Mathematica Slovaca

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Two corrections and a remark regarding my paper "Every normal linear system has a regular time-optimal synthesis"

Mathematica Slovaca, Vol. 29 (1979), No. 3, 319--320

Persistent URL: http://dml.cz/dmlcz/129353

ERRATUM

P. Brunovský, TWO CORRECTIONS AND A REMARK REGARDING MY PAPER "EVERY NORMAL LINEAR SYSTEM HAS A REGULAR TIME-OPTIMAL SYNTHESIS", Math. Slovaca 28, 1978, 81—100.

In the definition of regular synthesis it is required that if S is a k-dimensional cell of type I, $\Pi(S)$ should be k — 1-dimensional. As seen easily from the construction, this is not true in general. However, this property is not essential (e. g. Boltanski's sufficiency proof dispenses with it) and therefore it can be omitted in the definition without any loss.

Further, it is claimed that if $\bar{G}'$ admits a W-stratification $\mathcal{P}$ on dimension $<n$, then for any control $u(t)$, $t\in[0, T]$ in each neighbourhood of any point $x_0\in G$ there exists a point y_0 such that $x(t, y_0, u)$ meets $\bar{G}'$ for at most finitely many times. In fact, it is only proved that the set E_P of points for which $x(t, y_0, u)\in P$ for some $P\in\mathcal{P}$ is discrete but not that it is closed.

By a counterexample it can be shown that the properties of W-stratification as defined in the paper are not sufficient to prove this result. The proof can be completed if $\mathcal{P}$ has the dimension property: if P, $Q\in\mathcal{P}$, $P\subset\bar{Q}$, $P\neq Q$, then dim $P<$dim Q. By [1, 2] a subanalytic set admits a stratification with this property.

Assume thus that $\mathcal{P}$ has the dimension property. First note that if $y\in\tilde{V}$, then $E_P=\emptyset$ for every $P\in\mathcal{P}$, dim $P<n-1$. We prove that if dim $P=n-1$, then E_P is closed.

Indeed, let $t_m\in E_P$, $t_m\to t^*\notin E_P$. Then, $x(t^*, y, u)\in\bar{P}$, so $x(t^*, y, u)$ should belong to a stratum of $\mathcal{P}$ of dimension $<n-1$, which is impossible.

Finally, let us note that Whitney's property A is not needed at all and can be replaced by the dimension property. Indeed, the only place where it is used is the proof that in every neighbourhood of x_0 there is an y_0 such that $x(t, y_0, u)$ meets each stratum of $\mathcal{M}$ transversally.

Let $M\in\mathcal{M}$, $t_1, \ldots, t_k$ be the switching points of u, Since $\mathcal{M}$ is finite, it suffices to prove that if $\tilde{V}_M$ is the set of points y for which $x(t, y, u)$ meets $M\in\mathcal{M}$ transversally, then $\mu(\tilde{V}\setminus\tilde{V}_M)=0$ (μ being the Lebesgue measure). Denote $W_i=(t_{i-1}, t_i)\times M$. For $(t, z)\in W_i$ denote by $F_i(t, z)$ the unique y such that $x(t, y, u)=z$. Denote by W_i^0 the set of singular points of F_i, $Z_i=F_i(W_i^0)$. By Sard's theorem, $\mu(Z_i)=0$. We have

$$\tilde{V}\setminus\tilde{V}_M=(\tilde{V}\cap Z_1)\cup\ldots\cup(\tilde{V}\cap Z_k),$$

from which it follows that $\mu(\tilde{V}\setminus\tilde{V}_M)=0$

The author is indebted to prof. H. J. Sussmann who should be credited for the above observations.

REFERENCES

[1] HARDT, R. M.: Stratifications of real analytic mappings and images. Inventiones Math. 28, 1975, 193—208.

[2] HIRONAKA, H.: Subanalytic sets. In: Number Theory, Algebraic Geometry and Commutative Algebra, in honour of Y. Akizuki, Kinokuiya, Tokvo 1973, 453—493.

OZNÁMENIE

Výbor sekcie pre vedeckú a odbornú literatúru Slovenského literárneho fondu udelil RNDr. Štefanovi Porubskému, CSc. prémiu 500.— Kčs za články: On covering systems on rings; On theorems of Niven and Dressler, uverejnené v časopise Mathematica Slovaca v roku 1978.