(j_0 + t) = \frac{k'm^{j_0+t} + \sum_{i \in A} \#_{b_i}(w')n_i m^t}{km^{j_0+t} + \sum_{i=1}^{s} \#_{b_i}(w)n_i m^t}$$

for $t \in N$ and so

$$r_a(j) = r_a(j_0) \quad \text{for all} \quad j \geq j_0.$$

Remark 3.1. In the special case, when G' is the $D0L$-system with a subaxiom, the production rules of which have the following properties

$$\sum_{l=1}^{s} \#_{b_l}(\delta(b_i))=1,$$
$$\#_a(\delta(b_i))=0$$

for all $i=1, 2, \ldots, s$, the number j_0 can be determined exactly.

Denote by l_i the smallest index for which

$$\#_a(\delta^{l_i}(b_i))\neq 0$$

for all $i=1, 2, \ldots, s$ and define the values k_i, $i=1, 2, \ldots, s$, as

$$k_i=\#_a(\delta^{l_i}(b_i)).$$

Put now

$$j_0=\max_{i=1, 2, \ldots, s} l_i,$$

i.e. j_0 is the maximal index among the indices determining the first occurrence of the letter a by the application of the production rules to the letters b_i, $i=1, 2, \ldots, s$.

Since k_i is the number specifying the number of the occurrence of the letter a in the l_i-th step by the application of the production rules to b_i for each $i=1, 2, \ldots, s$, $a\in E^{(1)}$ and preserving all notations above, we obtain

$$n_i=\#_a(\delta^{j_0}(b_i))=k_i m^{j_0-l_i}.$$

It is clear that $j_0-l_i\geqslant 0$.

REFERENCES

[1] HERMAN, G. T., VITÁNYI, P. M. B.: Growth functions associated with biological development, Amer. Math. Monthly, 1976, 1—15.

[2] HROMKOVIČ, J., KELEMENOVÁ, A.: On kinetic models of cell population, Proc. of Simulation of Systems in Biology and Medicine '82, Prague, 1982, Microfishe No. 735.

[3] KELEMENOVÁ, A.: Levels in L-systems, Mathematica Slovaca, 33, 1983, 87—97.

[4] LINDENMAYER, A., ROZENBERG, G.: Automata, Languages, Development, North Holland, Amsterdam, 1976.

[5] ROZENBERG, G., SALOMAA, A.: The Mathematical Theory of L-systems, Academic Press, New York, 1980.

[6] SZILARD, A.: Growth functions of Lindenmayer systems, Univ. of Western Ontario Computer Science Department Technical Report No. 4, London, Canada, 1971.

[7] VITÁNYI, P. M. B.: Lindenmayer Systems: Structure, Languages and Growth functions, Mathematisch Centrum, Amsterdam, 1978.

Received April 15, 1983

Matematický ústav SAV
Obrancov mieru 49
814 73 Bratislava

ПОСТОЯННАЯ ДОЛЕВАЯ ФУНКЦИЯ СИСТЕМ ЛИНДЕНМАЙЕРА

Mária Kráľová

Резюме

$D0L$-система (т.е. детерминистическая система Линденмайера без взаимодействий) с подаксиомой есть четверка $G' = (W, \delta, w, w')$, где

1. $G = (W, \delta, w)$ — это $D0L$-система,
2. w' — подаксиома – это подслово аксиомы w.

Тогда долевая функция $r_a(j)$ системы G' как функция переменной j (j пробегает множество неотрицательных чисел) определена как функция, выражающая долю числа появлений символа a в слове, полученном из подаксиомы и числа появлений символа a в слове, полученном из аксиомы после j-того применения правила продукции δ. В статье характеризованы при помощи комбинаторического подхода через грамматические уровни такие $D0L$-системы с подаксиомой, долевые функции которых постоянны