Vladimír Baláž

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ON ALMOST AND WEAK FORMS OF CONTINUITY OF FUNCTIONS AND MULTIFUNCTIONS

VLADIMÍR BALÁŽ

The notion of the almost continuity of a function was introduced by M. K. Singal and A. R. Singal [13]. The class of almost continuous functions is contained in the class of weakly continuous functions. The notion of the weakly continuity was introduced by N. Levine [7]. If the range of a function is an almost-regular topological space, then any of the weak continuity notions coincides with the almost one and if the range is a semi-regular topological space, then any of the almost continuity coincides with the original one. In connection with this a characterization of almost-regular and semi-regular topological spaces may be obtained. Unless otherwise stated, T_1 is not assumed. On the other hand also multifunctions are considered and it is shown that the situation in such a case may be different.

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In what follows X, Y denote topological spaces. The notion $\bar{A}^X$ stands for the closure of a set A in X, $\operatorname{int}_X A$ denotes the interior of A in X. If there is no misunderstanding, X will be omitted. A is said to be regularly open if $A = \operatorname{int} \bar{A}$, and regularly closed if $A = \overline{\operatorname{int} A}$. Besides the mapping f: X → Y we consider also the multivalued mapping $F: X \to \mathscr{P}(Y)$, where $\mathscr{P}(Y)$ is the power set of Y. We write F: $X \to Y$ for shortness and suppose $F(x) \neq \emptyset$ for any $x \in X$. We give the definitions of various types of continuity and generalized continuity only for multifunctions. The corresponding notions of continuity and generalized continuity for single valued functions may be obtained in a natural way so that the function f: $X \to Y$ is considered as a multifunction which associates $\{f(x)\}$ with any $x \in X$. The continuity of a multifunction F: $X \to Y$ is defined by the means of its upper and lower continuity (see [6]).

Definition 1. *A multifunction F: $X \to Y$ is said to be upper semi-continuous, in brief u.c., (lower semi-continuous, in brief l.c.,) at a point $x \in X$ if for any open set V, $F(x) \subset V$ $(F(x) \cap V \neq \emptyset)$ there exists a neighbourhood U of x such that*

$F(y) \subset V$ $(F(y) \cap V \neq \emptyset)$ *for any* $y \in U$. *It is to be u.c. (l.c.) if it is u.c. (l.c.) at any* $x \in X$.

Definition 2. ([8]) *A multifunction* F: $F \to Y$ *is said to be upper semi-quasicontinuous, in brief u.qc., (lower semi-quasicontinuous, in brief l.qc,) at a point* $x \in X$ *if for any neighbourhood* U *of* x *and any open set* V, $F(x) \subset V$ $(F(x) \cap V \neq \emptyset)$ *there exists a nonempty open set* $G \subset U$ *such that* $F(y) \subset V$ $(F(y) \cap V \neq \emptyset)$ *for each* $y \in G$. *It is said to be u.qc. (l.qc.), if it is u.qc. (l.qc.) at any* $x \in X$.

Note that we can immediately obtain the corresponding almost notions substituting in Definition 1 and Definition 2 for the relations $F(y) \subset V$ or $F(y) \cap V \neq \emptyset$ the relations $F(y) \subset \operatorname{int} \bar{V}$ or $F(y) \cap \operatorname{int} \bar{V} \neq \emptyset$, respectively. The almost upper (lower) semi-continuity and almost upper (lower) semi-quasicontinuity will be abbreviated by a.u.c. (a.l.c.) and a.u.qc. (a.l.qc.), respectively.

The notion of the upper (lower) inverse image $F^+(A)$ $(F^-(A))$ is frequently used. It is defined for $A \subset Y$ as $F^+(A) = \{x: x \in X, F(x) \subset A\}$ $(F^-(A) = \{x: x \in X, F(x) \cap A \neq \emptyset\})$. In case of a single valued function f: $X \to Y$ the set $f^+(A) = f^-(A) = f^{-1}(A)$, where $f^{-1}(A)$ is the inverse image of A.

By means of F^+ and F^- one can define upper and lower somewhat continuity (see [8]).

Definition 3. *A multifunction* F: $X \to Y$ *is said to be upper somewhat continuous, in brief u.sc., (lower somewhat continuous, in brief l.sc.,) if for any open set* V *such that* $F^+(V) \neq \emptyset$ $(F^-(V) \neq \emptyset)$ *we have* $\operatorname{int} F^+(V) \neq \emptyset$ $(\operatorname{int} F^-(V) \neq \emptyset)$.

Again we can immediately obtain the corresponding almost notions substituting in Definition 3 for the relations $\operatorname{int} F^+(V) \neq \emptyset$ or $\operatorname{int} F^-(V) \neq \emptyset$ the relations $\operatorname{int} F^+(\operatorname{int} \bar{V}) \neq \emptyset$ or $\operatorname{int} F^-(\operatorname{int} \bar{V}) \neq \emptyset$ respectively. The almost upper (lower) somewhat continuity will be abbreviated by a.u.sc.(a.l. sc.).

Definition 4. [11] *A topological space* Y *is said to be semi-regular if the family of regularly open sets forms a base for the topology of* X.

It is well known that every regular space is semi-regular but the converse i not true.

For a single valued function the equivalence of the types of continuity and generalized continuities to their almost forms gives a characterization of semi-regular spaces.

Theorem 1. *A topological space* Y *is semi-regular if and only if any of the following conditions is satisfied for any topological space* X.

(*i*) *If* f: $X \to Y$ *is a one to one function, then* f *is continuous if and only if it is almost continuous.*

(*ii*) *If* f: $X \to Y$ *is an arbitrary function, then* f *is quasicontinuous if and only if it is almost quasicontinuous.*

(iii) If f: $X \to Y$ is an arbitrary function, then f is somewhat continuous if and only if it is almost somewhat continuous.

Proof. For the proof of the condition (i) see [10].

The proofs of necessity of either of the conditions (ii), (iii) are similar and straightforward. Thus we prove the necessity of (ii) only.

Let Y be semi-regular and f: $X \to Y$ almost quasicontinuous at $x \in X$. Let V be an open set containing $f(x)$ and U a neighbourhood of x. By semi-regularity there exists a regularly open set A such that $f(x) \in A \subset V$. Using the almost quasicontinuity at x there exists a non empty open set $G \subset U$ such that $f(z) \in A \subset V$ for every $z \in G$. Thus the quasicontinuity of f is proved because x was an arbitrary point.

To prove the sufficiency of (ii), (iii) it suffices to prove the following. If Y is not a semi-regular space, then there exist a topological space X and a function f: $X \to Y$ such that f is almost quasicontinuous but not somewhat continuous.

Now let Y be not semi-regular. Then there exist $y \in Y$ and an open set G, $y \in G$ such that for any regularly open set A containing y we have $A \cap (Y - G) \neq \emptyset$.

Put $X = Y$ and define the topology on X as follows. A set $S \subset X$ will be open in X if and only if $S \subset Y - G$ or $S = X$.

Define the function f: $X \to Y$ such that $f(x) = x$ for $x \in X - G$ and $f(x) = y$ for $x \in G$.

Let $x \in G$. Choose any regularly open set A in Y containing $f(x)$ and any open set U in X containing x. From the definition of the topology on X we have that $U = X$. Put $H = A \cap (X - G)$; then H is the nonempty open subset of X such that $f(z) \in A$ for any $z \in H$. If $x \notin G$, then f even is continuous in x because x is an isolated point. Thus the almost quasicontinuity of f is proved.

The function f is not somewhat continuous. Indeed if we consider the open set G, we have $f^{-1}(G) \neq \emptyset$ but $\operatorname{int} f^{-1}(G) = \emptyset$.

We show that if we consider the multifunctions, the situation is different. The equivalence of the types of introduced continuities to their almost forms in case of multifunctions depends on the semi-normality of the space of values. In fact we can give the characterization of semi-normal spaces. It may be formulated by means of notions of upper continuity, upper generalized continuities and their corresponding almost forms.

Definition 5. ([12]) *A topological space Y is said to be semi-normal if for any closed set K and open set S containing K there exists a regularly open set A such that $K \subset A \subset S$.*

It is evident that every normal space is semi-normal, but the converse is not true.

Theorem 2. *A T_1 topological space Y is semi-normal if and only if any of the following conditions is satisfied for any topological space X.*

(*i*) *If* F: $X \to Y$ *is a closed valued multifunction, then* F *is u.c. if and only if it is a.u.c.*

(*ii*) *If* F: $X \to Y$ *is a closed valued multifunction, then* F *is u.qc. if and only if it is a.u.qc.*

(*iii*) *If* F: $X \to Y$ *is a closed valued multifunction, then* F *is u.sc. if and only if it is a.u.sc.*

Proof. The proofs of necessity of the conditions (i), (ii), (iii) are again similar and straightforward. Thus we prove the necessity of (i) only.

Let Y be semi-normal and F: $X \to Y$ be an a.u.c. and a closed valued multifunction. Let $x \in X$ be any point, V an open set of Y such that $F(x) \subset V$. Since $F(x)$ is a closed set using semi-normality there exists a regularly open set A such that $F(x) \subset A \subset V$. By the a.u.c. of F there exists a neighbourhood U of x such that for any $y \in U$ we have $F(y) \subset A \subset V$. Thus the u.c. of F is proved.

For the sufficiency of (i) it suffices to show that if Y is not a semi-normal space, then there exist a topological space X and a closed valued multifunction F: $X \to Y$ such that F is a.u.c. but not u.c.

Hence let Y not be semi-normal. Then there exist a closed set $K \subset Y$ and an open set G, $K \subset G$, such that for any regularly open set A containing K we have $A \cap (Y - G) \neq \emptyset$.

Put $X = Y$ and define the topology on X as follows. A set S will be open in X if and only if $S \subset X - K$ or there exists a regularly open set A in Y such that $K \subset A \subset S$. Define the multifunction F: $X \to Y$ such that $F(x) = \overline{\{x\}}^Y \cup K$ for any $x \in X$. Then F is closed valued. We prove that F is a.u.c. at any $x \in X$. Let $x \in K$. Choose any regularly open set $A \subset Y$ containing $F(x)$ and put $U = A$; then U is the neighbourhood of x in X such that for any $y \in U$ we have $F(y) \subset A$. If $x \notin K$, the proof is evident because x is an isolated point. Thus F is a.u.c.

The multifunction F is not u.c. Suppose that F is u.c. at any $x \in K$. Then for each open set V containing $F(x)$ there exists a neighbourhood U of x such that for any $y \in U$ we have $F(y) \subset V$. Let us take $V = G$. Since $x \in U$ there exists a regularly open set A in Y such that $K \subset A \subset U$, and $A \cap (Y - G) \subset F(U) \cap$ $\cap\, (Y - G) \subset G \cap (Y - G) = \emptyset$. This is a contradiction.

To prove the sufficiency of (ii), (iii) it suffices to prove that if Y is not a semi-normal, then there exist a topological space X and a closed valued multifunction F: $X \to Y$ such that F is a.u.qc. but not u.sc.

Let Y not be semi-normal. Then there exist a closed set $K \subset Y$ and an open set G, $K \subset G$ such that for any regularly open set A containing K we have $A \cap (Y - G) \neq \emptyset$.

Put $X = Y$ and define the topology on X. A set S will be open in X if and only if $S \subset X - G$ or $S = X$. Define the multifunction F: $X \to Y$ such that $F(x) = \overline{\{x\}}^Y \cup K$ for any $x \in X$. Then F is closed valued.

Let $x \in G$. Choose any regularly open set A in Y containing $F(x)$ and any open set U in X containing x. Since $x \in G$ we have $U = X$. Put $H = A \cap (Y - G)$. Then H is the nonempty open subset of X such that $F(z) \subset A$ for any $z \in H$. Thus the a.u.qc. of F is proved because if $x \notin G$, then the situation is evident.

The multifunction F is not u.sc. For the nonempty open set $G \subset Y$ we have $F^+(G) \neq \emptyset$ but $\operatorname{int} F^+(G) = \emptyset$.

The following example gives an a.u.c. multifunction which is not u.sc. from a topological space X to a T_1 normal space Y. Thus the condition that F is a closed valued multifunction in Theorem 2 is essential.

Example 1. Let $X = Y = N$ where N represents the set of positive integers with the topology defined by $\{A \subset N: 1 \notin A$ or A has a finite complement$\}$. Then X and Y are both compact metric spaces. Define the multifunction F such that $F(1) = Y - \{1\}$ and $F(n) = \{1\}$ for $n \in N - \{1\}$.

Since $Y = \operatorname{int} \overline{F(1)}$ we can see that F is a.u.c. at 1. If $x \neq 1$, then F is u.c. at x because x is an isolated point. Thus F is a.u.c. but not u.sc. because for the open set $Y - \{1\}$ we have $F^+(Y - \{1\}) \neq \emptyset$ and $\operatorname{int} F^+(Y - \{1\}) = \emptyset$.

Remark 1. It may be easily seen that in case of multifunctions if we consider the notions of lower continuity, lower generalized continuities and their corresponding almost forms the same is true as in case of single valued functions.

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We discuss the analogies of the results obtained in the first part of the paper for weakly types of continuity and generalized continuities of functions and multifunctions.

Definition 6. *A multifunction F: $X \to Y$ is said to be weakly upper semi-continuous, in brief w.u.c., (weakly lower semi-continuous, in brief w.l.c.,) at a point $x \in X$ if for any open set V, $F(x) \subset V$ $(F(x) \cap V \neq \emptyset)$ there exists a neighbourhood U of x such that $F(y) \subset \bar{V}$ $(F(y) \cap \bar{V} \neq \emptyset)$ for any $y \in U$. It is said to be w.u.c. (w.l.c.) if it is w.u.c. (w.l.c.) at any $x \in X$.*

The notions of weak upper semi-quasicontinuity (w.u.qc.) and weak lower semi-quasicontinuity (w.l.qc.) can be immediately obtained by substituting in Definition 2 for the relations $F(y) \subset V$ or $F(y) \cap V \neq \emptyset$ the relations $F(y) \subset \bar{V}$ or $F(y) \cap \bar{V} \neq \emptyset$, respectively. Analogically the notions of weakly upper somewhat continuity (w.u.sc.) and weakly lower somewhat continuity (w.l.sc.) can be obtained by substituting in Definition 3 for the relation $\operatorname{int} F^+(V) \neq \emptyset$ or $\operatorname{int} F^-(V) \neq \emptyset$ the relations $\operatorname{int} F^+(\bar{V}) \neq \emptyset$ or $\operatorname{int} F^-(\bar{V}) \neq \emptyset$, respectively.

Definition 7. ([11]) *A topological space Y is said to be almost-regular if for each regularly closed set $A \subset Y$ not containing x there exist disjoint open sets U and V such that $x \in U$ and $A \subset V$.*

R e m a r k 2. Almost-regularity is weaker than regularity, but equivalent to it for a semi-regular space (see [11]).

For a single valued function the equivalence of the types of almost continuity and almost generalized continuities to their weakly forms gives a characterization of almost-regular spaces.

Theorem 3. *A topological space Y is almost-regular if and only if any of the following conditions is satisfied for any topological space X.*

(*i*) *If $f\colon X \to Y$ is a bijective function, then f is almost continuous if and only if it is weakly continuous.*

(*ii*) *If $f\colon X \to Y$ is an arbitrary function, then f is almost quasicontinuous if and only if it is weakly quasincontinuous.*

(*iii*) *If $f\colon X \to Y$ is an arbitrary function, then f is almost somewhat continuous if and only if it is weakly somewhat continuous.*

P r o o f. Again we prove the necessity of (i) only because the proofs of necessity of either of the conditions (ii), (iii) are straightforward and similar.

Let Y be almost-regular and $f\colon X \to Y$ weakly continuous at $x \in X$. Let A be a regularly open set containing $f(x)$. By almost regularity there exists an open set V such that $f(x) \in V \subset \bar{V} \subset A$. Using the weakly continuity at x there exists a neighbourhood U of x such that $f(z) \in \bar{V} \cap A$ for any $z \in U$. Thus the almost continuity of f is proved because x was an arbitrary point.

To prove the sufficiency of (i) it suffices to show that if Y is not an almost--regular space, then there exist a topological space X and a function $f\colon X \to Y$ such that f is weakly continuous but not almost continuous. Let Y be not almost-regular. Then there exist $y \in Y$ and a regularly open set A, $y \in A$ such that for any open set G containing y we have $\bar{G}^Y \cap (Y - A) \neq \emptyset$.

Put $X = Y$ and define the topology on X as follows. A set S will be open in X if and only if $S \subset X - \{y\}$ or there exists an open set P in Y such that $y \in P \subset \bar{P}^Y \subset S$. Let $f\colon X \to Y$ be the identity mapping.

We prove that f is weakly continuous at $y \in X$. Take any open set $V \subset Y$ containing y and put $U = \bar{V}^Y$; then U is a neighbourhood of y in X such that for any $x \in U$ we have $f(x) \in \bar{V}^Y$.
The weakly continuity of f at any $z \neq y$ is evident because z is an isolated point. Thus f is weakly continuous.

Suppose that f is almost continuous at $y \in X$. Then for any regularly open set B containing $f(y)$ there exists a neighbourhood U of y such that $f(x) \in B$ for any $x \in U$. Put $B = A$. Since $y \in U$ there exists an open set P in Y such that $y \in P \subset \bar{P}^Y \subset U$ and

$$\bar{P}^Y \cap (Y - A) \subset f(U) \cap (Y - A) \subset A \cap (X - A) = \emptyset.$$

This is a contradiction.

Sufficiency of (ii) and (iii). Let Y not be almost-regular. Then there exist $y \in Y$ and a regularly open set A, $y \in A$ such that for any open set G containing y we have $\bar{G}^Y \cap (Y - A) \neq \emptyset$. To prove the sufficiency of (ii) and (iii) it suffices to construct a topological space X and a function f: $X \to Y$ such that f is weakly quasicontinuous but not almost somewhat continuous.

Put $X = Y$ and define the topology on X as follows. A set S will be open in X if and only if $S \subset Y - A$ or $S = X$. Define the function f: $X \to Y$ such that $f(x) = x$ for $x \in Y - A$ and $f(x) = y$ for $x \in A$.

Let $z \in A$. Take open set G in Y containing $f(z)$ and any open set U in X containing z. $U = X$ follows from the definition of the topology on X. Put $H = \bar{G}^Y \cap (Y - A)$; then H is the nonempty open subset of X such that $f(x) \in \bar{G}^Y$ for any $x \in H$. If $z \notin A$, then f is even continuous because z is an isolated point. Thus the weakly quasicontinuity of f is proved. However, f is not almost somewhat continuous since for the regularly open set $A \subset Y$ we have $f^{-1}(A) \neq \emptyset$, but $\operatorname{int} f^{-1}(A) = \emptyset$.

Corollary 1. (see [1]) *A space Y is regular if and only if any of the following conditions is satisfied for any topological space X.*

(i) If f: $X \to Y$ is a bijective function, then f is continuous if and only if it is weakly continuous.

(ii) If f: $X \to Y$ is an arbitrary function, then f is quasicontinuous if and only if it is weakly quasicontinuous.

(iii) If f: $X \to Y$ is an arbitrary function, then f is somewhat continuous if and only if it is weakly somewhat continuous.

Proof. The proof is obvious if we use Theorem 1, Theorem 3 and Remark 2.

We show again that in case if we consider multifunctions the situation is different. In fact we can give a characterization of almost-normal spaces.

Definition 8. ([12]) *A topological space Y is said to be almost-normal if for each closed set $K \subset Y$ and each regularly closed set $A \subset Y$, $K \cap A = \emptyset$ there exist disjoint open sets U and V such that $K \subset U$ and $A \subset V$.*

Remark 3. Almost-normality is weaker than normality, but is equivalent to it for a semi-normal space (see [12]).

Theorem 4. *A topological space Y is almost-normal if and only if any of the following conditions is satisfied for any topological space X.*

(i) If F: $X \to Y$ is a closed valued multifunction, then F is a.u.c. if and only if it is w.u.c.

(ii) If F: $X \to Y$ is a closed valued multifunction, then F is a.u.qc. if and only if it is w.u.qc.

(iii) If F: $X \to Y$ is a closed valued multifunction, then F is a.u.sc. if and only if it is w.u.sc.

Proof. The proof is similar to that in Theorem 2. We restrict our attention to the proof of the sufficiency of (i) only.

Let Y not be almost-normal. It suffices to construct a topological space X and a multifunction F: $X \to Y$ such that F is w.u.c. but not a.u.c. Since Y is not almost-normal there exist a closed set $K \subset Y$ and a regularly open set A, $K \subset A$ such that for any open set G containing K we have $\bar{G}^Y \cap (Y - A) \neq \emptyset$.

Put $X = Y$ and define the topology on X as follows. A set S will be open in X if $S \subset X - K$ or there exists an open set G in Y such that $K \subset \bar{G}^Y \subset S$. Then the multifunction F defined in the same way as in Theorem 2 is a searched multifunction.

The following example gives a w.u.c. multifunction which is not a.sc. from a topological space X to a normal space Y. Thus the condition that F is a closed valued multifunction in Theorem 4 is essential.

Example 2. Let X and Y have the same meaning as in Example 1. Let F: $X \to Y$ be defined as $F(1) = \overline{Y - \{2k - 1\}_{k=1}^{\infty}}$ $F(n) = \{1\}$ for $n \in N - \{1\}$.

Since $1 \in \overline{Y - \{2k - 1\}_{k=1}^{\infty}}$ we can se that F is w.u.c. at 1. If $x \neq 1$, then F is u.c. at x because x is an isolated point. Thus F is w.u.c. but not a.sc. since for the open set $\overline{Y - \{2k - 1\}_{k=1}^{\infty}}$ we have $F^+(Y - \{2k - 1\}_{k=1}^{\infty}) \neq \emptyset$ and int $F^+(\text{int } \overline{Y - \{2k - 1\}_{k=1}^{\infty}}) = \emptyset$.

Remark 4. If in case of multifunction we consider the notions of lower almost continuity, lower almost generalized continuities and their corresponding weak forms, we can obtain the same results as in the case of single valued functions.

Collorary 2. (see [9], [1]) *A* T_1 *space* Y *is normal if and only if any of the following conditions is satisfied for any topological space* X.

(*i*) *If* F: $X \to Y$ *is a closed valued multifunction, then* F *is u.c. if and only if it is w.u.c.*

(*ii*) *If* F: $X \to Y$ *is a closed valued multifunction, then* F *is u.qc. if and only if it is w.u.qc.*

(*iii*) *If* F: $X \to Y$ *is a closed valued multifunction, then* F *is u.sc. if and only if it is w.u.sc.*

Proof. The proof is easily seen if we use Theore 2, Theorem 4 and Remark 3.

The Example 1 simultaneously shows that the condition F is closed valued in Collorary 2 is essential.

For other characterizations of regularity and normality by means of multifunctions see [2], [3], [4], [5], [14].

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Katedra matematiky CHTF SVŠT
Jánska 1
811 07 *Bratislava*

О ПОЧТИ И СЛАБЫХ ВИДАХ НЕПРЕРЫВНОСТИ ОТОБРАЖЕНИЙ И МНОГОЗНАЧНЫХ ОТОБРАЖЕНИЙ

Vladimír Baláž

Резюме

Исследуются соотношения между различными типами непрерывности и их почти и слабыми видами. В связи с тем характеризуются различные типы обобщенных регулярных и нормальных пространств.