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SEMIGROUPS SONTAINING SOVERED TWO-SIDED IDEALS

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In [5], semigroups containing one-sided covered ideals have been investigated. It turns out that a semigroup need not have one-sided covered ideals at all. As for twosided ideals, the situation is different. The purpose of the paper is to investigate the structure of semigroups containing two-sided covered ideals.

Definition 1. *A proper two-sided ideal M of a semigroup S is covered (briefly C-ideal) if $M \subset S(S-M)S$.*

$$I^a = \{x \in S/(a)_T = a \cup Sa \cup aS \cup SaS = x \cup Sx \cup xS \cup SxS = (x)_T\}$$

is the $\mathcal{I}$-class of S containing a.

The $\mathcal{I}$-class I^a is maximal, if $(a)_T$ is not a proper subset of any principal two-sided ideal of S.

It is known ([1]) that the $\mathcal{I}$-class I^a is maximal iff its complement is a maximal ideal of S.

When dealing with maximal ideals M_a, we shall denote the corresponding maximal $\mathcal{I}$-classes by M^a.

Theorem 1. *If S contains two different proper ideals M_1, M_2 such that $M_1 \cup M_2 = S$, then none of them is a C-ideal.*

Proof. If $M_1 \cup M_2 = S$, then $S - M_2 \subset M_1$, $S - M_1 \subset M_2$. If one of them were a C-ideal, e.g. M_1, then $M_1 \subset S(S-M_1)S \subset SM_2S \subset M_2$. Since $M_1 \cup M_2 = S$, it implies $M_2 = S$. Hence, we get a contradiction with our assumption that M_1, M_2 are proper two-sided ideals.

Corollary. *If S contains more than one maximal two-sided ideal, then none of them is a C-ideal of S.*

Theorem 2. *If M_1 and M_2 are two C-ideals of S, then $M_1 \cup M_2$ is a C-ideal of S.*

Proof. We show that if $M_1 \subset S(S-M_1)S$, $M_2 \subset S(S-M_2)S$, then $M_1 \cup M_2 \subset S[S-(M_1 \cup M_2)]S$.

Let $x \in M_1$, then $M_1 \subset S(S-M_1)S$ implies that there is $a \in S - M_1$ such that $x \in SaS$. There are two possibilities:

(1) $a \in S-(M_1 \cup M_2)$, then $x \in S[S-(M_1 \cup M_2)]S$.

(2) $a \in (S - M_1) \cap M_2$, then $a \in M_2 \subset S(S - M_2)S$. So there is $b \in S - M_2$ such that $a \in SbS$. The element b does not belong to M_1, since otherwise we would have $a \in SbS \subset SM_1S \subset M_1$ and it is contradicting with the choice of a. Therefore, $b \in S - M_2$, $b \in S - M_1$, so $b \in (S - M_1) \cap (S - M_2) = S - (M_1 \cup M_2)$. We have $x \in SaS \subset S(SbS)S \subset SbS \subset S[S - (M_1 \cup M_2)]S$. Hence $M_1 \subset S[S - (M_1 \cup M_2)]S$. And in the same way we can prove that $M_2 \subset S[S - (M_1 \cup M_2)]S$.

Theorem 3. *If M_1, M_2 are two C-ideals of S, then $M_1 \cap M_2$ is a C-ideal of S.*

Proof. It is well known ([7]) that $M_1 \cap M_2 \neq \emptyset$. It is enough to show that

$$M_1 \cap M_2 \subset S[S - (M_1 \cap M_2)]S.$$

From the ralation $M_1 \subset S(S - M_1)S$ we have

$$M_1 \cap M_2 \subset M_1 \subset S(S - M_1)S \subset S[S - (M_1 \cap M_2)]S.$$

If we consider both Theorem 2 and Theorem 3 we get:

Corollary. *The set of all C-ideals of S is a sublattice of the lattice of all ideals of S.*

We have seen that if S contains more than one maximal ideal, then none of them can be a C-ideal of S.

Now we shall consider the case that S contains only one maximal two-sided ideal.

Definition 2. *A two-sided ideal M of a semigroup S is said to be the greatest ideal of S, if any proper two-sided ideal of S is contained in M.*

If such an ideal in S exists, then we shall denote it by M^*.

Theorem 4. *Let a semigroup S contain only one maximal two-sided ideal M. If M is a C-ideal, then $M = M^*$.*

Proof. It is sufficient to show that any proper ideal of S is contained in M. If T is any proper two-sided ideal of S, then with regard to Theorem 1 we get that $T \subset M$. It means that $M = M^*$.

For one-sided ideals the converse statement holds too. The next example illustrates that for two-sided ideals it need not hold.

Example 1. Let $S = \{a, b, c, d\}$ be the semigroup with the multiplication table:

	a	b	c	d
a	a	b	a	b
b	b	a	b	a
c	a	b	a	b
d	b	a	b	c

$M=\{a, b, c\}$ is the only maximal two-sided ideal of S. Any proper ideal of S is contained in M, so $M=M^*$. $S-M^*=\{d\}$. $SdS=\{a, b\}$, so $M^*\not\subset S(S-M^*)S$, hence M^* is not a C-ideal of S.

Theorem 5. *The ideal M^* of a semigroup S is a C-ideal iff $S^2=S^3$.*

Proof. (a) Let M^* be a C-ideal, so $M^*\subset S(S-M^*)S$. Since M^* is at the same time a maximal ideal, then $S-M^*=I^a$ is the unique maximal $\mathcal{J}$-class in S. Then either $S^2\subsetneqq S$, or $S^2=S$. If $S^2=S$, then $S^3=S^2$.

If $S^2\subsetneqq S$, then either $S^3=S^2$, or $S^3\subsetneqq S^2$.

If $S^3\subsetneqq S^2$, then $M^*\subset S(S-M^*)S\subset S^3$, hence $S-M^*$ would contain at least two different $\mathcal{J}$-classes, one in S^2-S^3 and another in $S-S^2$. But this is a contradiction, since $S-M^*$ contains just one maximal $\mathcal{J}$-class. So, we have $S^2=S^3$.

(b) Suppose that S contains M^* and $S^2=S^3$. We show that M^* is a C-ideal.

Let $x\in M^*$ be any element. Then for any element $a\in I^a=S-M^*$, $(a)_T=S$, therefore $x\in(a)_T$. However, $x\in M^*$ and $a\in I^a=S-M^*$, hence $x\neq a$. Then $x\in(aS\cup Sa\cup SaS)$.

If $x\in aS$ or $x\in Sa$, then $x\in S^2$. If $x\in SaS$, then $x\in S^3$. But according to the assumption $S^2=S^3$, therefore $x\in S^3$.

Then there is $c\in S$ such that $x\in ScS$. Since $(a)_T=S$, we have $c\in(a)_T$. If $c=a$, then $x\in SaS$. If $c\neq a$ then $c\in(aS\cup Sa\cup SaS)$. If $c\in aS$, then $ScS\subset SaS$. The same relation can be obtained if $c\in Sa$, or $c\in SaS$. Hence, $x\in ScS\subset SaS$, $a\in I^a=S-M^*$ in all three cases. This implies: for any $x\in M^*$, $x\in SaS$ and $a\in I^a=S-M^*$, therefore $M^*\subset S(S-M^*)S$ i.e. M^* is a C-ideal of S.

It was already mentioned that S need not have one-sided covered ideals at all. For two-sided ideals we have:

Theorem 6. *If S is not a simple semigroup, then S contains at least one C-ideal of S.*

Proof. Let T be any proper two-sided ideal of S. Consider $S(S-T)S$. $S(S-T)S$ is an ideal of S, and it is known [7] that the intersection of two two-sided ideals is non-empty. Therefore, $T\cap S(S-T)S\neq\emptyset$. If we denote $M=T\cap S(S-T)S$, then M is an ideal of S and for M we get $M\subset S(S-T)S$. Since $S-T\subset S-M$, then the relation $M\subset S(S-T)S$ implies

$$M\subset S(S-T)S\subset S(S-M)S,$$

hence M is a C-ideal of S.

We now investigate the case that S contains more than one maximal ideal.

Definition 3. *A two-sided ideal M of S will be called the greatest covered ideal of S if any covered ideal of S is contained in M.*

If S contains the greatest covered ideal of S, this ideal will be denoted by M^g.

Remark. Let a semigroup S contain maximal two-sided ideals. If M_α, $\alpha\in\lambda$ are all maximal two-sided ideals of S, then $\bigcap_{\alpha\in\lambda} M_\alpha \neq \emptyset$ ([7]). Denote $\hat{M}=\bigcap_{\alpha\in\lambda} M_\alpha$. If S contains M^g, then necessarily $M^g \subset \hat{M}$ holds. For if there is at least one M_γ such that $M^g \not\subset M_\gamma$, then by Theorem 1 M^g is not a C-ideal and it is a contradiction.

However, we can show that if S contains maximal two-sided ideals, it does not mean that S must contaim M^g.

Example 2. Let $N=\{0, 1, 2, ..., n, ...\}$. Let $S=N\cup\{z_1\}\cup\{z_2\}$. Define the binary operation $\circ$ in S as follows:

$$a\circ b=\begin{cases}\min\{a, b\} & \text{if } a, b\in N\\ 0 & \text{if } (1)\ a\in N,\ b\in\{z_1, z_2\},\ (2)\ a=z_1, b=z_2\\ z_i & \text{if } a=b=z_1,\quad i=1, 2.\end{cases}$$

It can be easily verified that S with the operation $\circ$ is a semigroup.

$$(z_1)_T=\{0, z_1\},\quad (z_2)_T=\{0, z_2\}.\quad M_1=S-\{z_1\},\quad M_2=S-\{z_2\}$$

are all maximal ideals of S. $\hat{M}=M_1\cap M_2=N$. The subset $T=\{0, 1, ..., k\}$ is an ideal of S and moreover $S(S-T)S=\{0, 1, 2, ..., k, k+1, ...\}\supset\{0, 1, 2, ..., k\}$, so T is a C-ideal of S. But S does not contain M^g.

For one-sided covered ideals the following statement holds: If $\hat{L}=\bigcap_{\alpha\in\lambda} L_\alpha\neq\emptyset$, where L_α, $\alpha\in\lambda$ are all maximal left ideals, then $\hat{L}$ is the greatest covered left ideal of S.

The following example illustrates that for two-sided ideals this need not be true.

Example 3. Let $S=\{a, b, c, d\}$ with the multiplication table:

	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

S contains two maximal ideals: $M_1=\{a, b, c\}$, $M_2=\{a, b, d\}$. $\hat{M}=M_1\cap M_2=\{a, b\}$. Then $S-\hat{M}=\{c, d\}$. But $S(S-\hat{M})S=S\{c, d\}S=\{a\}\not\supset\{a, b\}$. Therefore, $\hat{M}$ is not a C-ideal of S.

If we want to describe conditions for existence of M^g we need the notion of a two-sided base of a semigroup.

Definition 4. *A non-empty subset A of S is a two-sided base of S if*

(1) $(A)_T=A\cup SA\cup AS\cup SAS=S$

(2) *There is no proper subset $B\subsetneqq A$ such that $(B)_T=S$.*

The condition (2) implies that a two-sided base A of S does not contain two different elements of $\mathcal{I}$-class I^a. Hence, for $a, b \in A$, $a \neq b$, $I^a \cap I^b = \emptyset$.
Further, we shall denote maximal $\mathcal{I}$-class as a complement of a maximal two-sided ideal M_α by M^α.

Theorem 7. *If a semigroup S, which is not simple, contains a two-sided base A of S, then S contains the ideal M^g. Moreover, $M^g = S^3 \cap \hat{M}$, where $\hat{M} = \bigcap_{\alpha \in \lambda} M_\alpha$.*

Proof. The existence of a two-sided base A implies the existence of maximal ideals ([3]) and $M_\alpha = S - M^\alpha$ where M^α is a maximal $\mathcal{I}$-class. Since $\emptyset \neq \hat{M} = \bigcap_{\alpha \in \lambda} M_\alpha = \bigcap_{\alpha \in \lambda} (S - M^\alpha) = S - \bigcup_{\alpha \in \lambda} M^\alpha$, then $\hat{M}$ is an ideal of S ([7]). And S^3 is an ideal of S too. Denote by $M = \hat{M} \cap S^3 \neq \emptyset$. We shall show that M is a C-ideal of S.

Let $x \in M$ be any element. Hence, $x \in S^3$ and therefore there is $c \in S$ such that $x \in ScS$. If $c \bar{\in} A$, then there is $b \in A$ such that $c \in (b)_T$, hence $c \in (Sb \cup bS \cup SbS)$ and c is at least in one of the subsets: Sb, bS, SbS. Then $ScS \subset SbS$ and $x \in ScS \subset SbS$ implies $x \in SbS$ for $b \in A$. We have got that for any $x \in M$, there is $b \in A$ such that $x \in SbS \subset SAS \subset S(S - \hat{M})S \subset S(S - M)S$, therefore $M \subset S(S - M)S$. It remains to show that M is the greatest C-ideal of S.

We shall show that any C-ideal of S is contained in $M = S^3 \cap \hat{M}$, i.e. that M is the greatest covered ideal of S.

Let T be any C-ideal of S. Then $T \subset S(S - T)S \subset S^3$, therefore $T \subset S^3$. Since T is a C-ideal of S, then T cannot contain any maximal $\mathcal{I}$-class. It means, $T \subset S - M^\alpha$ for every $\alpha \in \lambda$. Hence we have $T \subset \bigcap_{\alpha \in \lambda} (S - M^\alpha) = \bigcap_{\alpha \in \lambda} M_\alpha = \hat{M}$. The relations $T \subset S^3$ and $T \subset \hat{M}$ imply $T \subset S^3 \cap \hat{M} = M$, therefore, any C-ideal T is contained in M, i.e. $M = M^g$.

Lemma 1. *Let S contain M^g and $M^g \subsetneqq S^3$. Then every $\mathcal{I}$-class in $S^3 - M^g$ is a maximal one and for any $a \in S^3 - M^g$ $(a)_T = SaS$.*

Proof. Let $S^3 - M^g \neq \emptyset$. Since both M^g and S^3 are ideals in S, then $S^3 - M^g$ consists of some $\mathcal{I}$-classes of S. Let M^γ be any one of them. Since $M^\gamma \subset S^3$ it implies that any $a \in M^\gamma$ is of the form: $a = xby$ for $x, y, b \in S$. Then $a \in SbS$. Next we show that $b \in M^\gamma$. If $b \in M^\delta$, $\delta \neq \gamma$, then $a \in SbS$ would imply $(a)_T \subseteqq (b)_T$. The element b does not belong to $(a)_T$ (otherwise $b \in (a)_T$ would imply $(b)_T \subset (a)_T$ and we get $(a)_T = (b)_T$ which is contradicting with $\delta \neq \gamma$). Therefore $(a)_T$ is a C-ideal and $M^g \cup (a)_T$ is a C-ideal too, properly containing M^g, what is impossible.

So we have got: for any $a \in M^\gamma$ there is $b \in M^\gamma$ such that $a \in SbS$. This implies $(a)_T \subset SbS \subset (b)_T = (a)_T$, hence $(a)_T = SbS$. We show that $(a)_T = SaS$. Since $(a)_T = SbS = (b)_T$, from there we get: if $a = b$, $(a)_T = SaS$. If $a \neq b$, then $b \in (Sa \cup aS \cup SaS)$. If $b \in Sa$, then $SbS \subset SaS$. If $b \in aS$ or $b \in SaS$ we get again

$SbS \subset SaS$. The relation $(a)_T = (b)_T = SbS \subset SaS \subset (a)_T$ implies $(a)_T = SaS$. This is true for any $\mathcal{J}$-class M^γ from $S^3 - M^g$.

If we suppose that for $a \in M^\gamma \subset S^3 - M^g$, $(a)_T = SaS \subset (c)_T$ for $c \in S$, then $a \in (Sc \cup cS \cup ScS)$ and a is contained in at least one of the sets: Sc, cS, ScS. Then $SaS \subset ScS$ and $(a)_T \subset ScS$. But $c \bar{\in} (a)_T$, i.e. $(a)_T$ is a C-ideal and $M^g \cup (a)_T$ is a C-ideal too, properly containing M^g and we have a contradiction.

We have proved that any $\mathcal{J}$-class in $S^3 - M^g$ is maximal.

Theorem 8. *If a semigroup S contains the ideal M^g, then S contains a two-sided base.*

Proof.

$$M^g \subset S(S - M^g)S \subset S^3 \subset S^2 \subset S.$$

Denote by M^α $\mathcal{J}$-classes from $S - S^2$, by M^β $\mathcal{J}$-classes from $S^2 - S^3$ and by M^γ $\mathcal{J}$-classes from $S^3 - M^g$. Construct a subset A in the following way: from each $\mathcal{J}$-class M^α and M^γ choose just one element into A. Denote by

$$(A)_T = A \cup SA \cup AS \cup SAS.$$

$\mathcal{J}$-classes $M^\alpha \subset S - S^2$ are of the form: $M^\alpha = \{a\}$, where a is undecomposable element, so any $M^\alpha \subset S - S^2$ is a maximal $\mathcal{J}$-class.

Equally, any $\mathcal{J}$-class $M^\gamma \subset S^3 - M^g$ is a maximal $\mathcal{J}$-class (by Lemma 1).

We wish to show that for any $x \in M^\beta \subset S^2 - S^3$ there is some $u \in M^\alpha \subset S - S^2$ such that $x \in (u)_T$. Note that $S^2 - S^3 \neq \emptyset$ implies $S - S^2 \neq \emptyset$. Since $x \in M^\beta \subset S^2 - S^3$, then $x = uv$, where u, $v \in S - S^2$, it means that both u and v are undecomposable. Now $x = uv$ implies $x \in (u)_T$, $u \in S - S^2$, i.e. $u \in M^\alpha$.

Till now we have got: $M^g \subset S(S - M^g)S$, it means for any $y \in M^g$ there is $z \in S - M^g$ such that $y \in (z)_T$. Since $z \in S - M^g$, then z is from M^α, or M^β, or M^γ. If $z \in M^\alpha$ or $z \in M^\gamma$, then in both cases we can choose $z \in A$. If $z \in M^\beta$, then there is $u \in M^\alpha$ such that $z \in (u)_T$, hence, $y \in (z)_T \subset (u)_T$, and $u \in A$.

We have shown that for any $y \in M^g$ there is $a \in A$ such that $y \in (a)_T$, hence $M^g \subset (A)_T$. And with regard to the construction of A we have: $S^3 - M^g \subset (A)_T$, $S^2 - S^3 \subset (A)_T$ and $S - S^2 \subset (A)_T$ so together we have $S \subset (A)_T$, therefore

$$(A)_T = S,$$

hence, the subset A generates S.

To prove that A is a two-sided base of S, it remains to show that there is no proper subset $B \subsetneqq A$ with the property

$$(B)_T = B \cup SB \cup BS \cup SBS = S.$$

But this is evident, because A has been constructed by means of elements of maximal $\mathcal{J}$-classes of S, and from each maximal $\mathcal{J}$-class just one element was chosen into A. Therefore, A is a two-sided base of S.

Theorem 7 and Theorem 8 imply:

Corollary. *If S is not simple, then S contains the ideal M^g iff S has a two-sided base.*

The question arises, whether the relation $M^g \subset S^3$ does not mean $M^g = S^3$ always. Next example illustrates that it need not be so.

Example 4. T is the multiplicative semigroup of real numbers: $T = \{x^{-1} / 0 \leq x \leq \frac{1}{2}\}$, G an arbitrary commutative group. Define in $S = T \cup G$ an associative binary operation $\circ$ as follows: $x \circ y = 0$ if $x \in T$, $y \in G$ and the products in T and G remain old ones. Then S is a semigroup. T is a maximal ideal in S. However, there is an infinite number of further maximal ideals of the form: $M_\alpha = S - \{\alpha\}$, where $\frac{1}{4} < \alpha \leq \frac{1}{2}$. The intersection of all maximal ideals in S is:

$$\hat{M} = \bigcap_{\alpha} M_\alpha \cap T = \left\langle 0, \frac{1}{4} \right\rangle.$$

For S, $S \neq S^2$, $S^3 = \left\langle 0, \frac{1}{8} \right\rangle \cup G$. Then $M^g = S^3 \cap \hat{M} = \left\langle 0, \frac{1}{8} \right\rangle$. Hence, $M^g \subsetneqq S^3$.

Theorem 9. *Let any proper two-sided ideal of a semigroup S be covered. Then just one of the following conditions holds:*

(1) *S contains the ideal M^*.*

(2) *$S = S^2$ and for any proper two-sided ideal M and for any principal two-sided ideal $(a)_T \subset M$, there is a principal proper two-sided ideal $(b)_T$ such that $(a)_T \subsetneqq (b)_T$ and $b \in S - M$.*

Proof. First we show that if any proper two-sided ideal of S is covered, then S cannot contain even two different maximal $\mathcal{J}$-classes.

If M^α, M^β were maximal $\mathcal{J}$-classes, $M^\alpha \neq M^\beta$, then as we know $M_\alpha = S - M^\alpha$, $M_\beta = S - M^\beta$ would be maximal proper twosided ideals of S and none of them is a C-ideal of S.

Let S contain just one maximal $\mathcal{J}$-class M^α. Then $M_\alpha = S - M^\alpha$ is again a maximal proper two-sided ideal of S and moreover it is a C-ideal. By Theorem 4 we have that $M_\alpha = M^*$.

Suppose that S does not contain maximal $\mathcal{J}$-classes. First we show that $S^2 = S$. If $S^2 \subsetneqq S$, then for $y \in S - S^2$ $(y)_T \neq S$, because if $(y)_T = S$, then S would contain a maximal $\mathcal{J}$-class. So $(y)_T \subsetneqq S$, and since any proper two-sided ideal is covered, then $(y)_T \subset S[S - (y)_T]S$, hence $y \in SzS$ for $z \in S - (y)_T$, therefore $y \in S^3$. But $S^3 \subset S^2$, it means $y \in S^2$, which is a contradiction with the fact that $y \in S - S^2$. Therefore, $S = S^2$.

Let M be any proper two-sided ideal of S. Then $M \subset S(S - M)S$. Let $a \in M$ be any element. Then there is $b \in S - M$ such that $a \in SbS$. This implies $(a)_T \subset SbS \subset (b)_T$. The ideal $(b)_T \neq S$, since S does not contain maximal $\mathcal{J}$-classes. Moreover $(a)_T \neq (b)_T$, because $a \in M$, $b \in S - M$. Therefore, $(b)_T \not\subset M$.

Theorem 10. *Let a semigroup S satisfy just one of the following conditions:*

(1) *S contains M^* and it is a C-ideal.*

(2) *$S=S^2$ and for any proper two-sided ideal M and for every principal two-sided ideal $(a)_T \subset M$, there is a principal two-sided proper ideal $(b)_T$, whose generator $b \in S-M$ and $(a)_T \subseteqq (b)_T$.*

Then any two-sided proper ideal is covered.

Proof. Let M be any proper two-sided ideal of S. If (1) holds, then $M \subset M^*$, and $S-M^* \subset S-M$. Then $M \subset M^* \subset S(S-M^*)S \subset S(S-M)S$. Hence, M is a C-ideal.

Let (2) be satisfied. If $x \in M$, then $(x)_T \subset M$. Then there is $b \in S-M$ such that $(x)_T \subset (b)_T \subset S$. It is evident that $(x)_T \neq (b)_T$. Since $S=S^2$, then $S=S^3$, and $b \in SdS$ for some $d \in S$. We show that $d \bar{\in} M$. If $d \in M$, then $SdS \subset M$, and $b \in SdS \subset M$, hence, $b \in M$, which is a contradiction with the fact $b \in S-M$. Therefore, for arbitrary $x \in M$, there is $d \in S-M$ such that $x \in SdS$. This means that

$$M \subset S(S-M)S,$$

hence, M is a C-ideal.

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ПОЛУГРУППЫ, СОДЕРЖАЩИЕ ЗАКРЫТЫЕ ДВУСТОРОННИЕ ИДЕАЛЫ

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Резюме

Двусторонний идеал M полугруппы S называется закрытым, если $M \subset S(S-M)S$.

В работе доказано, что множество всех двусторонних закрытых идеалов полугруппы S является подструктурой структуры всех идеалов в S.

Приведено необходимое и достаточное условие для того, чтобы:

(1) полугруппа S содержала наибольший закрытый идеал.

(2) каждый идеал полугруппы S был закрытым.