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RECONSTRUCTION OF GRAPHS WITH SPECIAL DEGREE-SEQUENCES

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Our note concerns the problem of reconstructing simple finite graphs from the collection of their point-deleted subgraphs.

For any graph G let $V(G)$ denote the set of all vertices of G. We say that the graph H is a reconstruction of the graph G iff there exists a bijective map f: $V(G) \to V(H)$ such that for any $u \in V(G)$ the point-deleted subgraphs $G-u$ and $H-f(u)$ are isomorphic. The graph G is said to be reconstructible iff any reconstruction of G is isomorphic to G. The famous reconstruction conjecture (see [1], [2], [3]) states that any simple finite graph with more than two vertices is reconstructible.

All other graph-theoretical terms are used in their usual sense (cf. [2], [3]).

In [3] J. A. Bondy and R. L. Hemminger define a vertex v of a graph G to be bad if there exists a vertex in G of degree $d(v)-1$. They remark that a simple finite graph G with more than two vertices is reconstructible provided that G contains a vertex with no bad neighbours. This result can be extended as follows.

Theorem. *Let G be a simple finite graph with more than two vertices. Suppose that there exists a vertex $v \in V(G)$ such that for any its neighbour w all vertices of G of degree $d(w)-1$ that are distinct from v are neighbours of v. Then G is reconstructible.*

Proof. Let a graph H be a reconstruction of G where G is a graph satisfying all assumptions of our theorem. Obviously there is a vertex $u \in V(H)$ such that the point-deleted subgraphs $G-v$ and $H-u$ are isomorphic. Denote by $O_G(v)$, $O_H(u)$ the set of all neighbours of the vertices $v \in V(G)$, $u \in V(H)$ in G, H respectively. Further let $d_G(x)$, $d_H(y)$ denote the degrees of the vertices $x \in V(G)$, $y \in V(H)$ respectively.

To show that G and H are isomorphic it suffices to show that any graph isomorphism g: $G-v \to H-u$ maps $O_G(v)$ onto $O_H(u)$.

Assume the contrary. Then we can choose a vertex $x \in O_G(v)$ of minimum degree such that $g(x) \notin O_H(u)$. Clearly $d_H(g(x)) = d_G(x)-1$. Let M denote the set of all vertices of G of degree $d = d_G(x)-1$. Put $K = M - \{v\}$. Since H is a reconstruction of G, the graphs G and H must have the same degree-sequences (cf. [3]) Moreover $d_G(v) = d_H(u)$. This together with our assumptions guarantee that $K \neq \emptyset$ and $K \subset O_G(v)$. Now if $g(K) \subset O_H(u)$ the number of vertices in H of degree $d = d_G(x)$ 1 would be greater than the number of vertices of the same degree n G (because of the vertex $g(x)$), i.e the degree-sequences of G and H would be distinct. Therefore there exists a vertex $y \in K$ such that $g(y) \notin O_H(u)$. But this is a contradiction with the choice of the vertex x since $d_G(y) = d < d_G(x)$. The theorem follows

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РЕКОНСТРУКЦИЯ ГРАФОВ СО СПЕЦИАЛЬНЫМИ ПОСЛЕДОВАТЕЛЬНОСТЯМИ СТЕПЕНЕЙ ВЕРШИН

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Резюме

В статье доказана следующая теорема: Пусть v — вершина графа G и W — множество вершин соседних с v Если каждая вершина графа G степени $d(w) - 1$ (для некоторой вершины w из W) принадлежит множеству W, то G — реконструируемый граф.